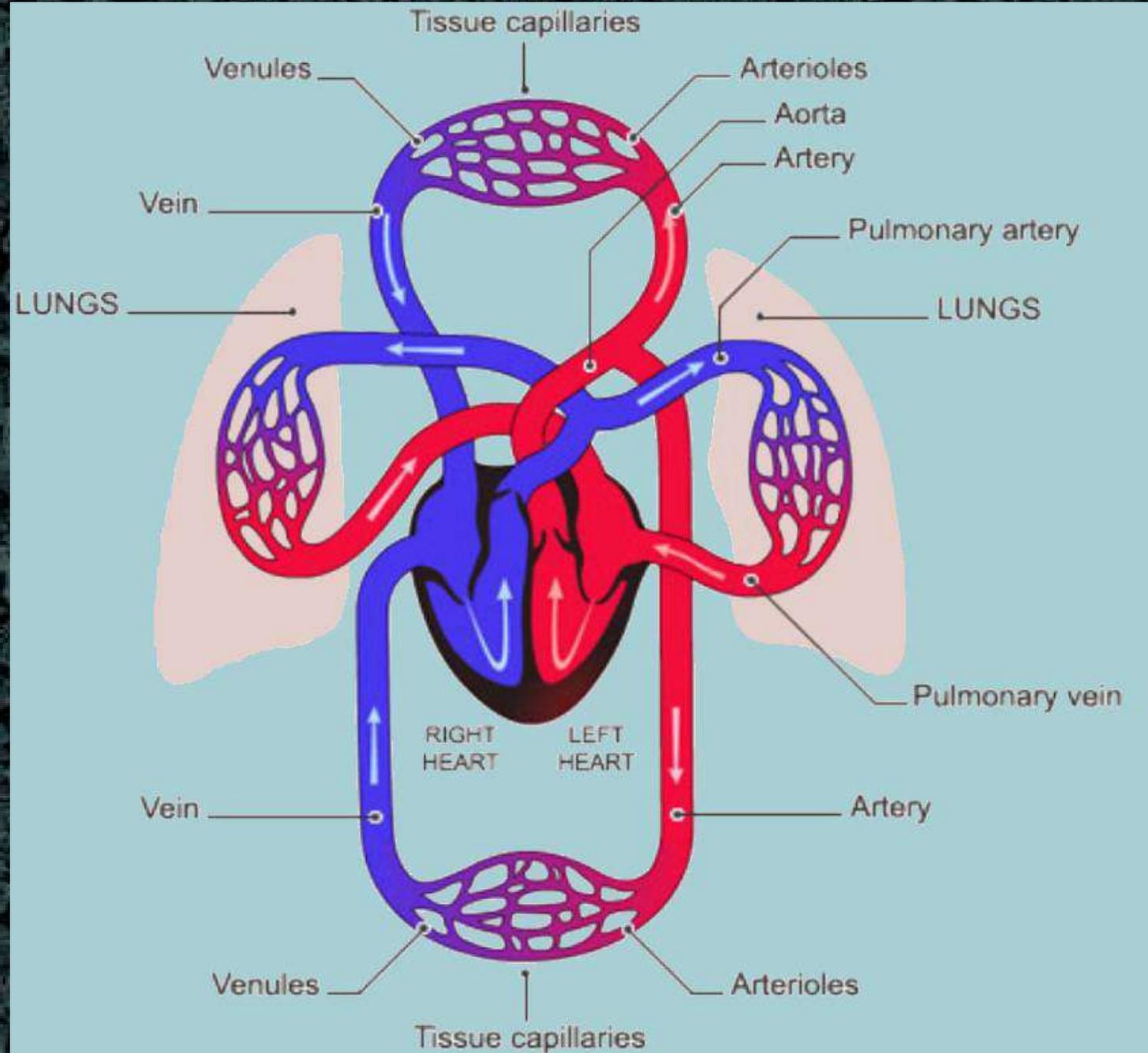
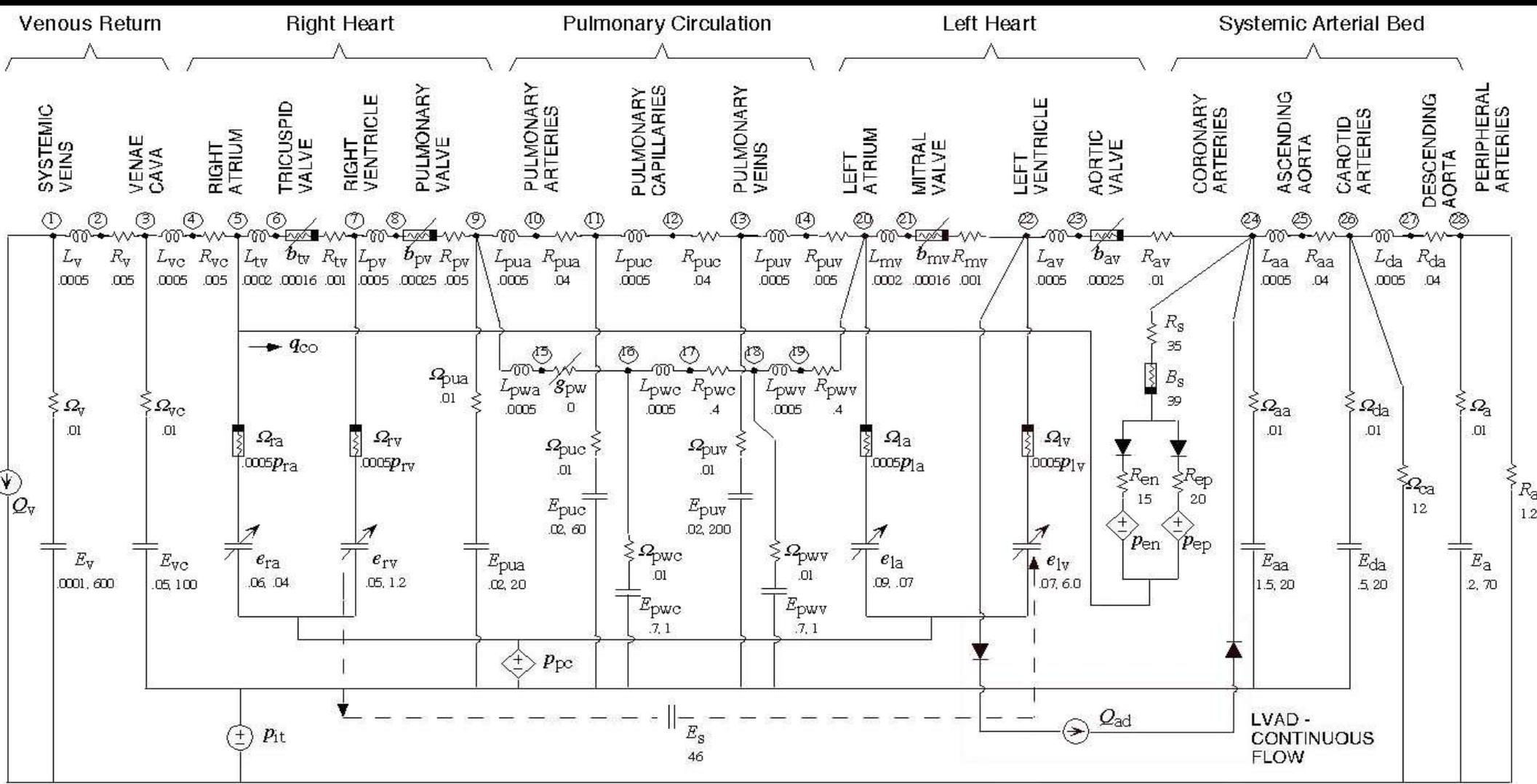


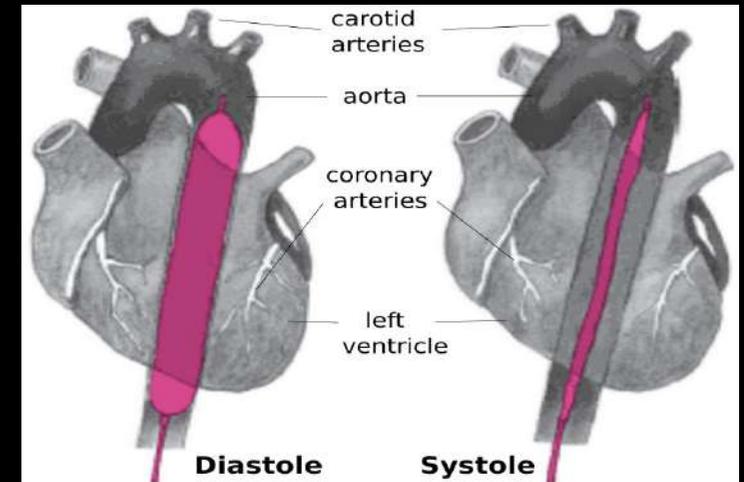
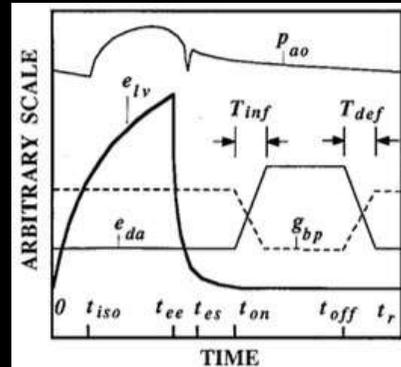
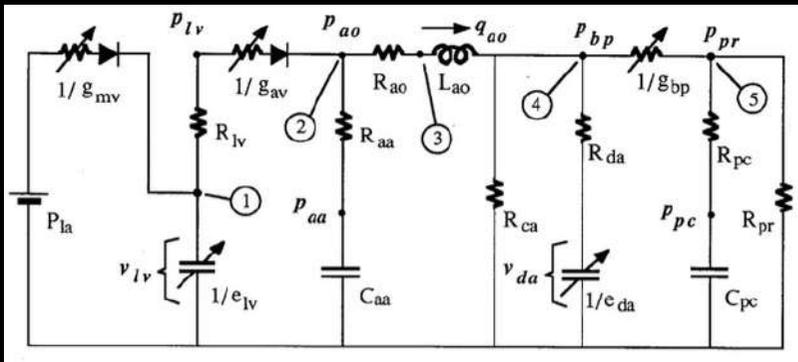
*Mathematical
Modeling and
Simulation of the
Cardiovascular
System*



