

# Chapter 8 Operational Amplifier as A Black Box

- **8.1 General Considerations**
- **8.2 Op-Amp-Based Circuits**
- **8.3 Nonlinear Functions**
- **8.4 Op-Amp Nonidealities**
- **8.5 Design Examples**

# Chapter Outline

## General Concepts

- Op Amp Properties

## Linear Op Amp Circuits

- Noninverting Amplifier
- Inverting Amplifier
- Integrator and Differentiator
- Voltage Added

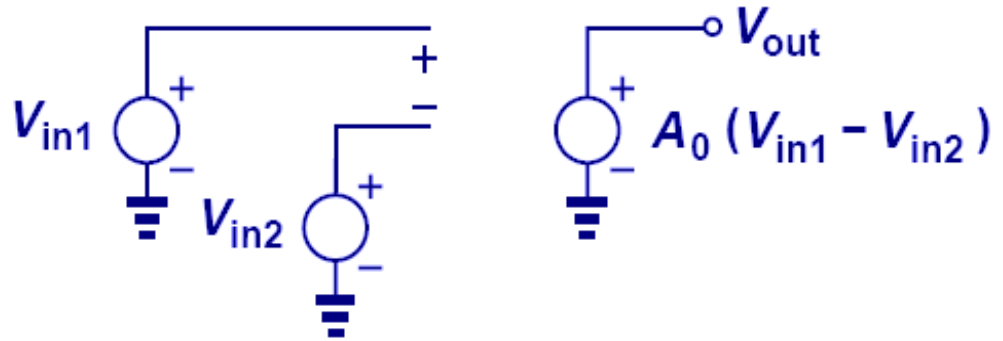
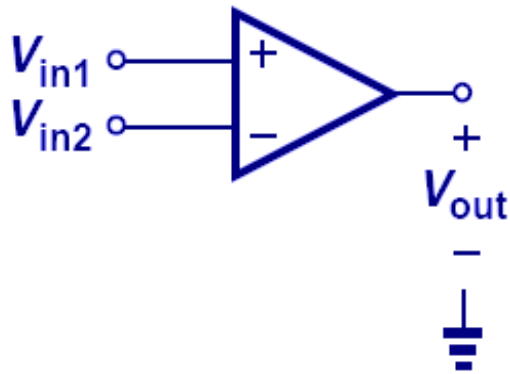
## Nonlinear Op Amp Circuits

- Precision Rectifier
- Logarithmic Amplifier
- Square Root Circuit

## Op Amp Nonidealities

- DC Offsets
- Input Bias Currents
- Speed Limitations
- Finite Input and Output Impedances

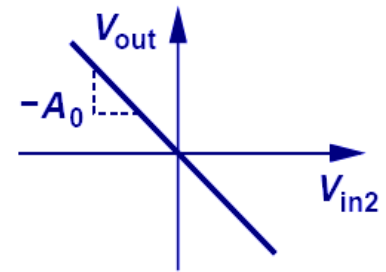
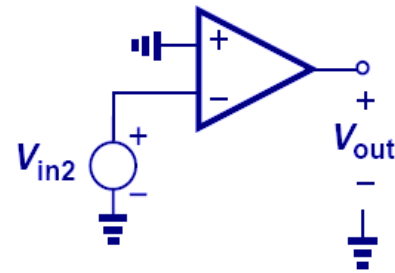
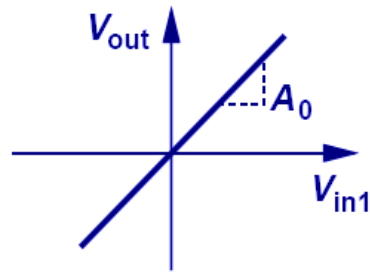
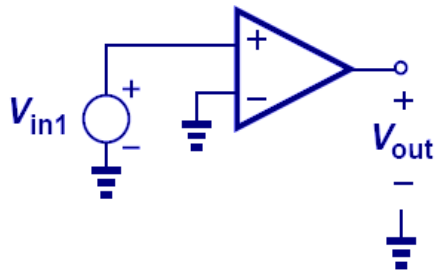
# Basic Op Amp



$$V_{out} = A_0 (V_{in1} - V_{in2})$$

- Op amp is a circuit that has two inputs and one output.
- It amplifies the difference between the two inputs.

# Inverting and Non-inverting Op Amp

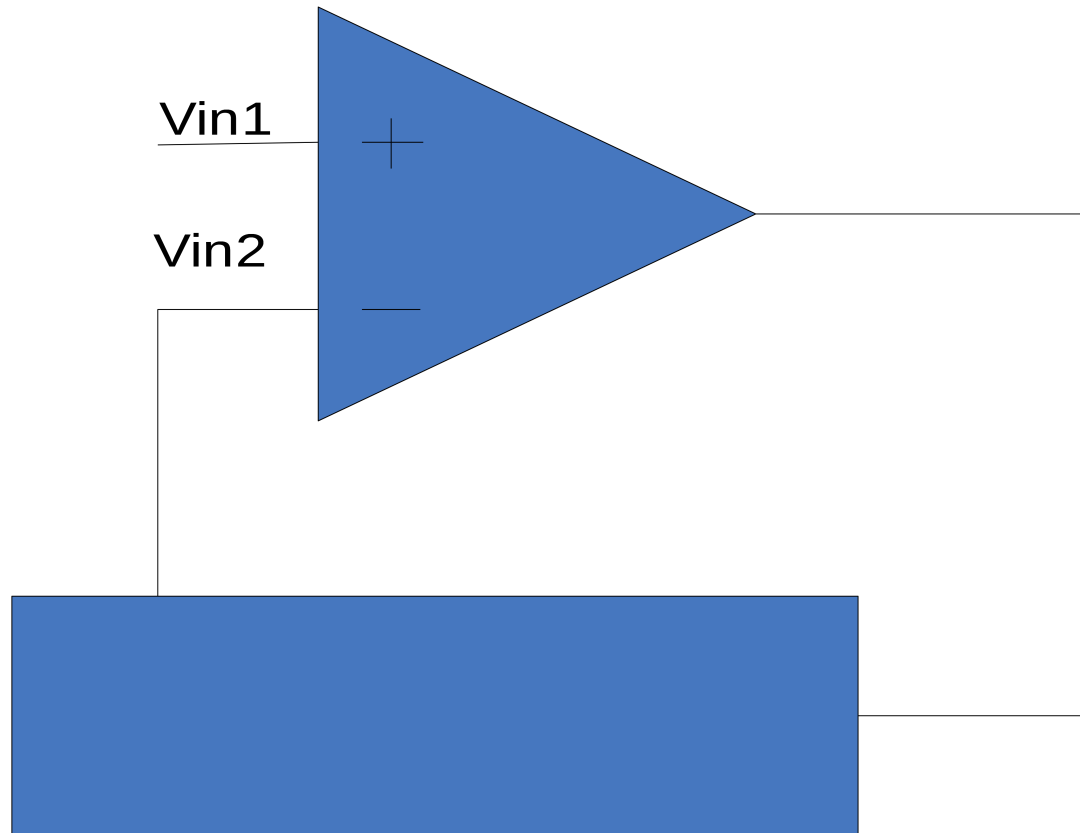


- If the negative input is grounded, the gain is positive.
- If the positive input is grounded, the gain is negative.

# Ideal Op Amp

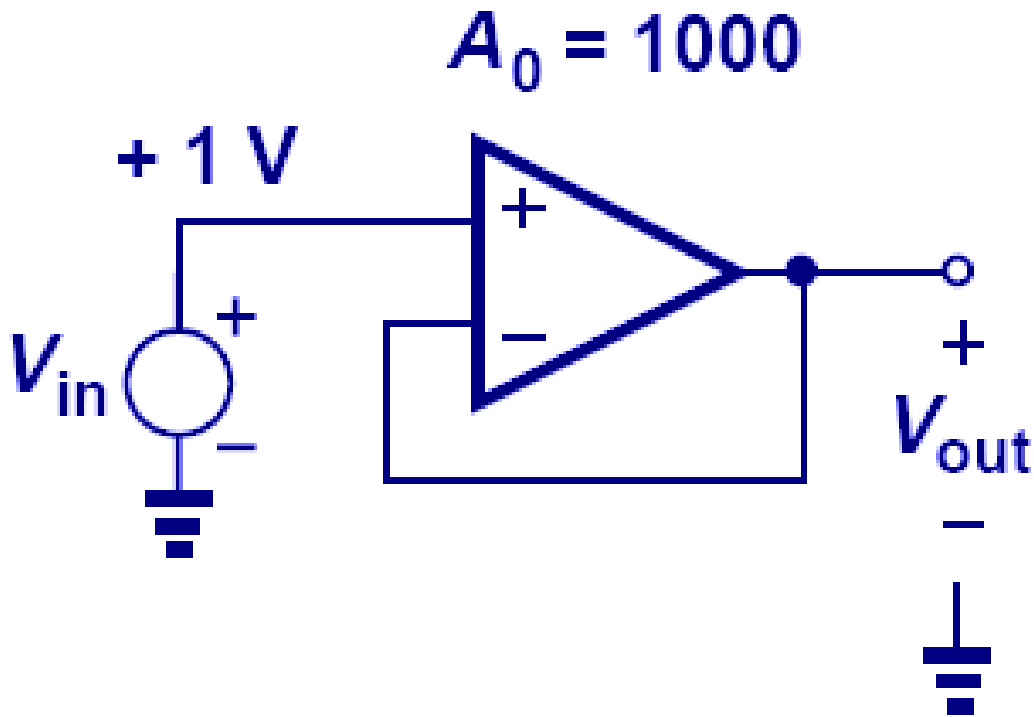
- **Infinite gain**
- **Infinite input impedance**
- **Zero output impedance**
- **Infinite speed**