Electrical Analog Model of the Cardiovascular System



Intra-Aortic Balloon Pump (IABP)



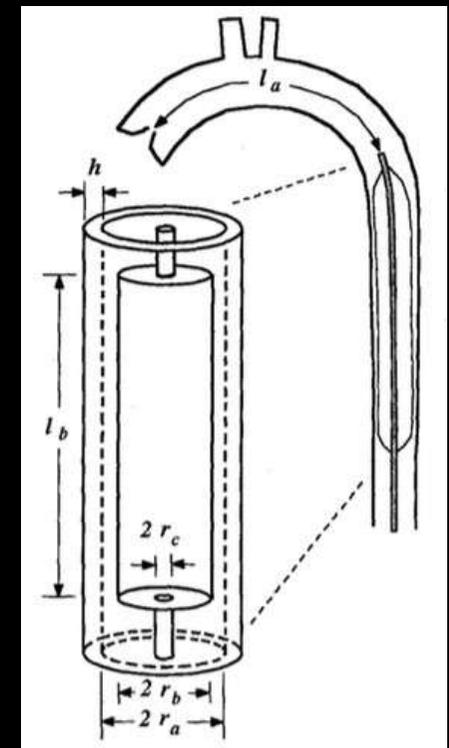
$$\frac{dv_{lv}}{dt} = -\left(g_{mv} + \frac{g_{av}}{1 + g_{av}R_{lv}}\right)e_{lv}v_{lv} + g_{mv}P_{la} + \left(p_{aa} + R_{aa}C_{aa}\frac{dp_{aa}}{dt}\right)\frac{g_{av}}{1 + g_{av}R_{lv}}$$

$$\frac{dp_{aa}}{dt} = \left(\frac{g_{av}}{1 + g_{av}R_{lv}}e_{lv}v_{lv} - \frac{g_{av}}{1 + g_{av}R_{lv}}p_{aa} - q_{ao}\right) / \left[C_{aa}\left(1 + \frac{g_{av}}{1 + g_{av}R_{lv}}R_{aa}\right)\right]$$

$$\frac{dq_{ao}}{dt} = \frac{p_{aa}}{L_{ao}} - \frac{e_{da}}{L_{ao}}v_{da} - \frac{R_{ao}}{L_{ao}}q_{ao} + \frac{R_{aa}C_{aa}}{L_{ao}}\frac{dp_{aa}}{dt} - \frac{R_{da}}{L_{ao}}\frac{dv_{da}}{dt}$$

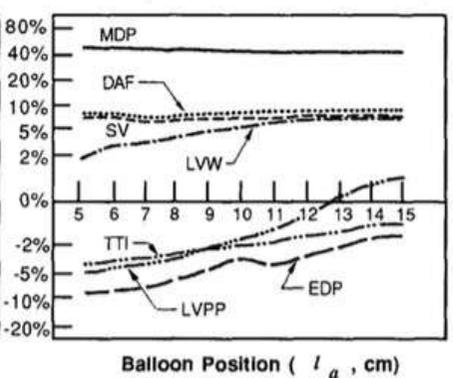
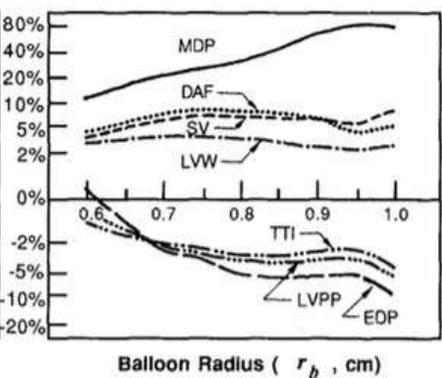
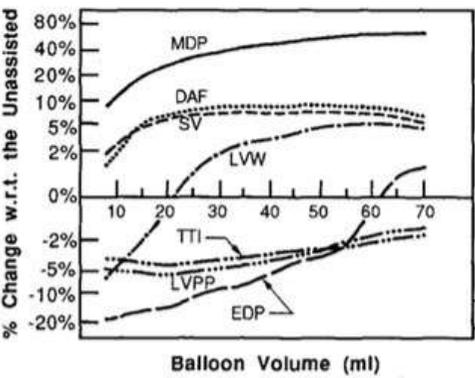
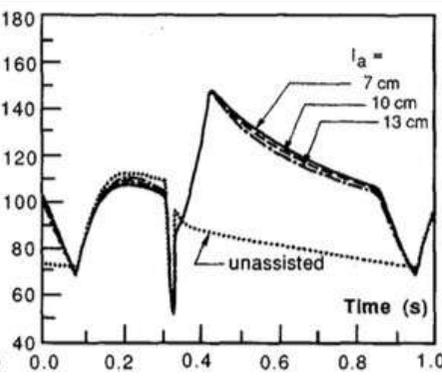
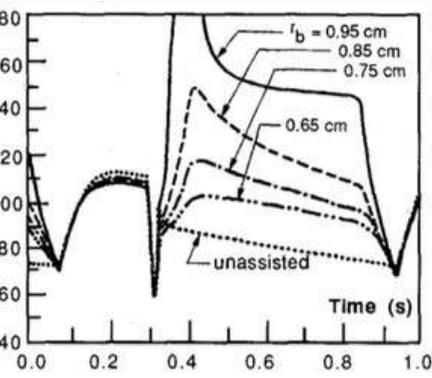
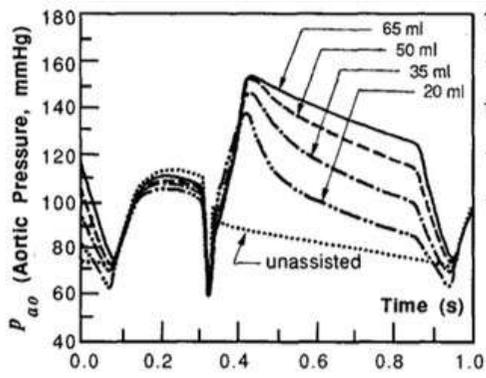
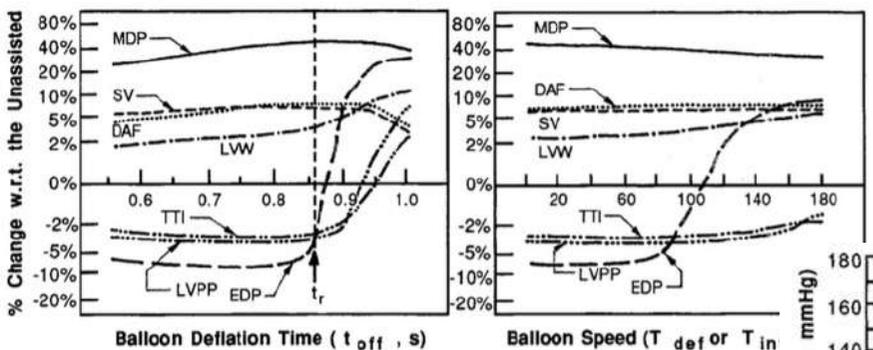
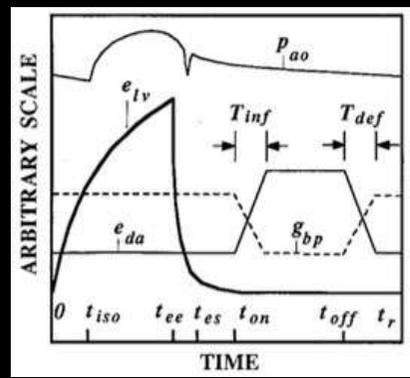
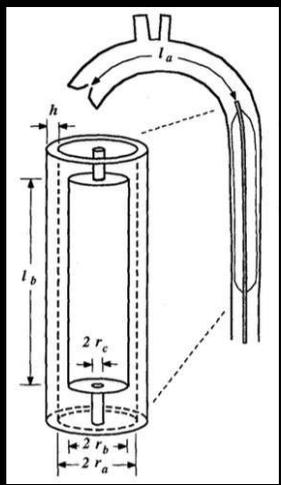
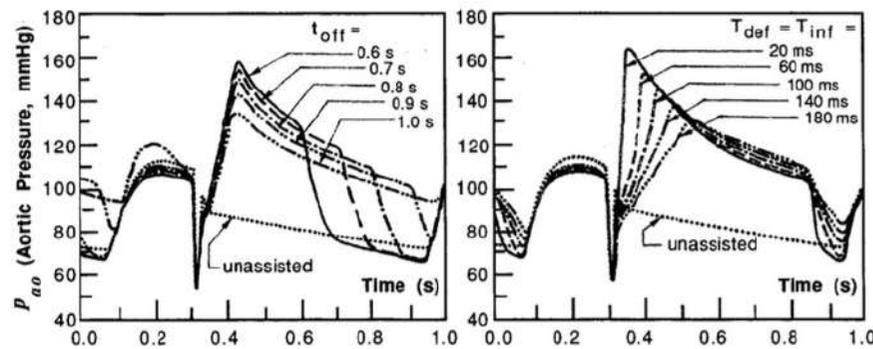
$$\frac{dv_{da}}{dt} = \left[q_{ao} - \left(g_{bp} + \frac{1}{R_{ca}}\right)e_{da}v_{da} + g_{bp}p_{pc} + g_{bp}R_{pc}C_{pc}\frac{dp_{pc}}{dt}\right] / \left[1 + R_{da}\left(g_{bp} + \frac{1}{R_{ca}}\right)\right]$$

$$\frac{dp_{pc}}{dt} = \left\{ \frac{g_{bp}R_{da}q_{ao}}{1 + R_{da}(g_{bp} + 1/R_{ca})} + \left[1 - \frac{R_{da}(g_{bp} + 1/R_{ca})}{1 + R_{da}(g_{bp} + 1/R_{ca})}\right]g_{bp}e_{da}v_{da} - \left[g_{bp} + \frac{1}{R_{pr}} - \frac{g_{bp}^2R_{da}}{1 + R_{da}(g_{bp} + 1/R_{ca})}\right]p_{pc} \right\} / \left\{ C_{pc}\left[1 + g_{bp}R_{pc} + \frac{R_{pc}}{R_{pr}} - \frac{g_{bp}^2R_{da}R_{pc}}{1 + R_{da}(g_{bp} + 1/R_{ca})}\right] \right\}$$