# Virtual Short



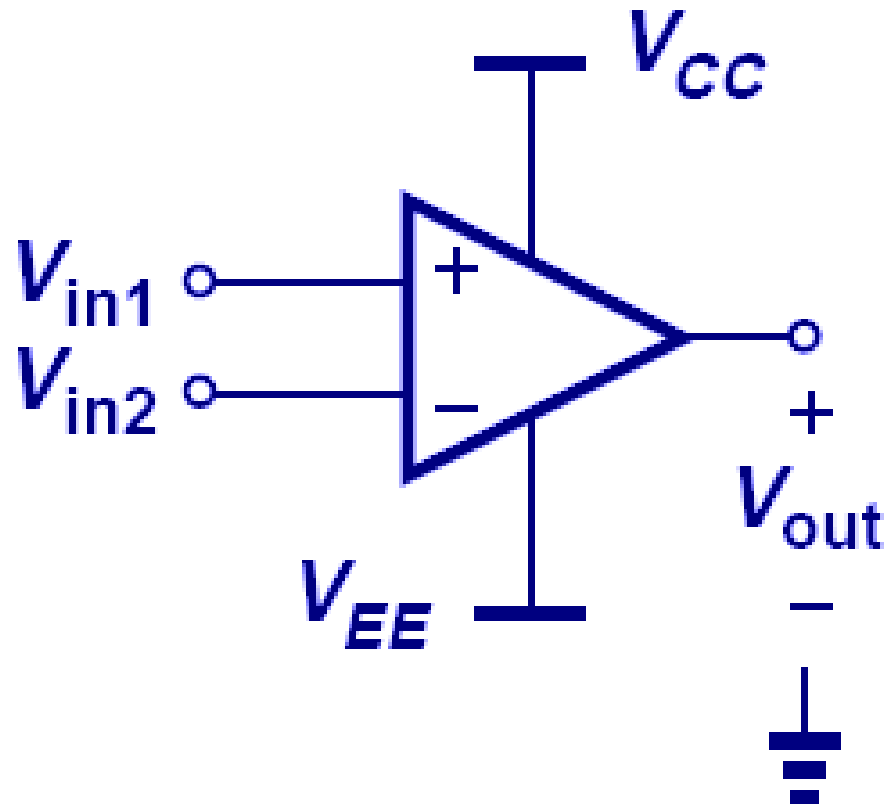
- Due to infinite gain of op amp, the circuit forces  $V_{in2}$  to be close to  $V_{in1}$ , thus creating a virtual short.

# Unity Gain Amplifier



$$V_{out} = A_0(V_{in} - V_{out})$$
$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + A_0}$$

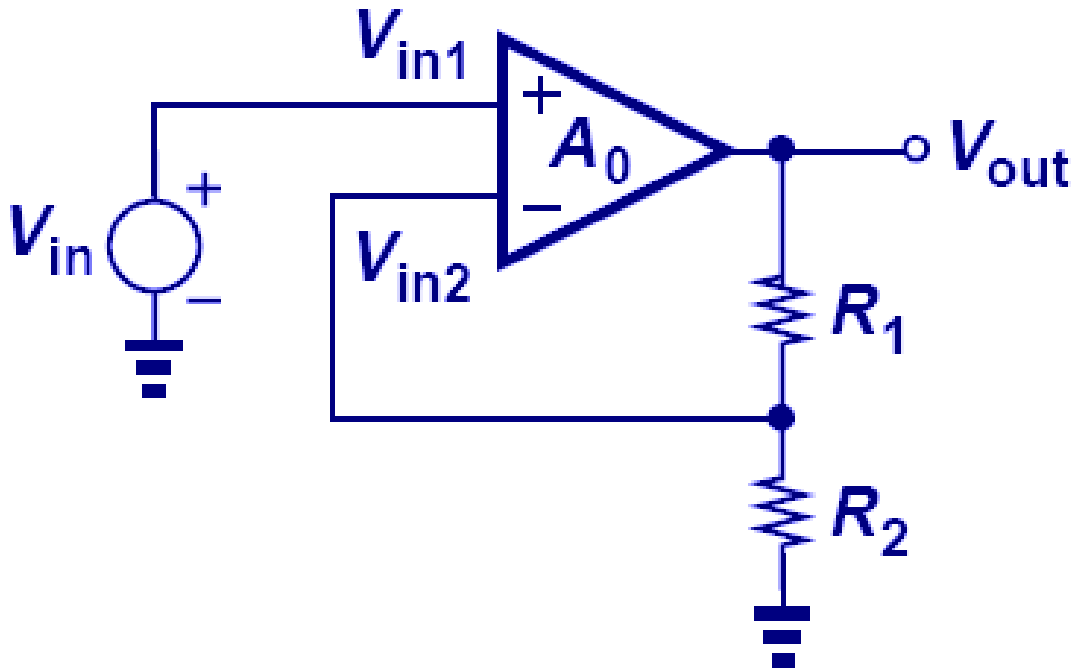
# Op Amp with Supply Rails



- To explicitly show the supply voltages,  $V_{CC}$  and  $V_{EE}$  are shown.
- In some cases,  $V_{EE}$  is zero.



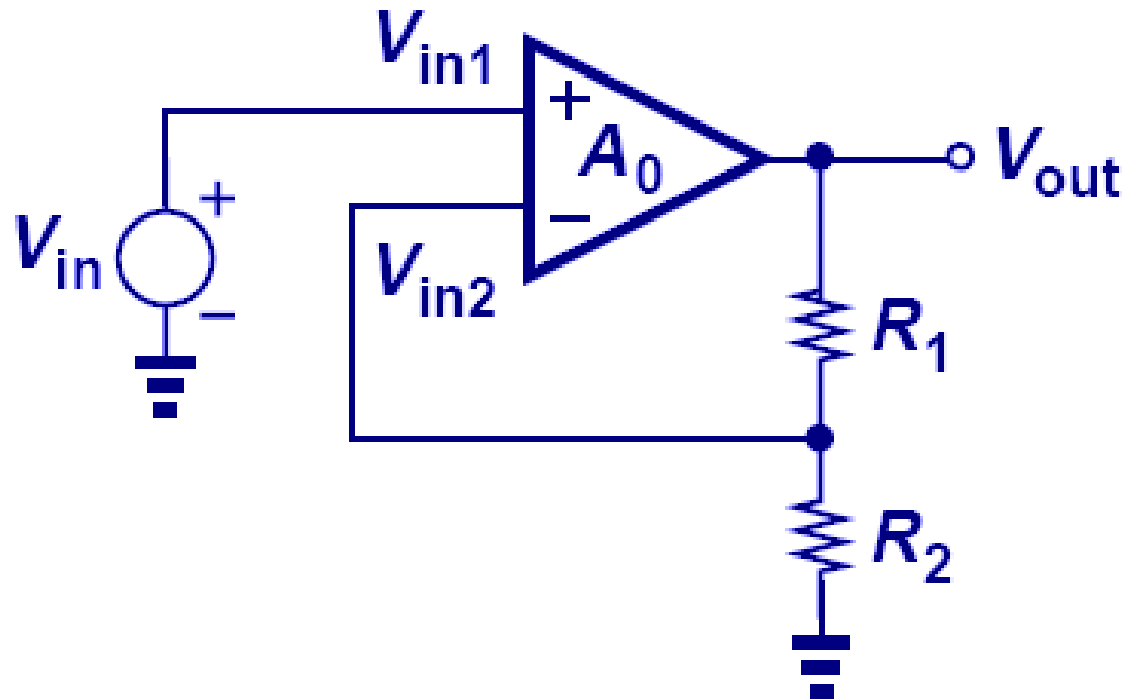
# Noninverting Amplifier (Infinite $A_0$ )



$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_2}$$

- A noninverting amplifier returns a fraction of output signal thru a resistor divider to the negative input.
- With a high  $A_0$ ,  $V_{out}/V_{in}$  depends only on ratio of resistors, which is very precise.