Optimization of IABP Parameters

Balloon Configurations



Timing

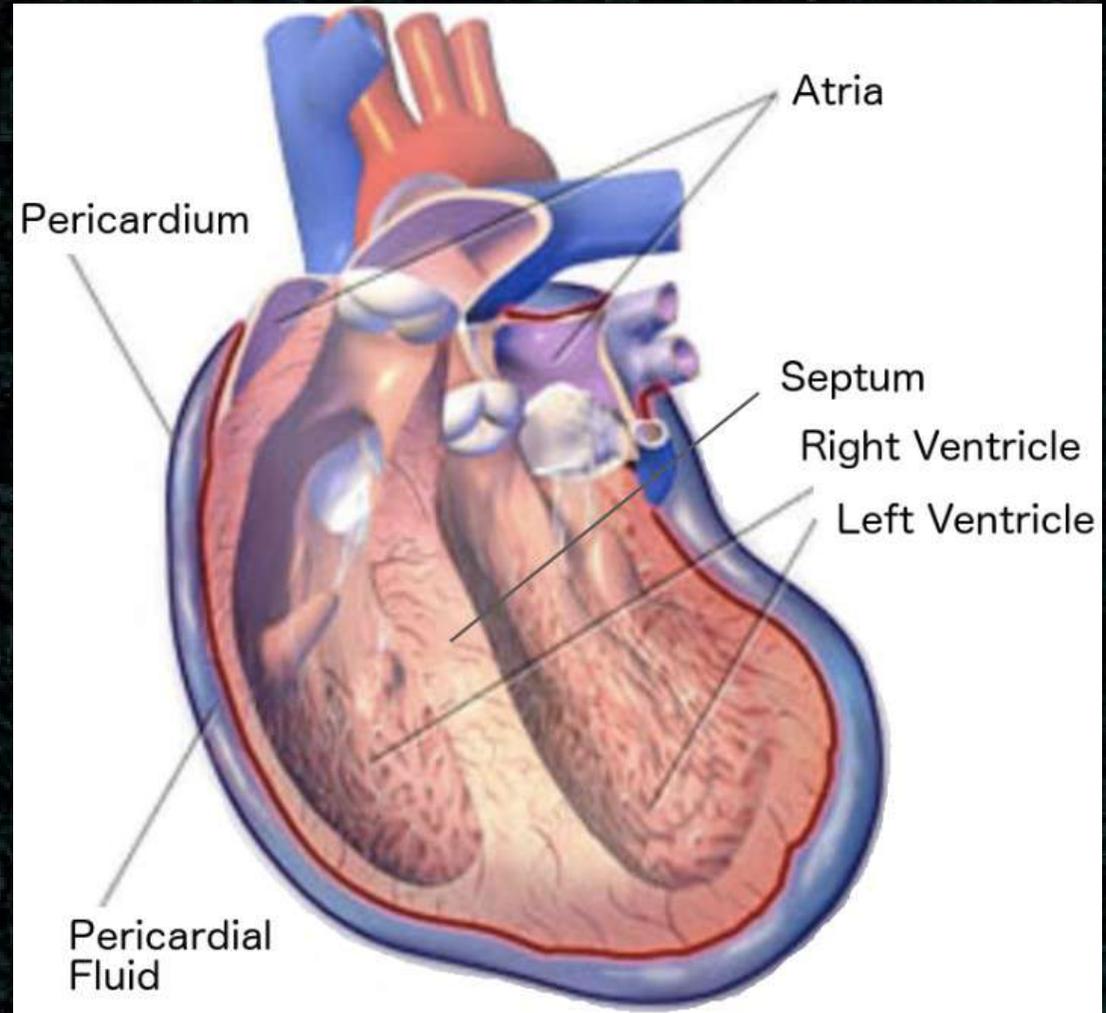
Sun Y. Modeling the dynamic interaction between left ventricle and intraaortic balloon pump. American Journal of Physiology 261(4) (Heart Circulatory Physiology 30): H1300-H1311, 1991.

Coupling Between Right and Left Ventricles

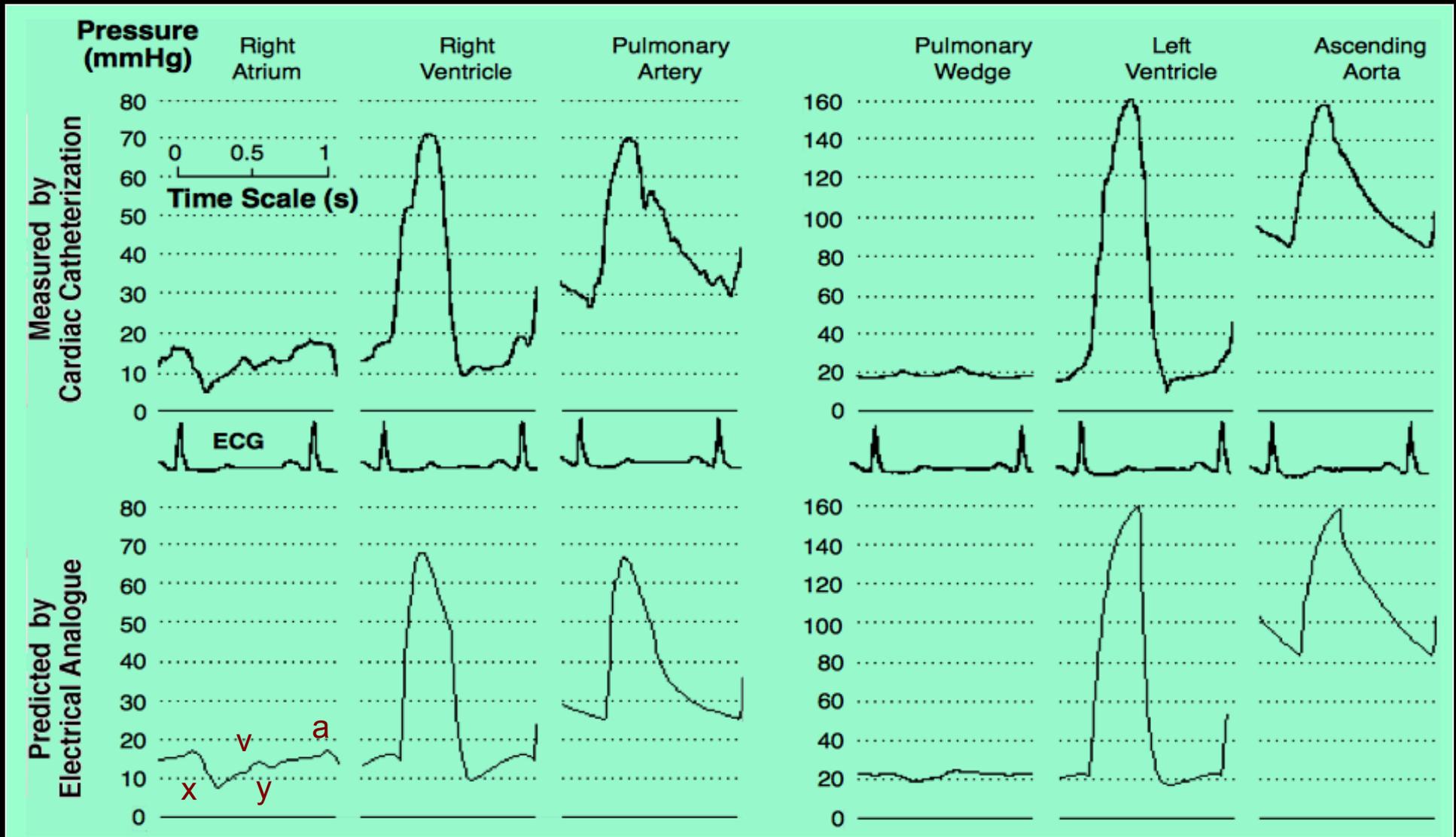
- *Hemodynamic*
- *Transseptal*
- *Pericardial*

Which one is more important?