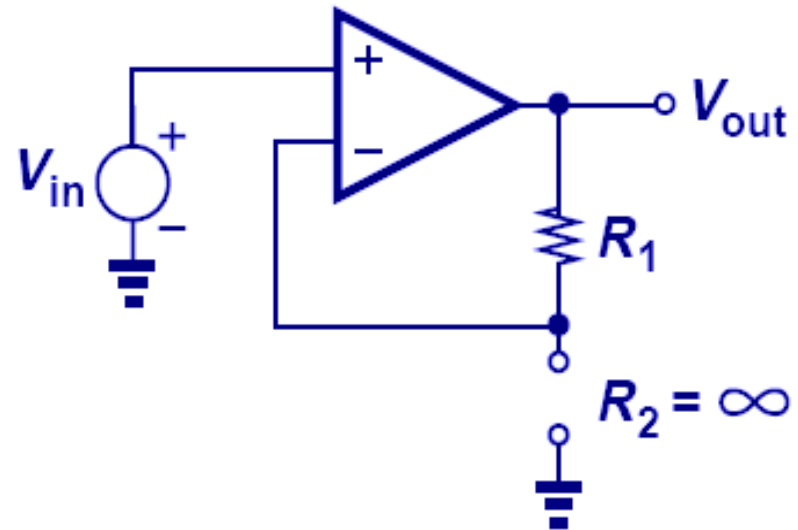
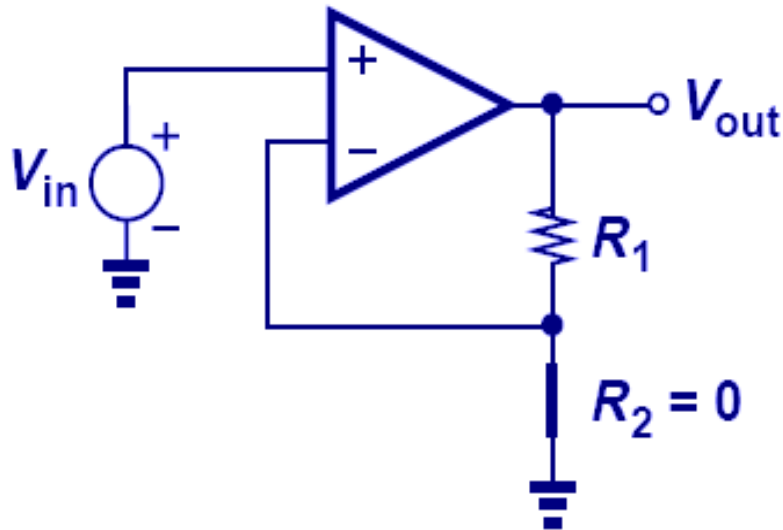
## Noninverting Amplifier (Finite $A_0$ )



$$\frac{V_{out}}{V_{in}} \approx \left(1 + \frac{R_1}{R_2}\right) \left[1 - \left(1 + \frac{R_1}{R_2}\right) \frac{1}{A_0}\right]$$

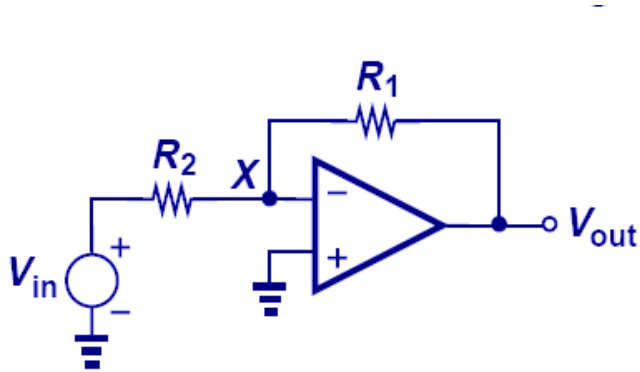
➤ The error term indicates the larger the closed-loop gain, the less accurate the circuit becomes.

## Extreme Cases of $R_2$ (Infinite $A_0$ )

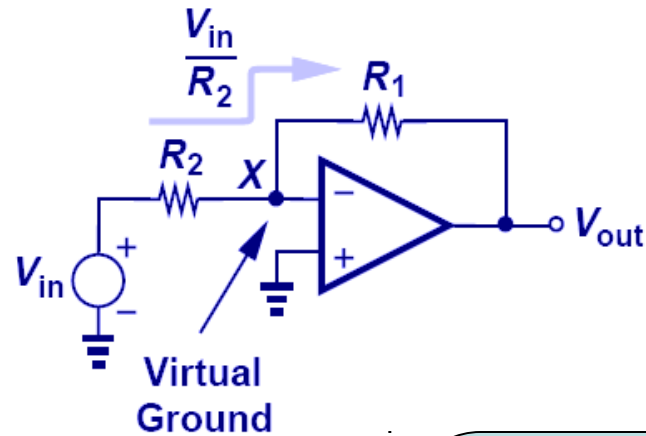


- If  $R_2$  is zero, the loop is open and  $V_{out}/V_{in}$  is equal to the intrinsic gain of the op amp.
- If  $R_2$  is infinite, the circuit becomes a unity-gain amplifier and  $V_{out}/V_{in}$  becomes equal to one.

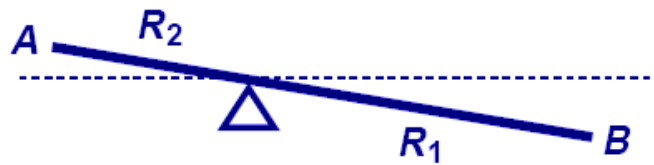
# Inverting Amplifier



(a)



(b)



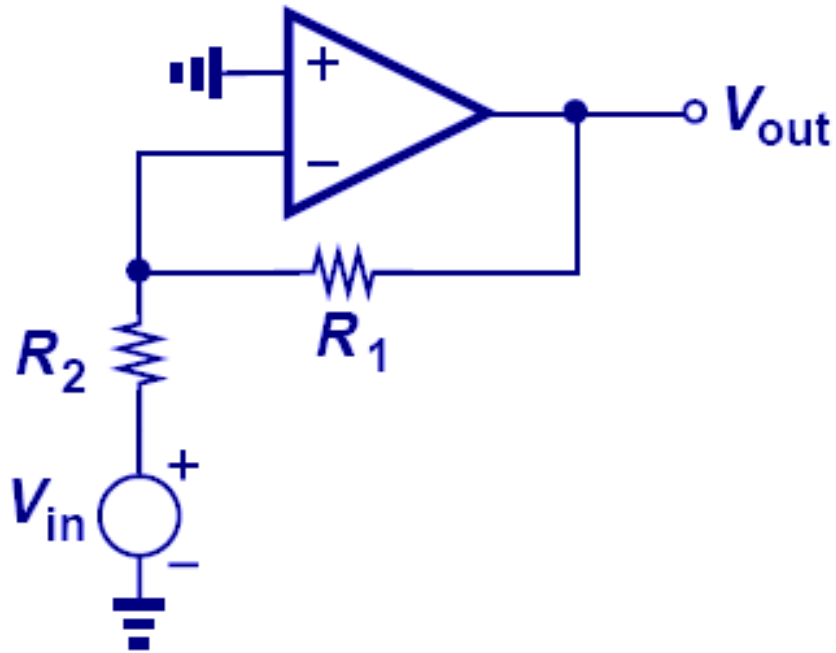
(c)

$$\frac{0 - V_{out}}{R_1} = \frac{V_{in}}{R_2}$$

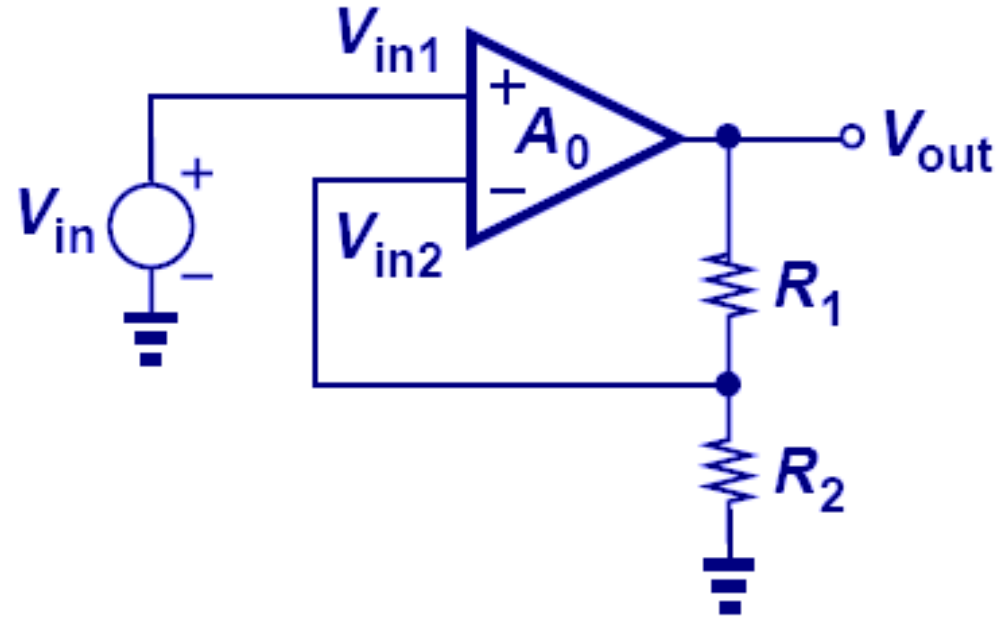
$$\frac{V_{out}}{V_{in}} = \frac{-R_1}{R_2}$$

➤ Infinite  $A_0$  forces the negative input to be a virtual ground.

## Another View of Inverting Amplifier

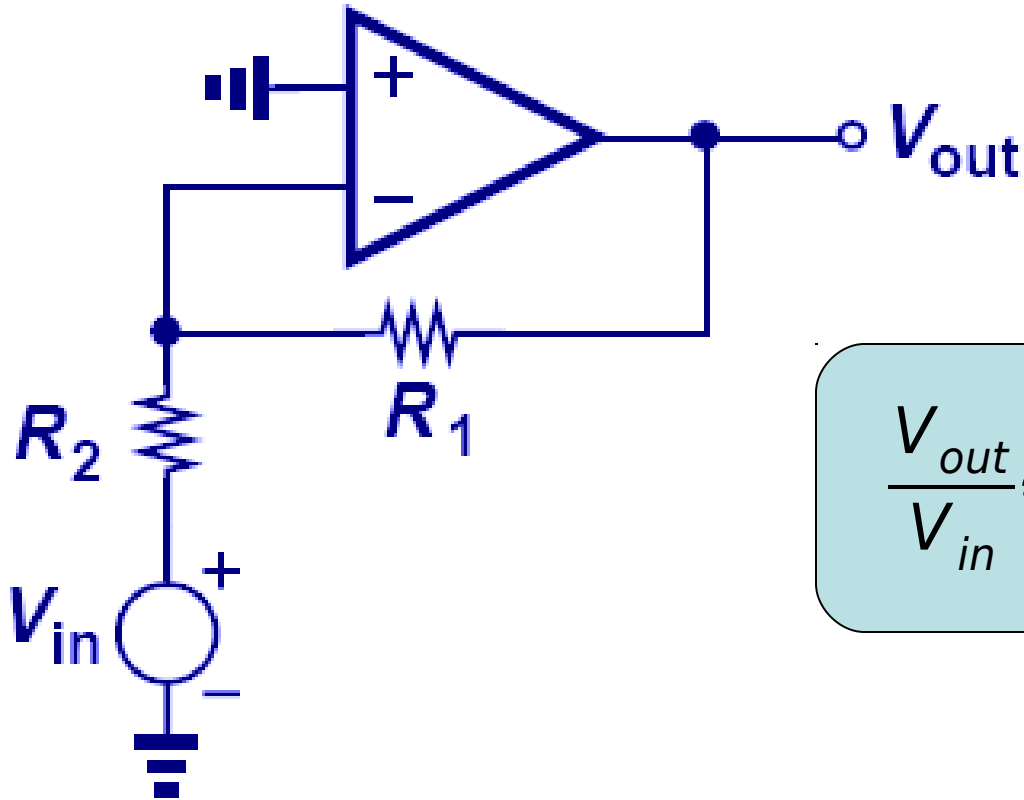


**Inverting**



**Noninverting**

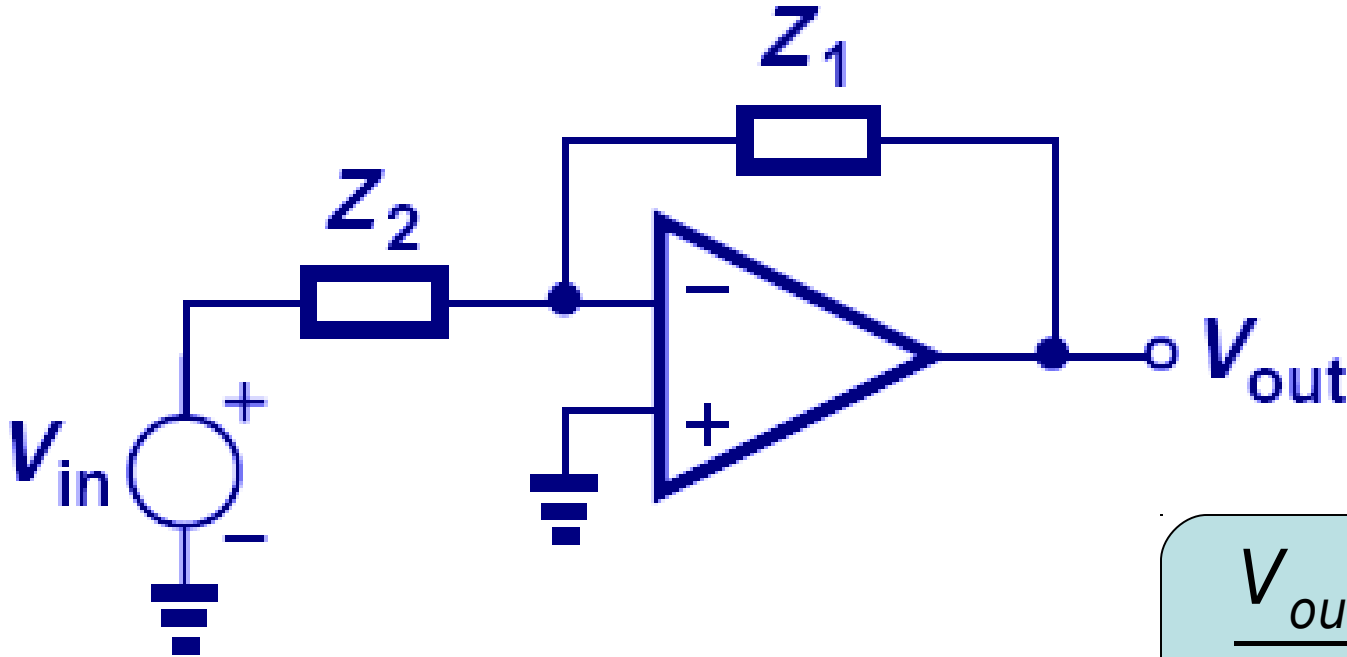
## Gain Error Due to Finite $A_0$



$$\frac{V_{out}}{V_{in}} \approx -\frac{R_1}{R_2} \left[ 1 - \frac{1}{A_0} \left( 1 + \frac{R_1}{R_2} \right) \right]$$

➤ The larger the closed loop gain, the more inaccurate the circuit is.

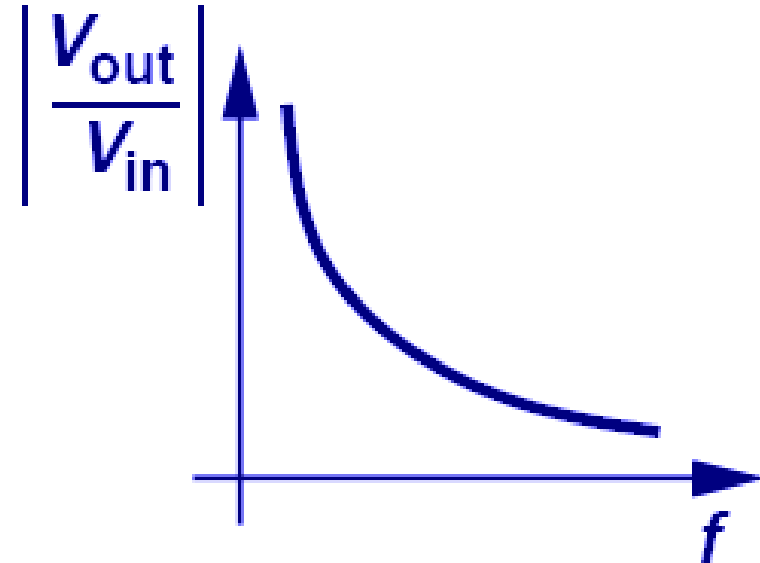
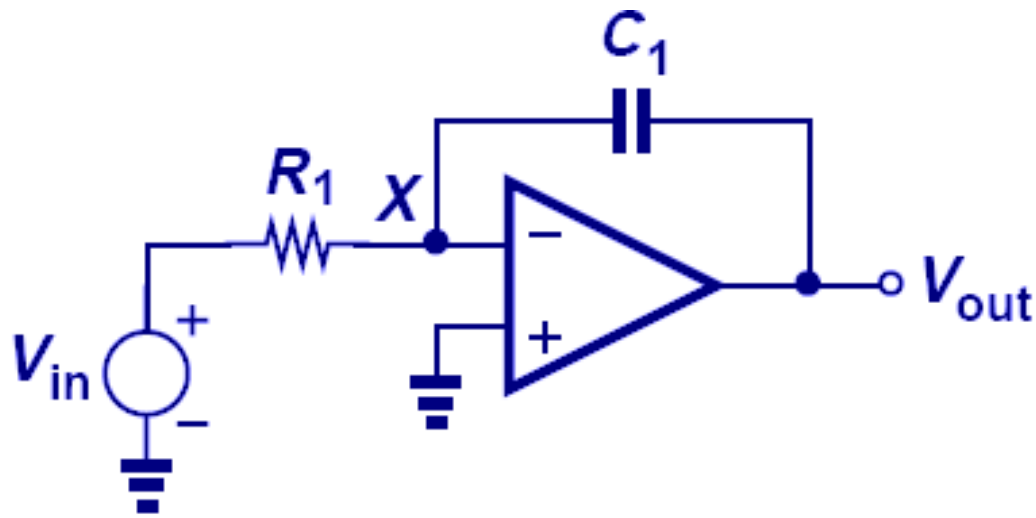
## Complex Impedances Around the Op Amp



$$\frac{V_{out}}{V_{in}} \approx -\frac{Z_1}{Z_2}$$

➤ The closed-loop gain is still equal to the ratio of two impedances.