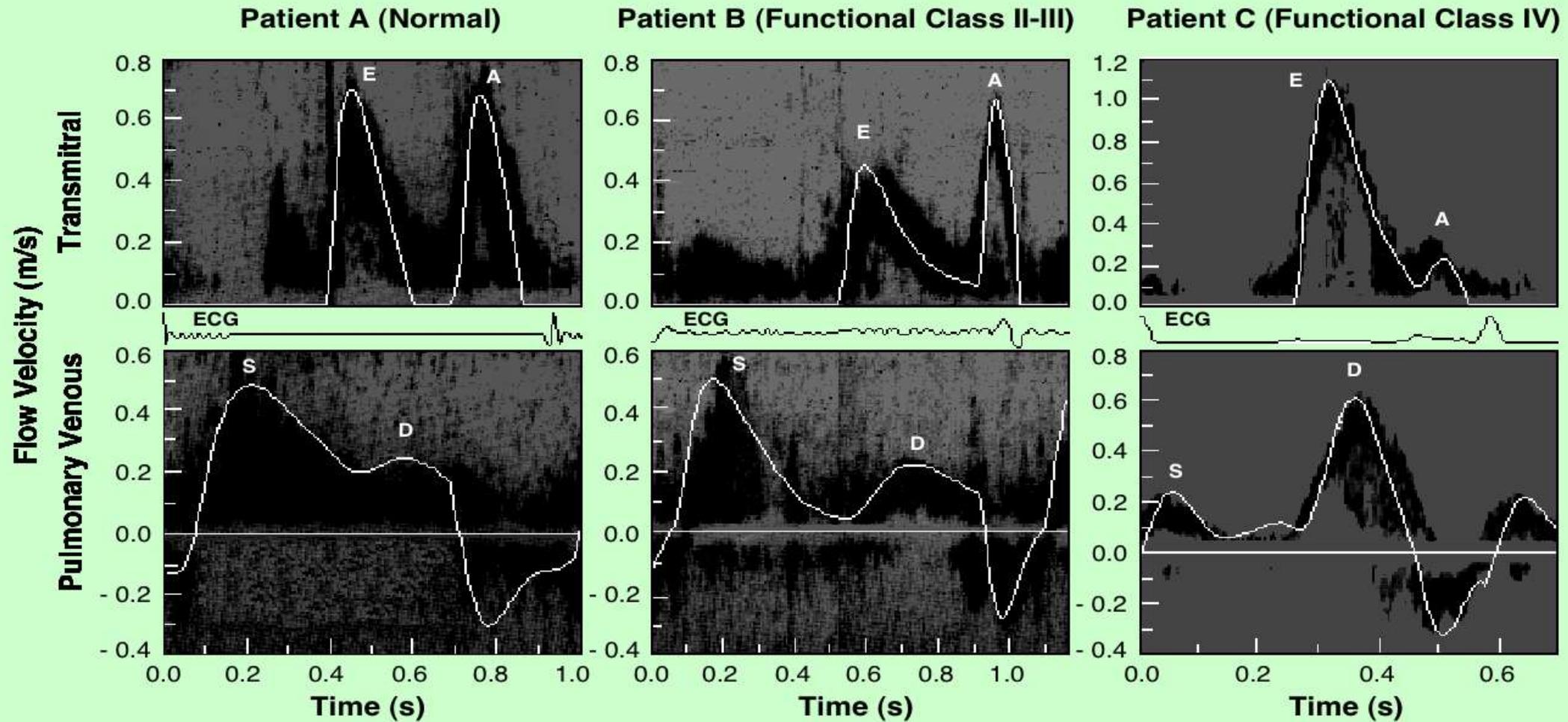
Sun Y, Beshara M, Lucariello RJ, Chiaramida SA. A comprehensive model for right-left heart interaction under the influence of pericardium and intrathoracic pressure. *American J. Physiology* 272 (3 Pt 2; Heart Circ Physiol 41): H1499-H1515, Mar. 1997.



Validation – Cath Lab Data (Pressures)

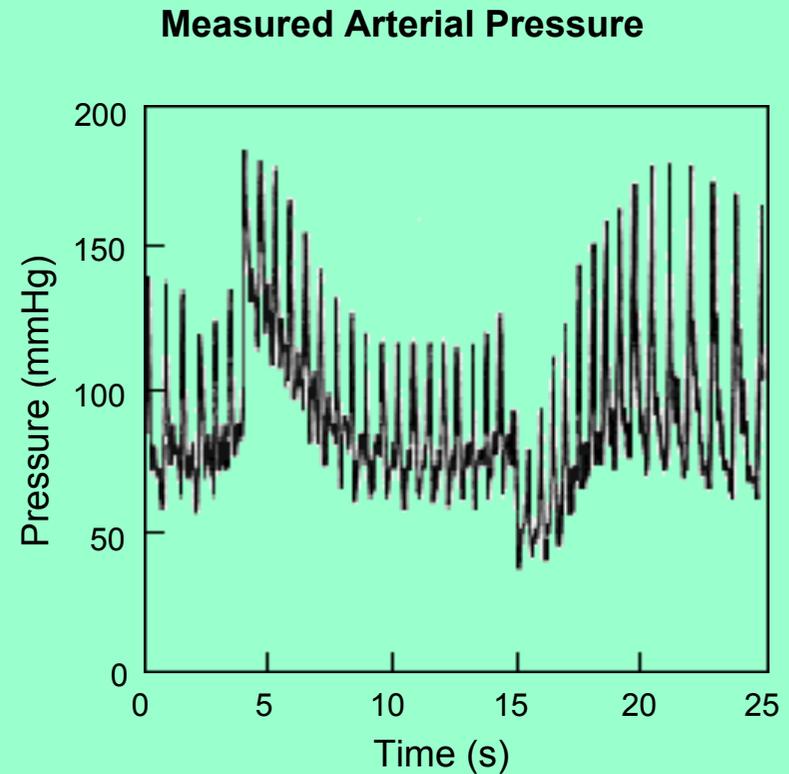
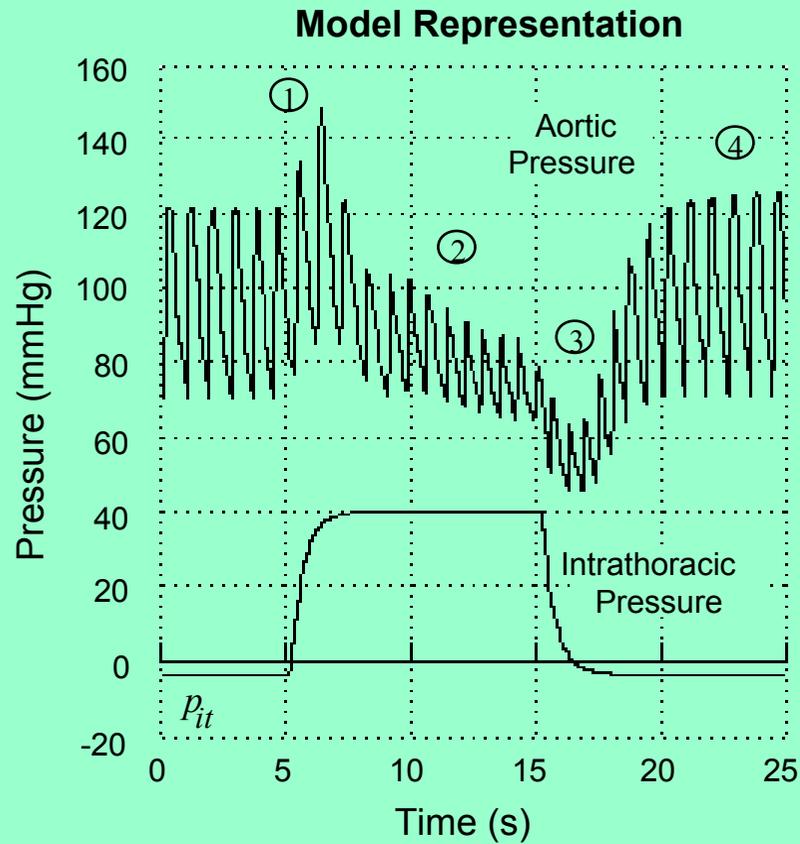