# Integrator

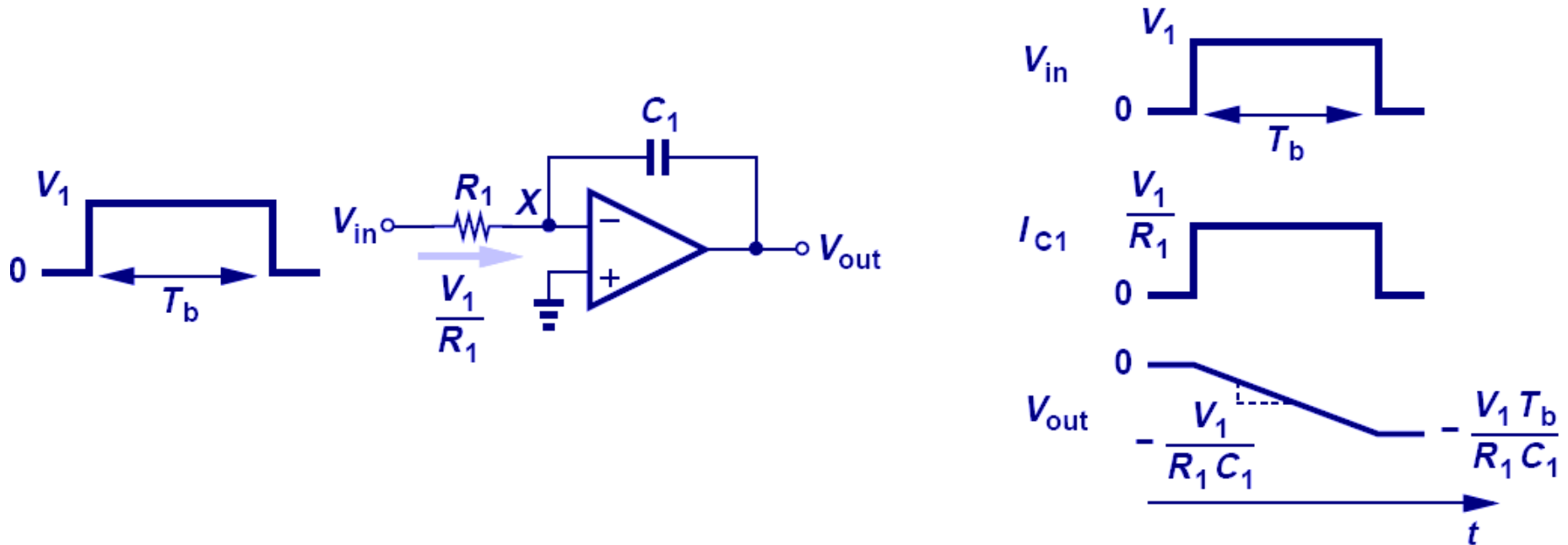


$$\frac{V_{out}}{V_{in}} = -\frac{1}{R_1 C_1 s}$$

$$V_{out} = -\frac{1}{R_1 C_1} \int V_{in} dt$$

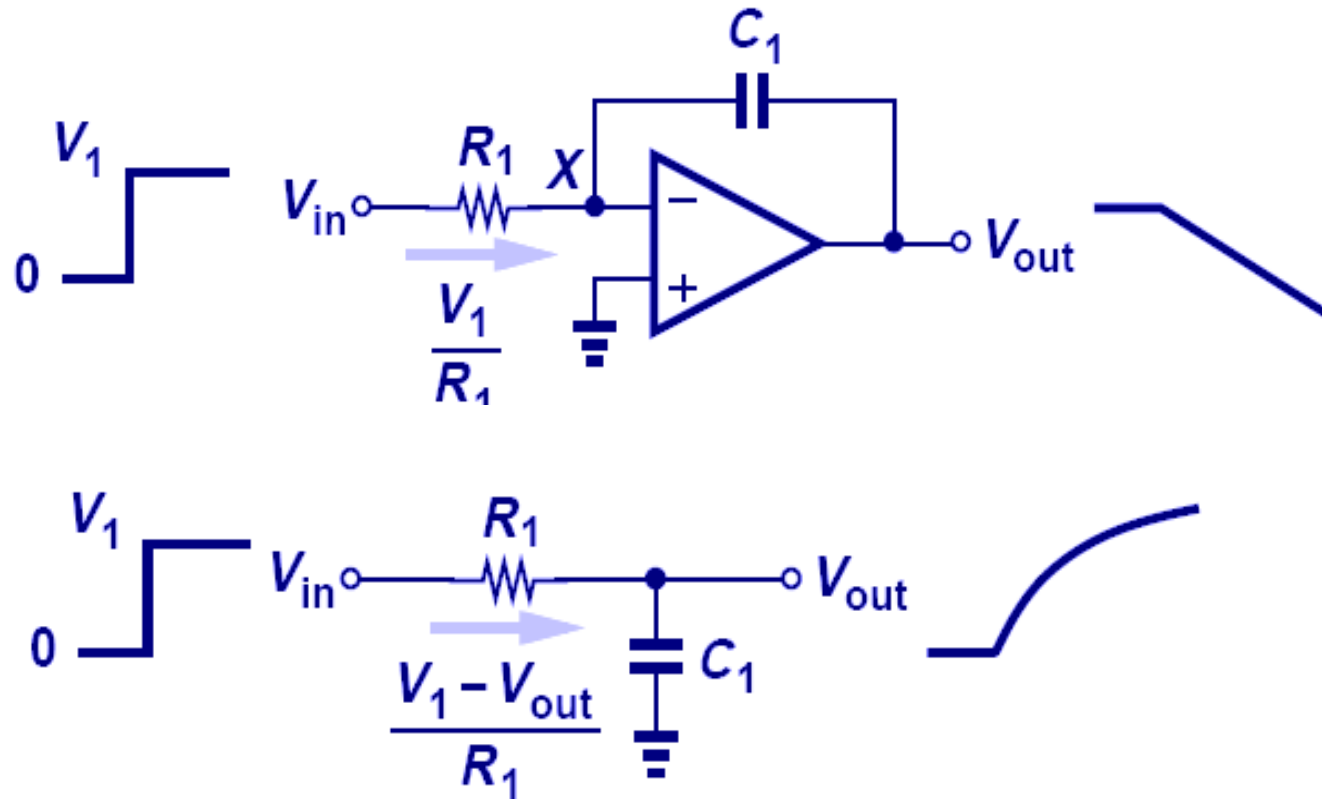


# Integrator with Pulse Input



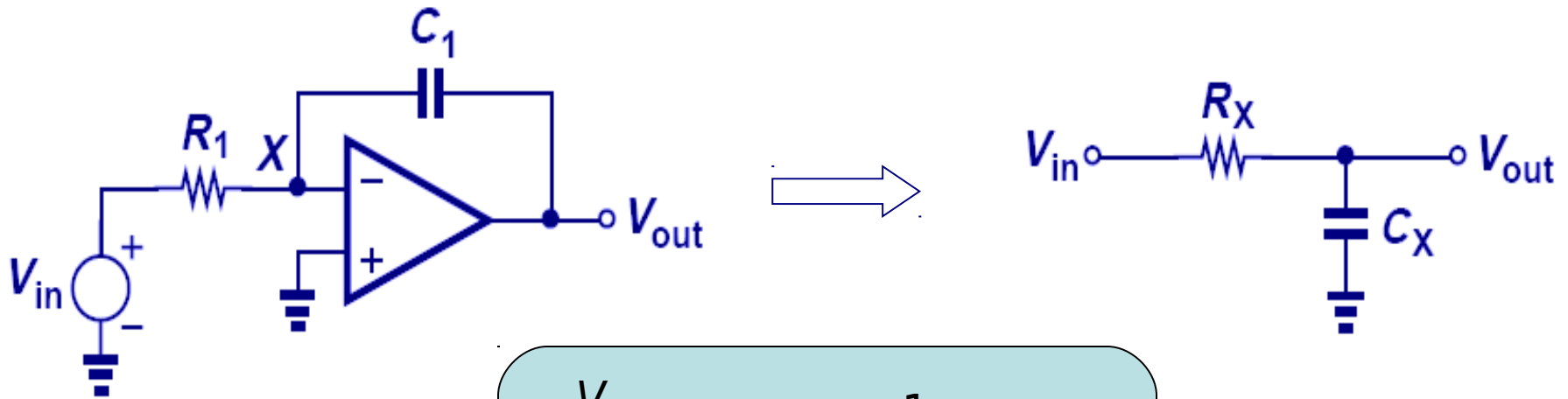
$$V_{out} = -\frac{1}{R_1 C_1} \int V_{in} dt = -\frac{V_1}{R_1 C_1} t \quad 0 < t < T_b$$

# Comparison of Integrator and RC Lowpass Filter



- The RC low-pass filter is actually a “passive” approximation to an integrator.
- With the RC time constant large enough, the RC filter output approaches a ramp.

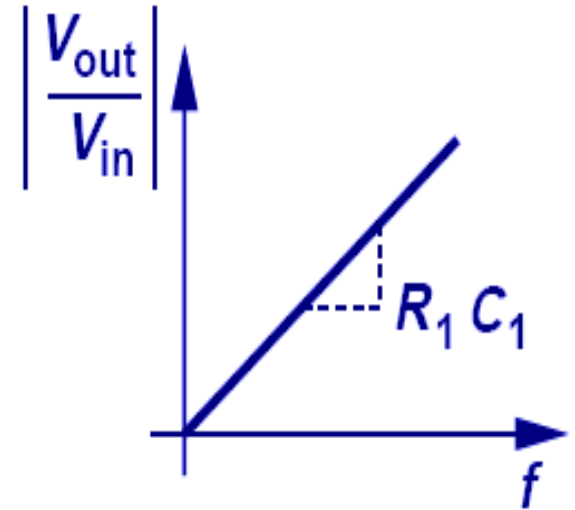
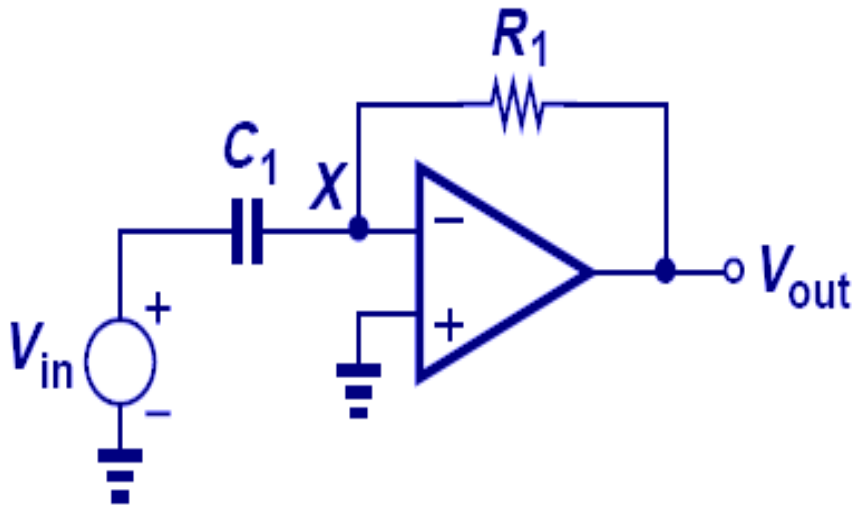
# Lossy Integrator



$$\frac{V_{out}}{V_{in}} = \frac{-1}{\frac{1}{A_0} + \left(1 + \frac{1}{A_0}\right) R_1 C_1 s}$$