Validation – Doppler Echocardiographic Data



Validation – Valsalva Maneuver

Valsalva Maneuver



Coupling Between Right and Left Ventricles

- **Hemodynamic**
- **Transseptal** ~2%
- **Pericardial** ~20%

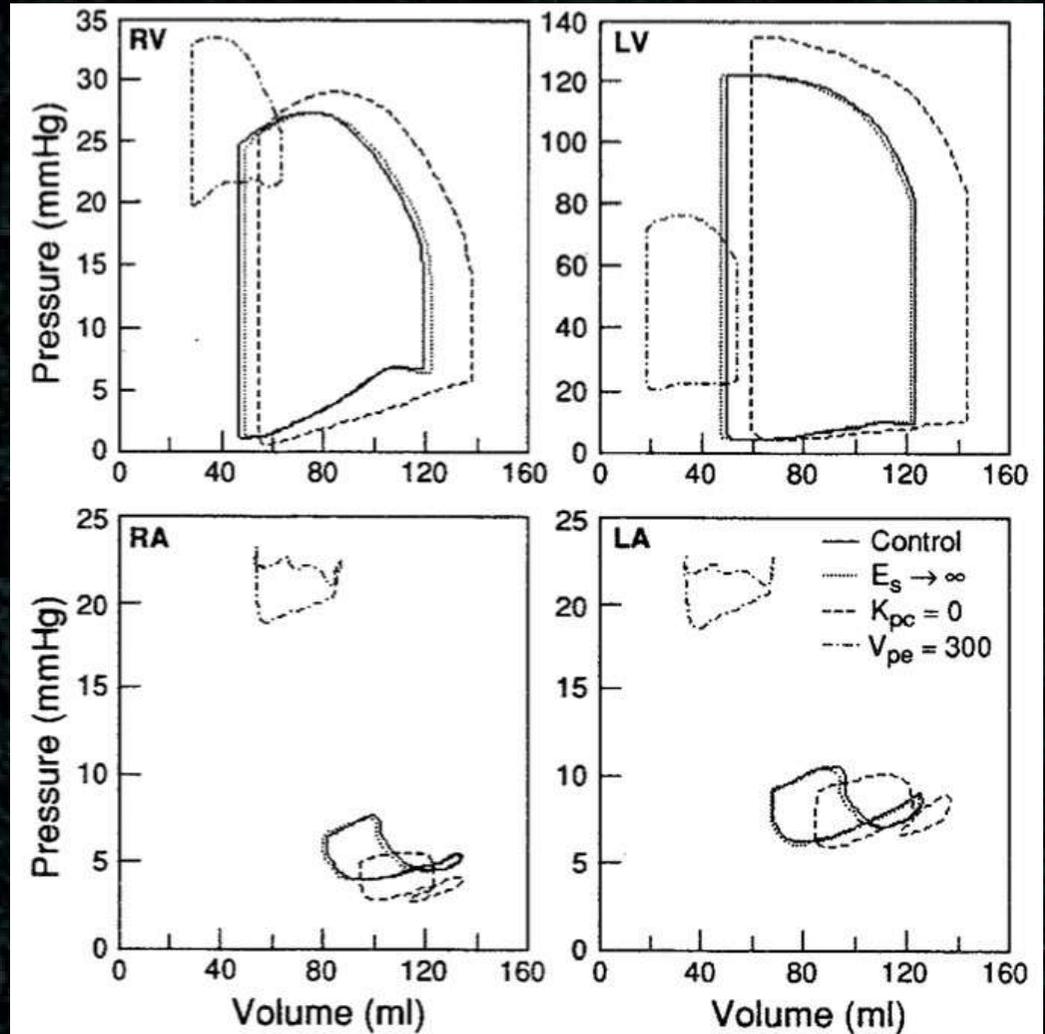
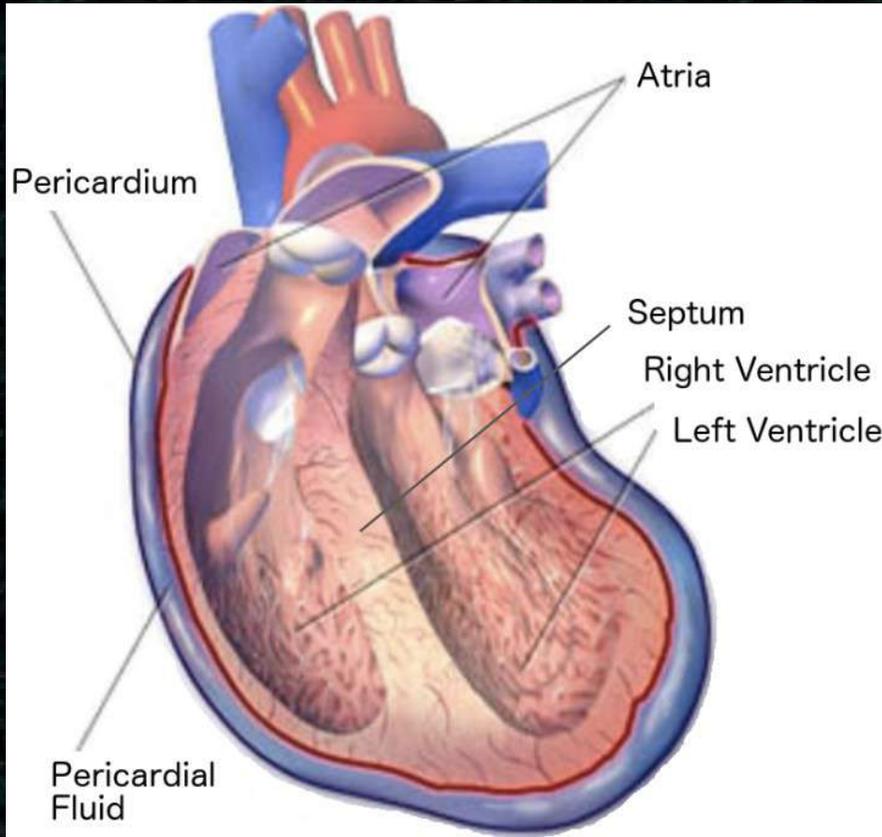
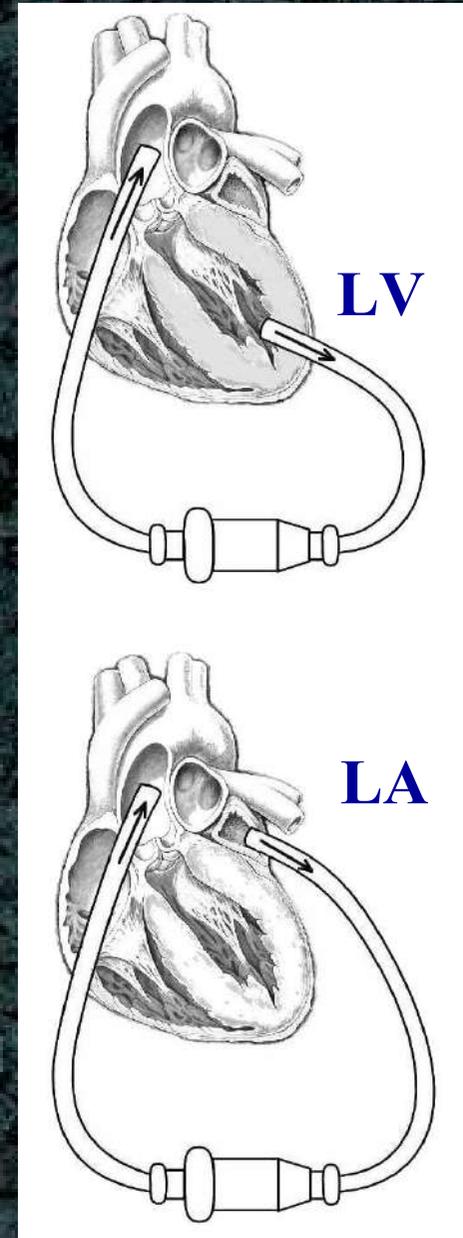
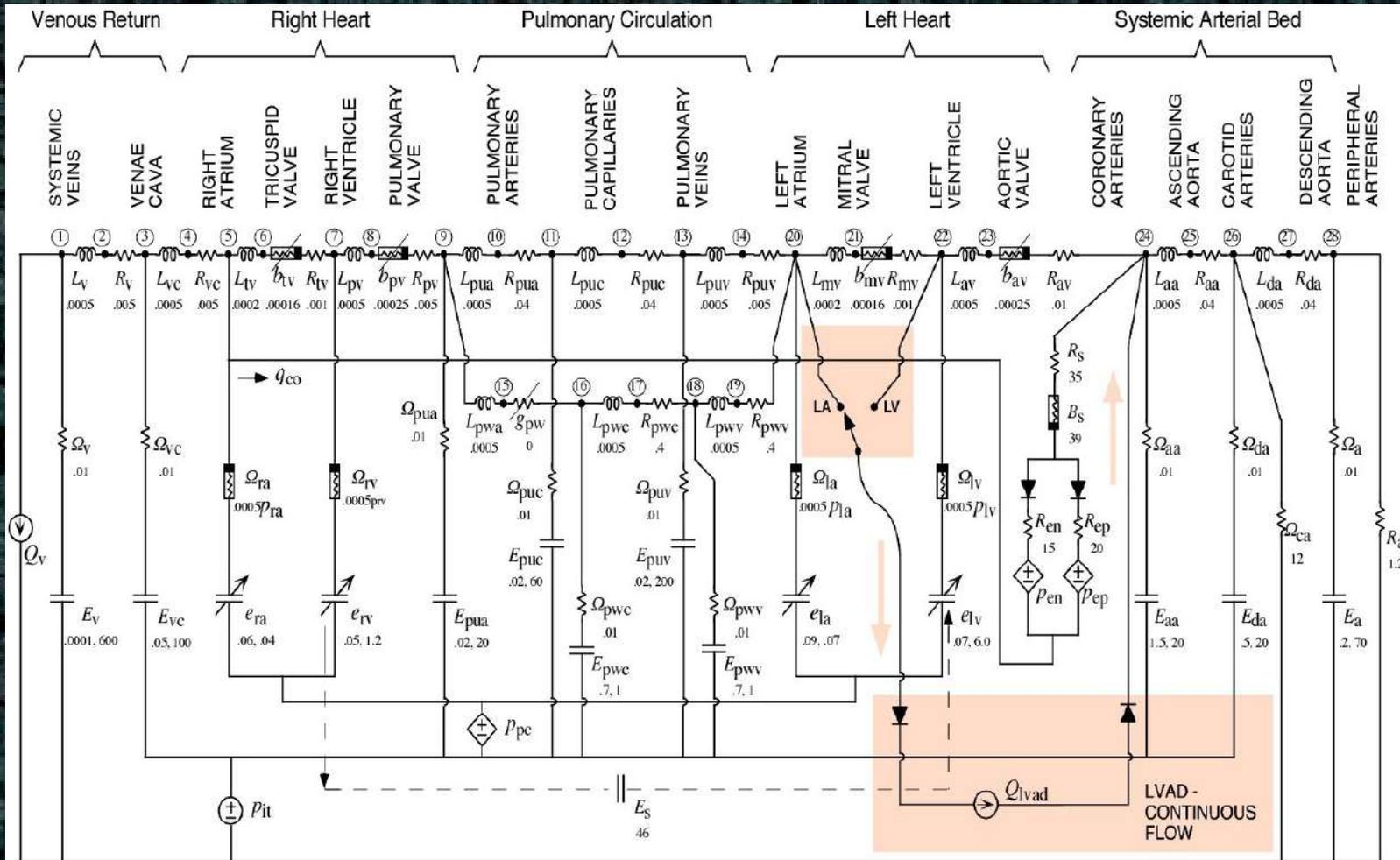


Fig. 11. Effects of removing transseptal coupling ($E_s \rightarrow \infty$), removing pericardial coupling ($K_{pc} = 0$), and cardiac tamponade ($V_{pe} = 300$ ml) on pressure-volume loops in 4 cardiac chambers. See *Glossary* for definition of abbreviations.