- When finite op amp gain is considered, the integrator becomes lossy as the pole moves from the origin to  $-1/[(1+A_0)R_1C_1]$ .
- It can be approximated as an RC circuit with C boosted by a factor of  $A_0+1$ .

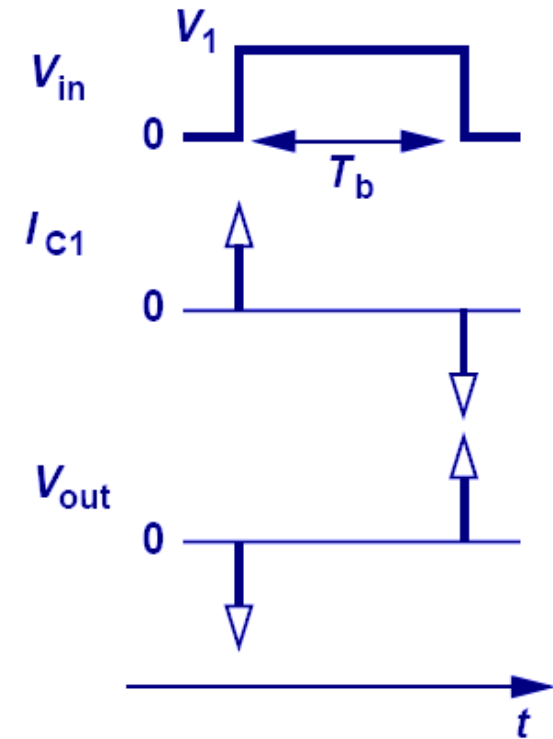
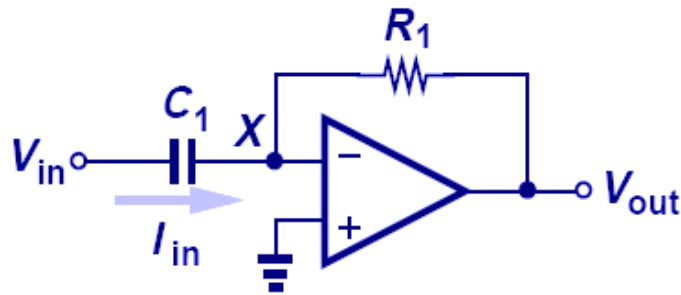
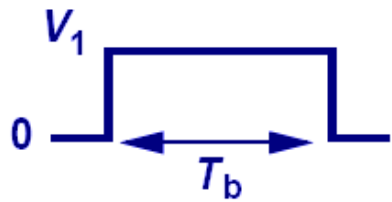
# Differentiator



$$V_{out} = -R_1 C_1 \frac{dV_{in}}{dt}$$

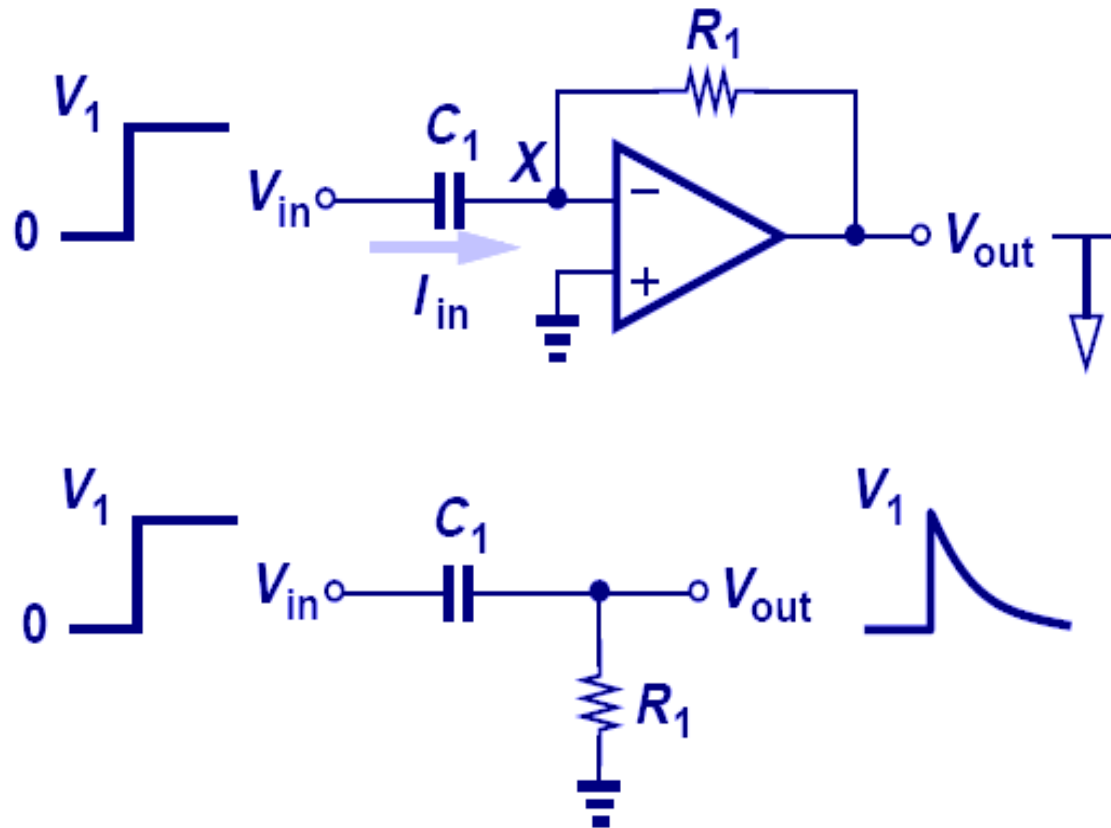
$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{\frac{1}{C_1 s}} = -R_1 C_1 s$$

# Differentiator with Pulse Input



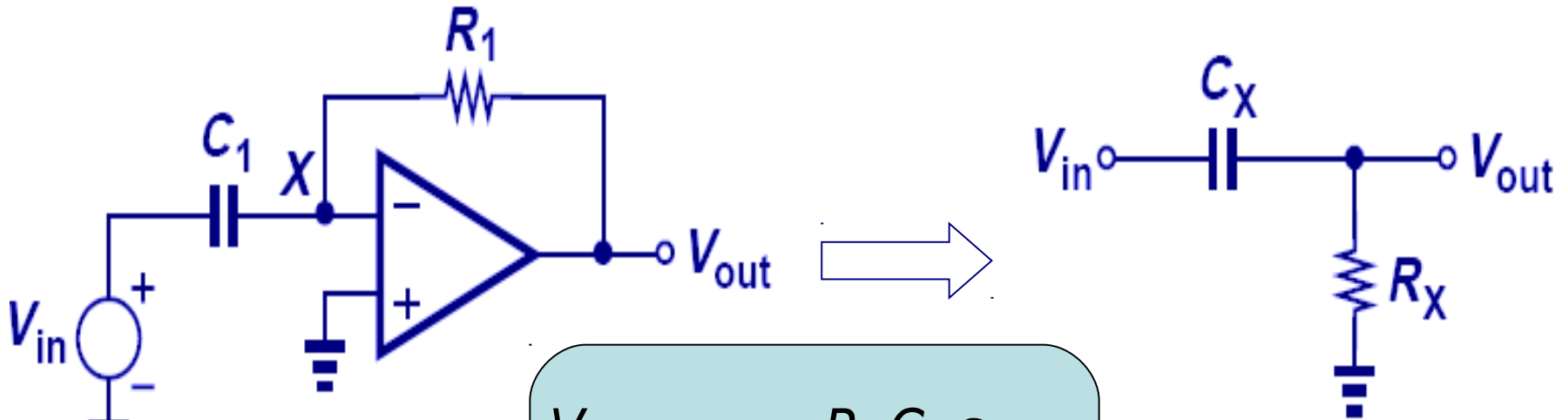
$$V_{out} = -R_1 C_1 V_1 \delta(t)$$

# Comparison of Differentiator and High-Pass Filter



- The RC high-pass filter is actually a passive approximation to the differentiator.
- When the RC time constant is small enough, the RC filter approximates a differentiator.