Left Ventricular Assist Device (LVAD)



Left Ventricular Assist Device (LVAD)

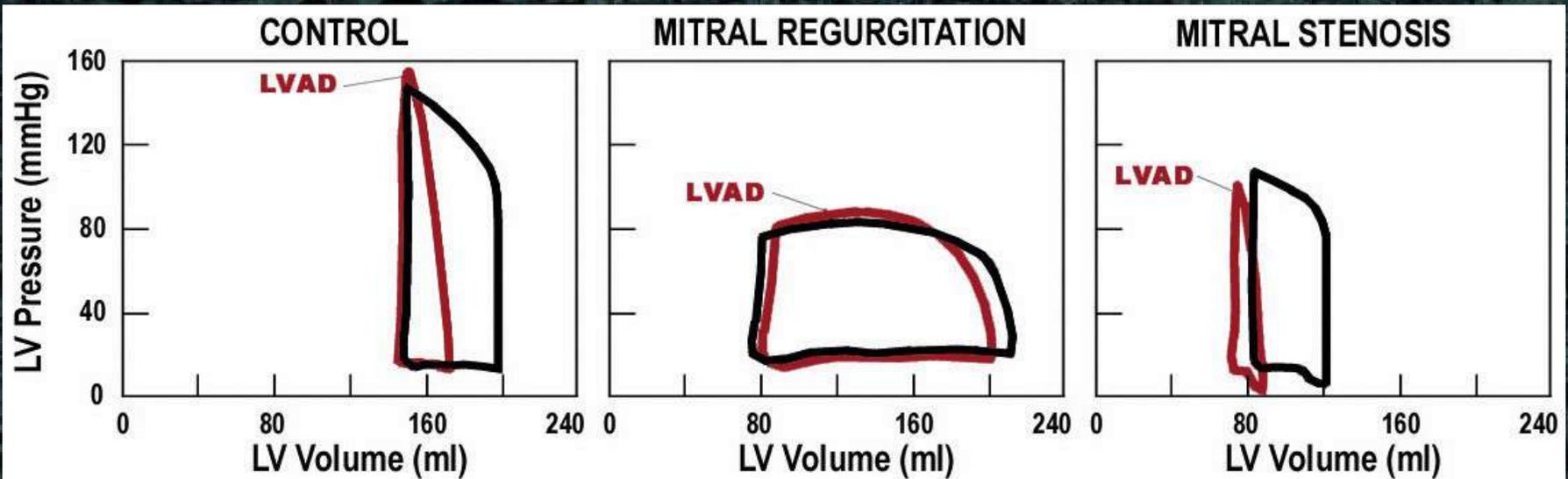


Table 1

	Control	Mitral Regurgitation	Mitral Stenosis
Cardiac Output (l/min)	3.4 → 4.6 (+33%)	2.1 → 2.5 (+20%)	2.6 → 3.0 (+15%)
LV Stroke Work Index (g·m/m ² /beat)	45 → 19 (-58%)	60 → 57 (-6%)	25 → 14 (-43%)

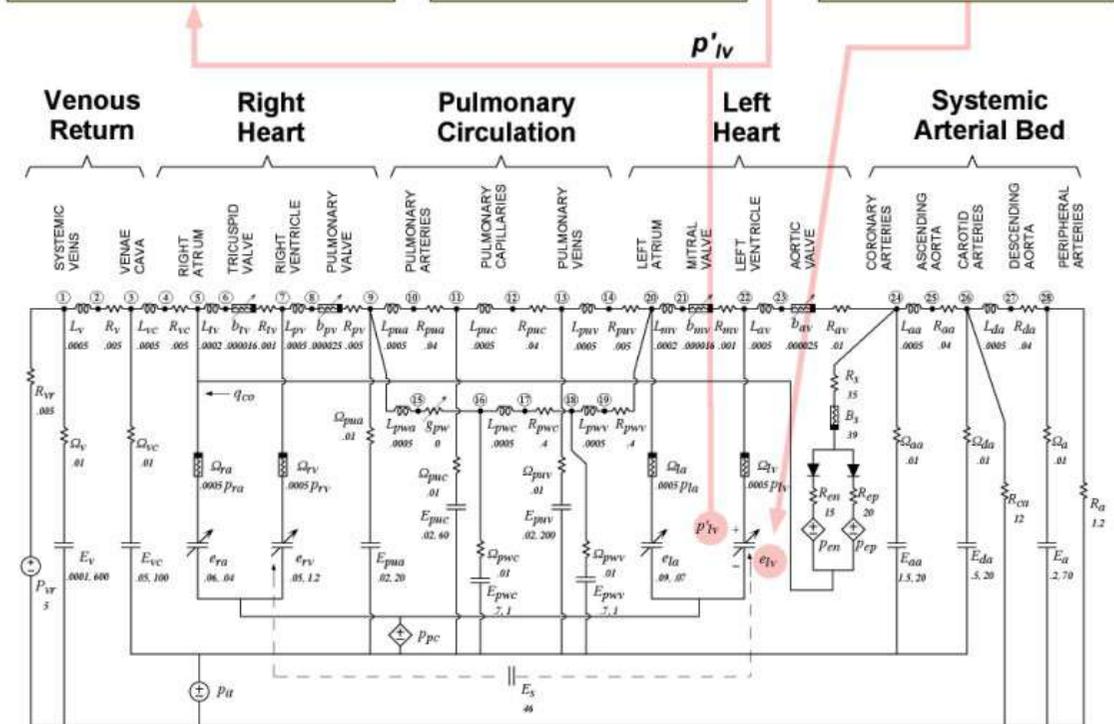
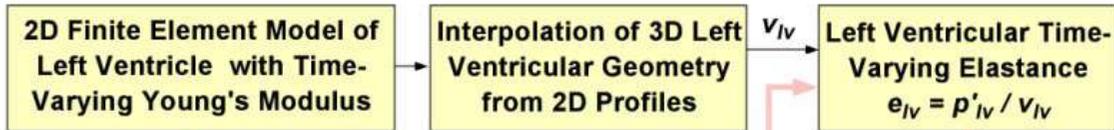
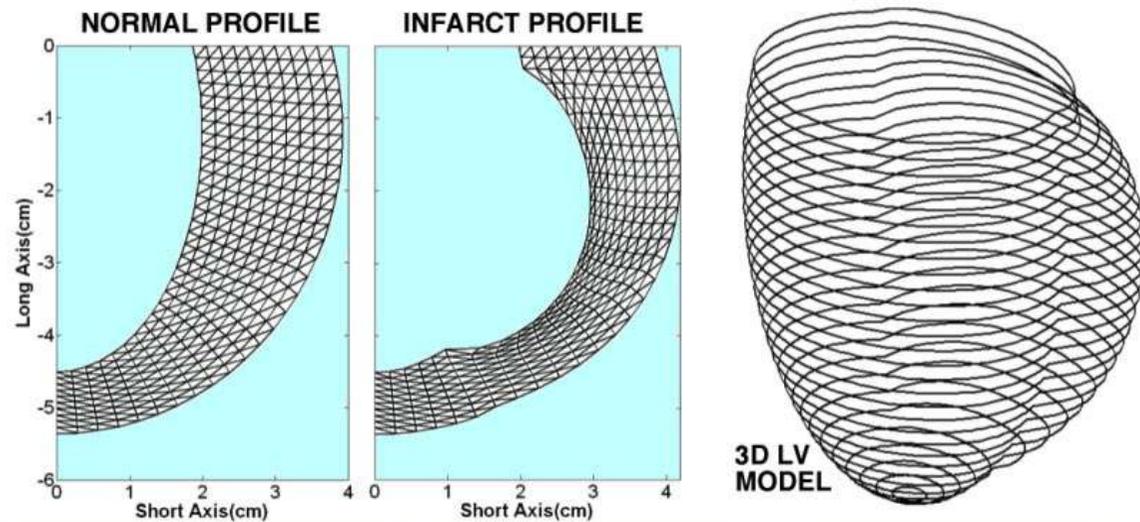
LV