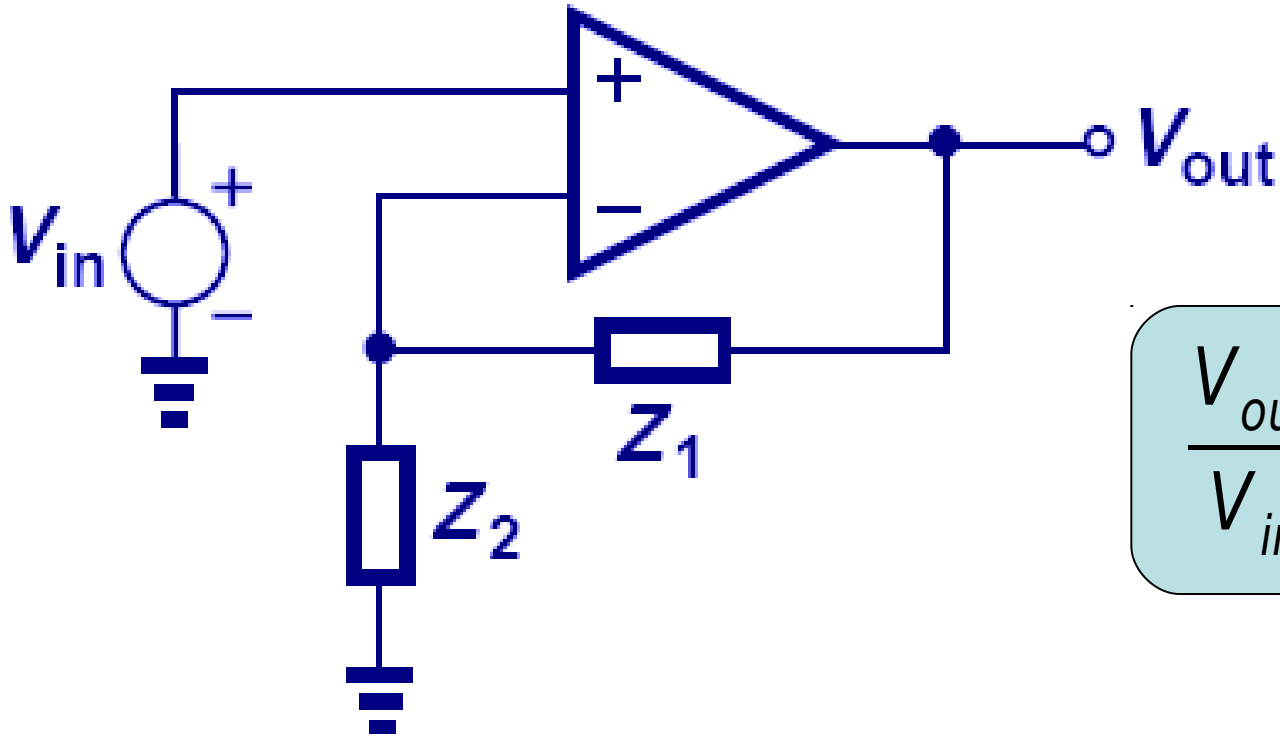
# Lossy Differentiator



$$\frac{V_{out}}{V_{in}} = \frac{-R_1 C_1 s}{1 + \frac{1}{A_0} + \frac{R_1 C_1 s}{A_0}}$$

- When finite op amp gain is considered, the differentiator becomes lossy as the zero moves from the origin to  $-(A_0+1)/R_1 C_1$ .
- It can be approximated as an RC circuit with R reduced by a factor of  $(A_0+1)$ .

# Op Amp with General Impedances

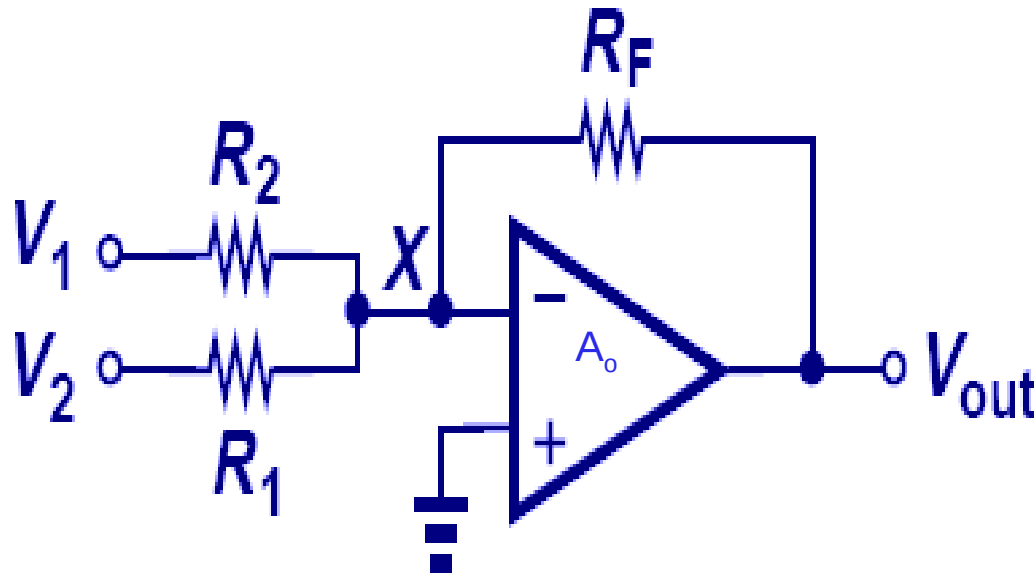


$$\frac{V_{out}}{V_{in}} = 1 + \frac{Z_1}{Z_2}$$

➤ This circuit cannot operate as ideal integrator or differentiator.



# Voltage Adder



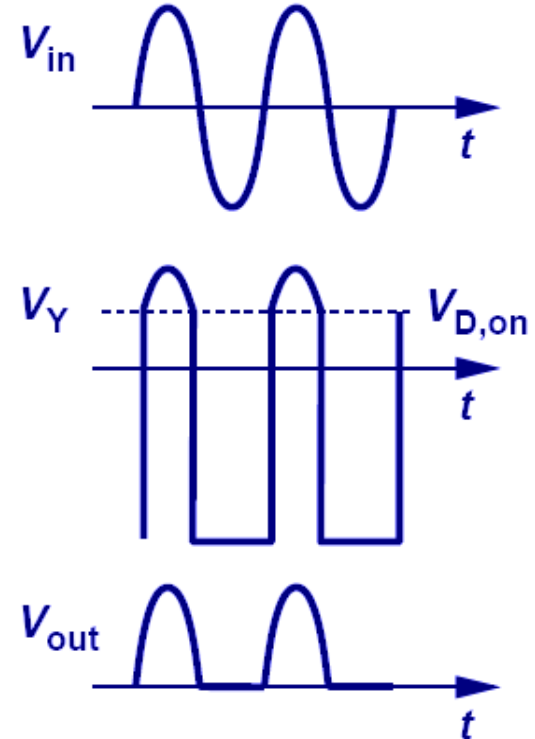
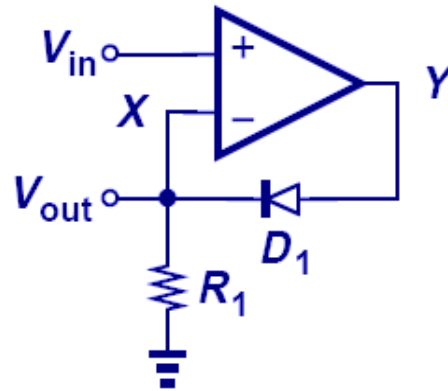
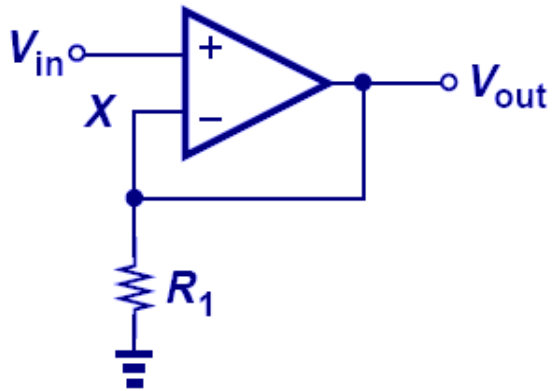
$$V_{out} = -R_F \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