LA

3D Model of the Left Ventricle with Infarction

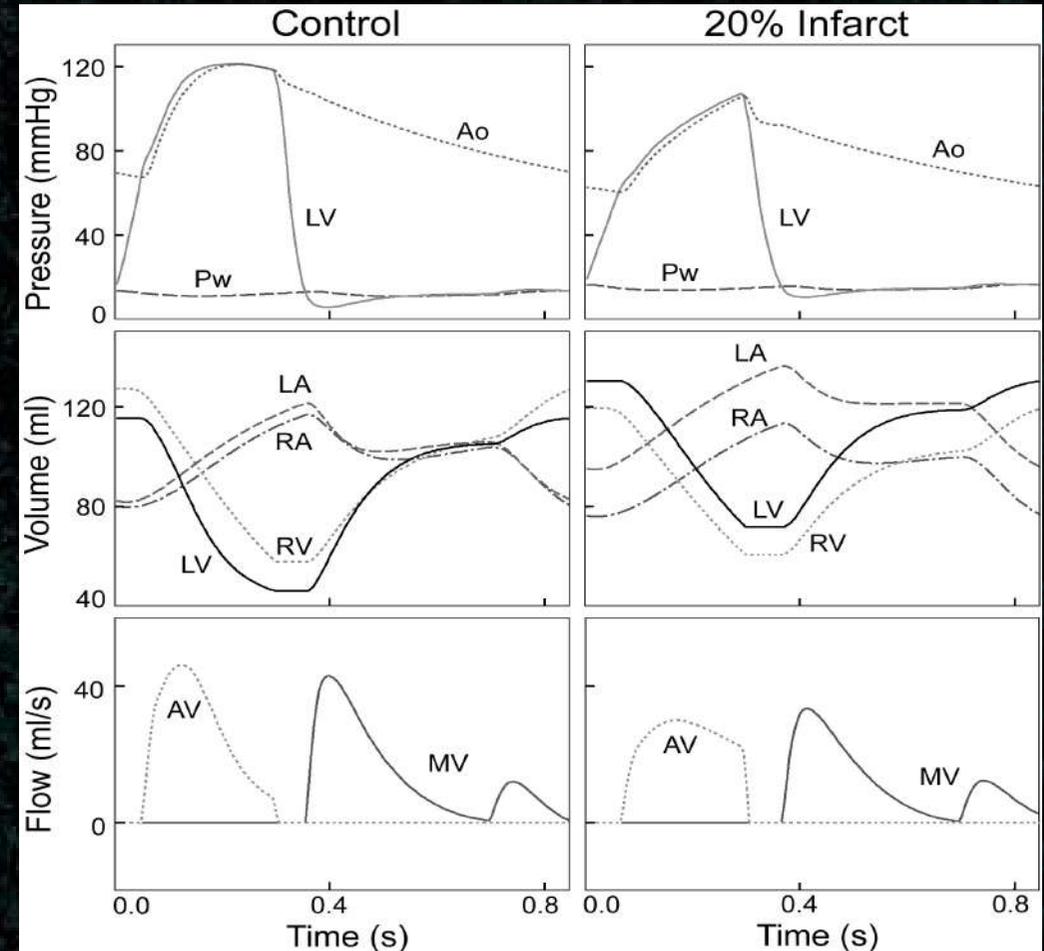
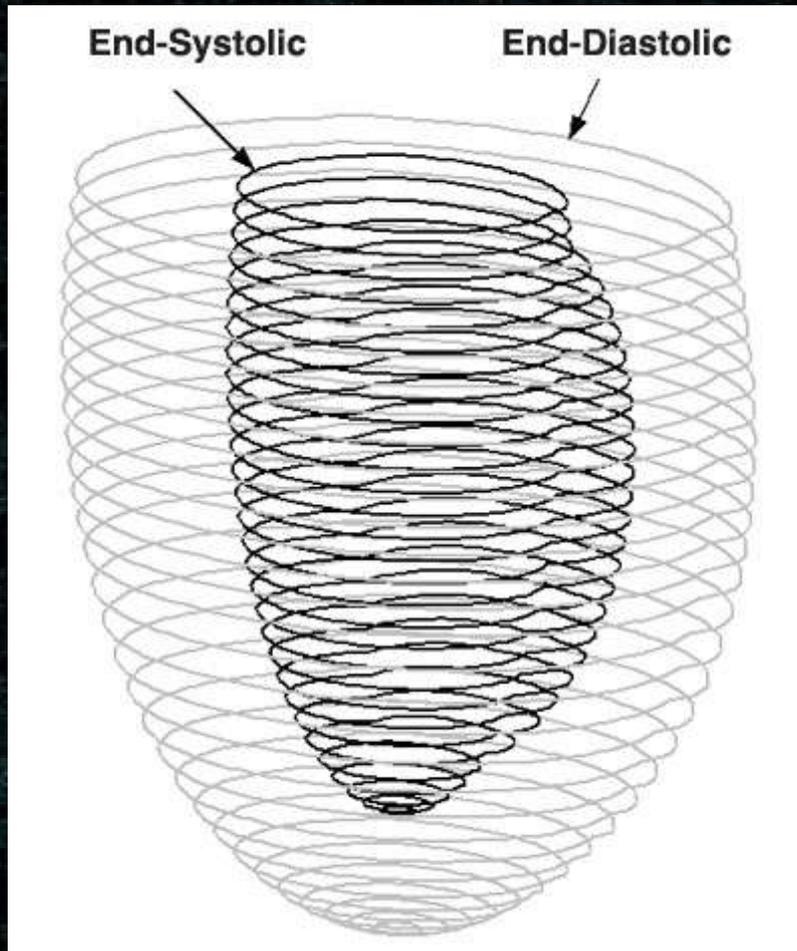
A finite element model of the left ventricle interacts dynamically with the circulatory model at a 5-ms time step.

US Patent No. US 8,295,907 B2,
October 23, 2012



ELECTRICAL ANALOG MODEL OF THE CARDIOPULMONARY SYSTEM

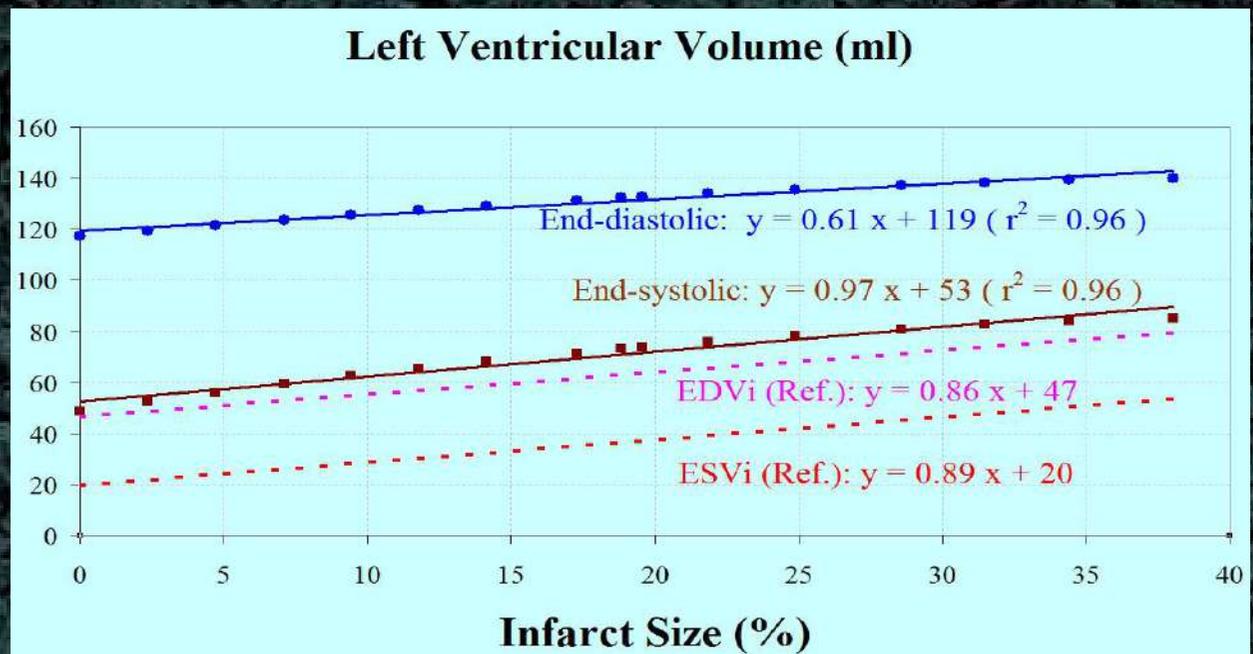
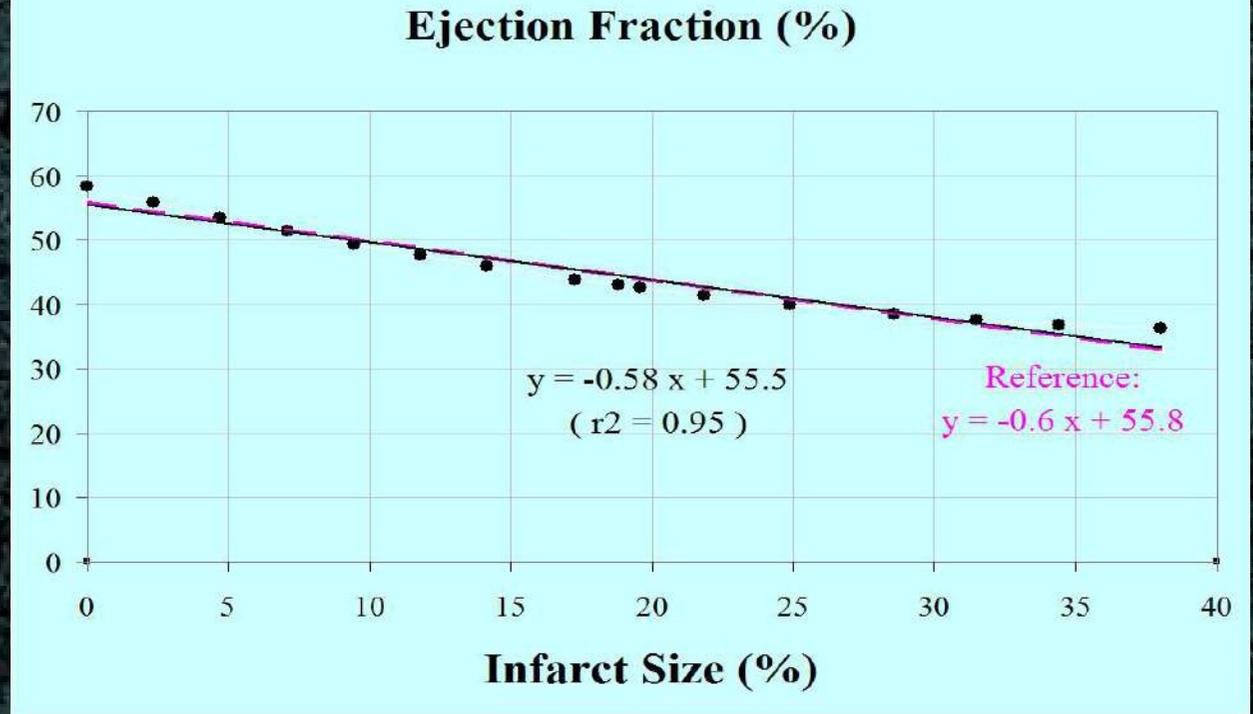
3D LV Model and Hemodynamic Waveforms



For an infarct size of 20% of the total left ventricular mass, the LV ejection fraction reduces from 59% to 41%.

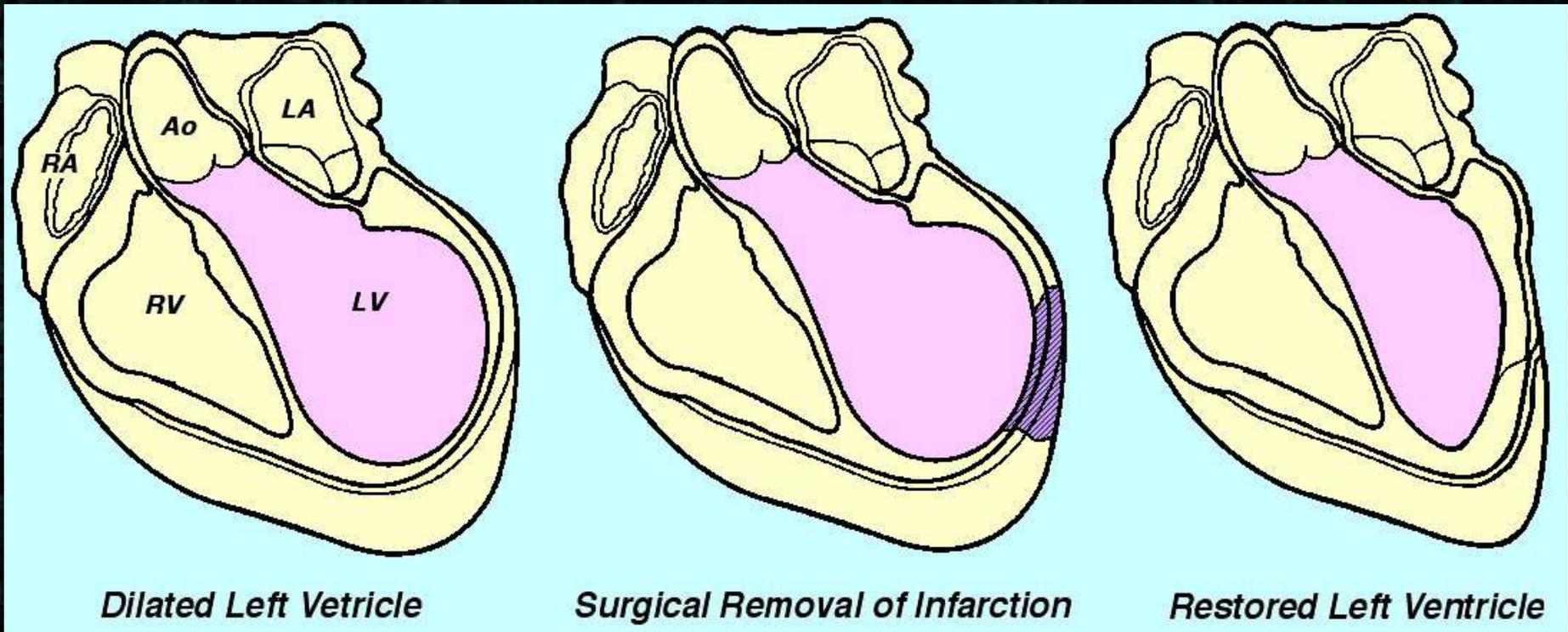
Validation

The integrated model predicts the decrease of LV ejection fraction and the increase of LV volumes as infarct size increases in consistence with clinical data (Sciagra et al. European J Nuclear Medicine & Molecular Imaging 31: 969-974, 2004).



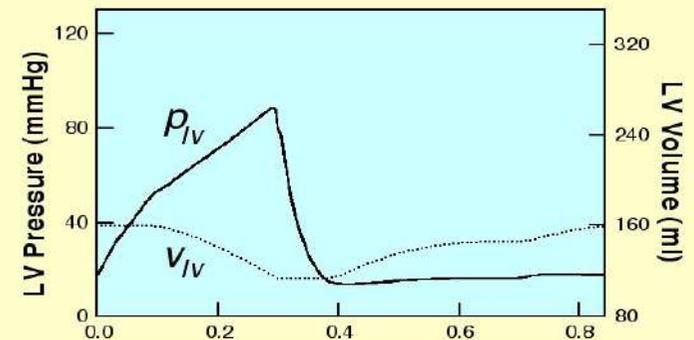
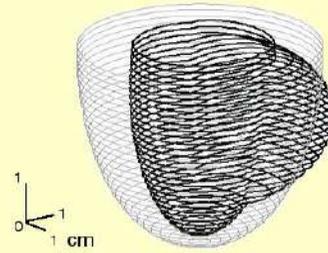
Surgical Ventricular Restoration

To remove infarction and to restore a remodeled left ventricle to its optimal size and shape for a patient with congestive heart failure

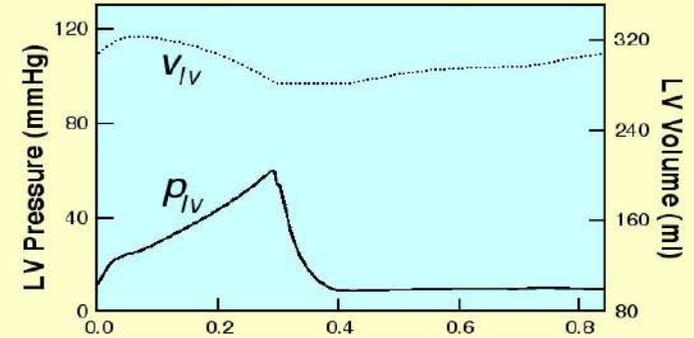
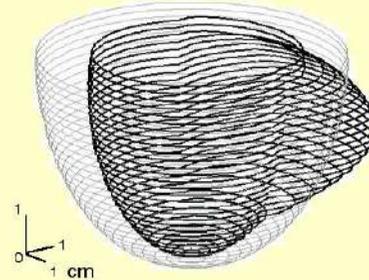


*Model Predictions
of LV pressure
and volume
waveforms for a
40% infarction
under various
preoperative LV
geometries*

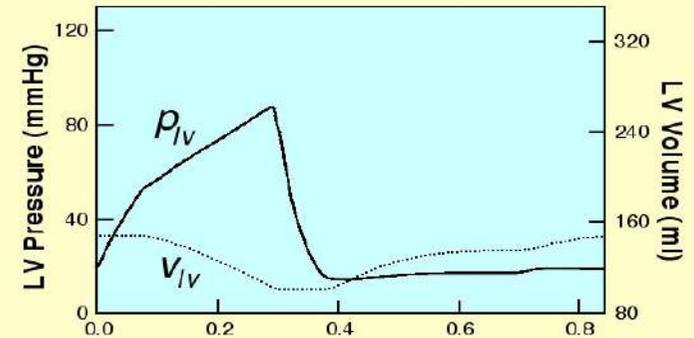
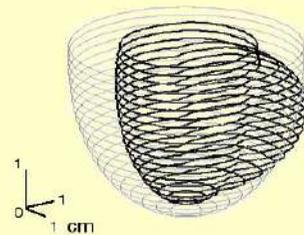
A Control
LV EDV = 146 ml, LV Mass = 160 g



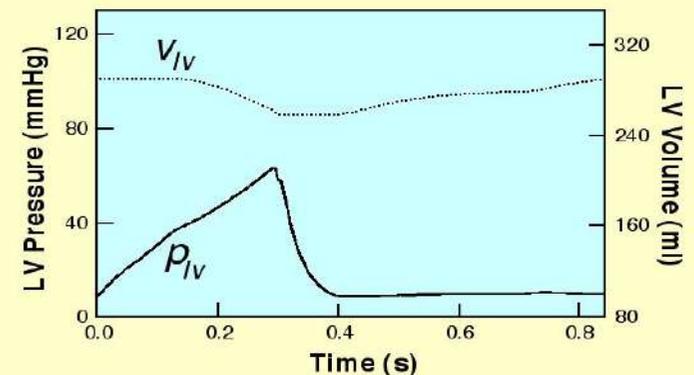
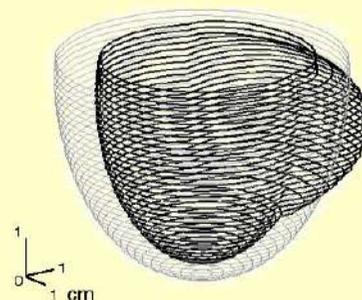
B Dilation
LV EDV = 305 ml, LV Mass = 152 g



C Hypertrophy
LV EDV = 148 ml, LV Mass = 301 g



D Dilation & Hypertrophy
LV EDV = 307 ml, LV Mass = 291 g



*Optimal Infarct Removal for
Various LV Geometries with a 40%
Preoperative Infarction*

LV EDV LV Mass	150 ml	225 ml	300 ml
150 g	50%	76%	100%
225 g	40%	75%	100%
300 g	39%	76%	88%

Summary

Mathematical models and simulations can be useful for biomedical research for the following purposes:

- *Data regression*
- *Mechanism explanation*
- *Hypothesis formulation*
- *Outcome prediction*