$$V_{out} = \frac{-R_F}{R} (V_1 + V_2)$$

*If  $R_1 = R_2 = R$*

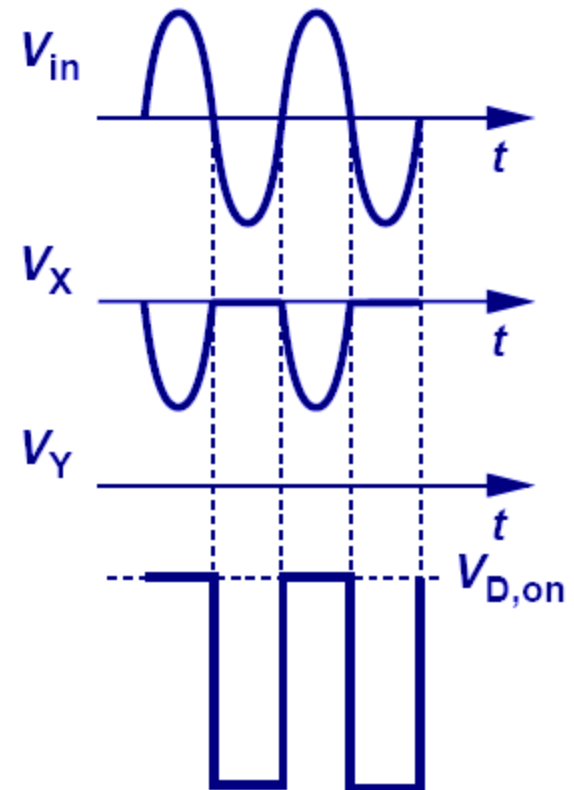
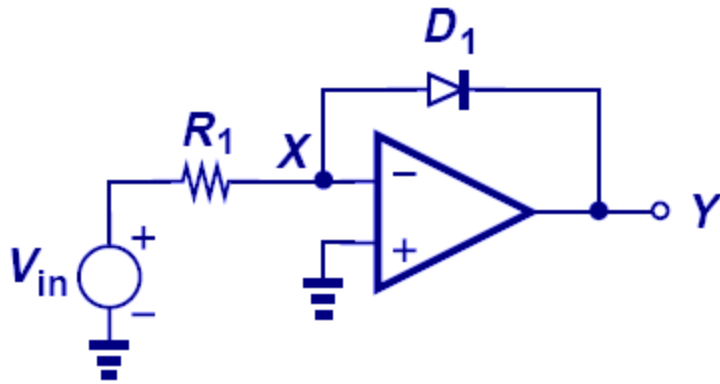
- If  $A_o$  is infinite, X is pinned at ground, currents proportional to  $V_1$  and  $V_2$  will flow to X and then across  $R_F$  to produce an output proportional to the sum of two voltages.

# Precision Rectifier



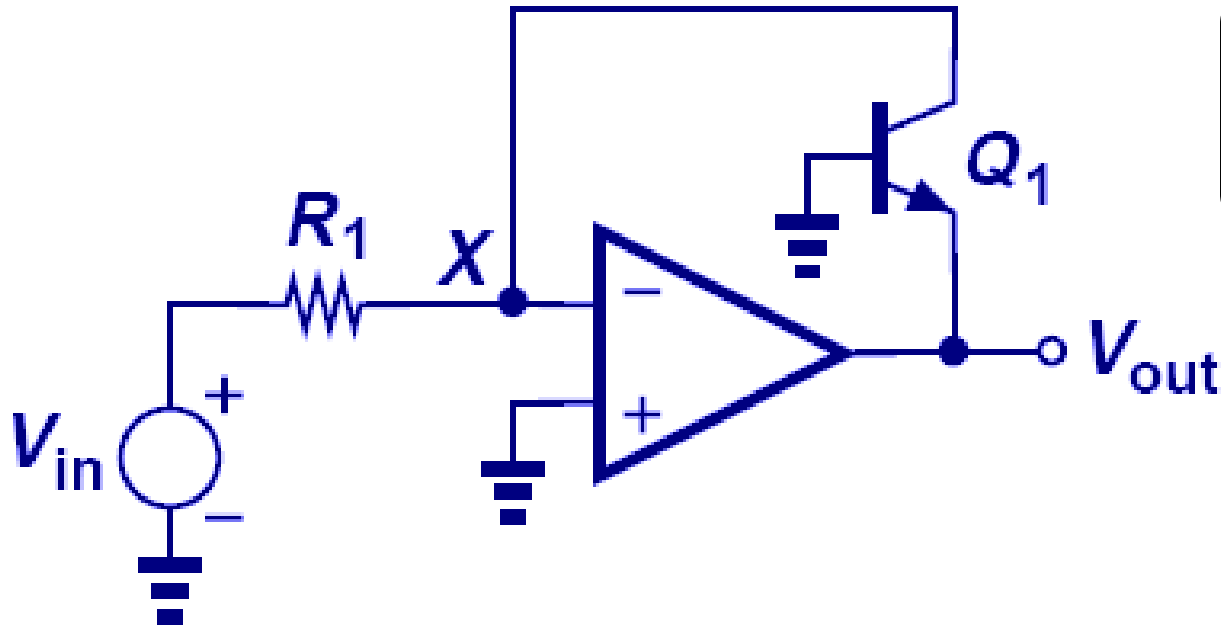
- When  $V_{in}$  is positive, the circuit in b) behaves like that in a), so the output follows input.
- When  $V_{in}$  is negative, the diode opens, and the output drops to zero. Thus performing rectification.

# Inverting Precision Rectifier



- When  $V_{in}$  is positive, the diode is on,  $V_y$  is pinned around  $V_{D,on}$ , and  $V_x$  at virtual ground.
- When  $V_{in}$  is negative, the diode is off,  $V_y$  goes extremely negative, and  $V_x$  becomes equal to  $V_{in}$ .

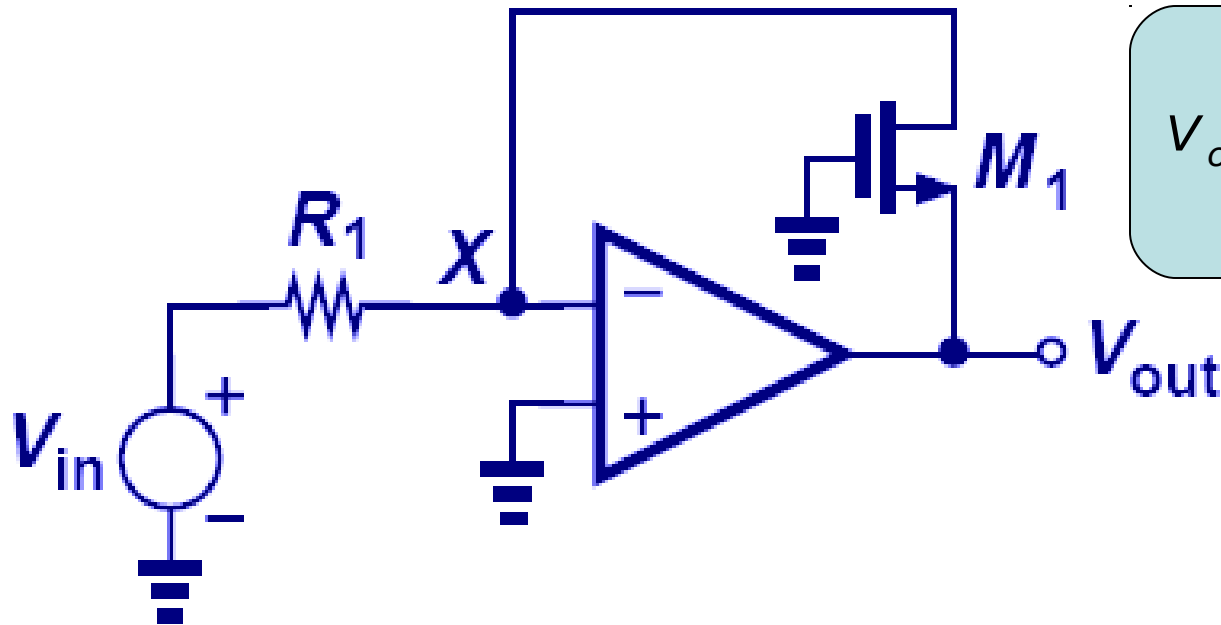
# Logarithmic Amplifier



$$V_{out} = -V_T \ln \frac{V_{in}}{R_1 I_S}$$

- By inserting a bipolar transistor in the loop, an amplifier with logarithmic characteristic can be constructed.
- This is because the current to voltage conversion of a bipolar transistor is a natural logarithm.

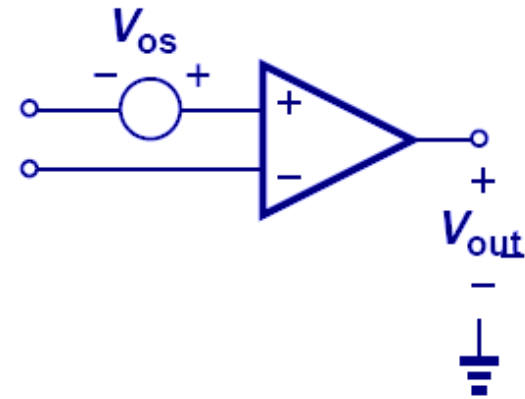
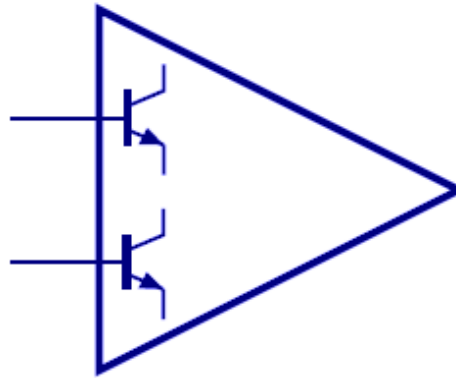
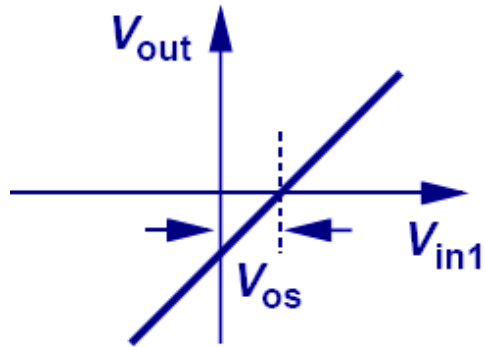
# Square-Root Amplifier



$$V_{out} = - \sqrt{\frac{2V_{in}}{\mu_n C_{ox} \frac{W}{L} R_1}} - V_{TH}$$

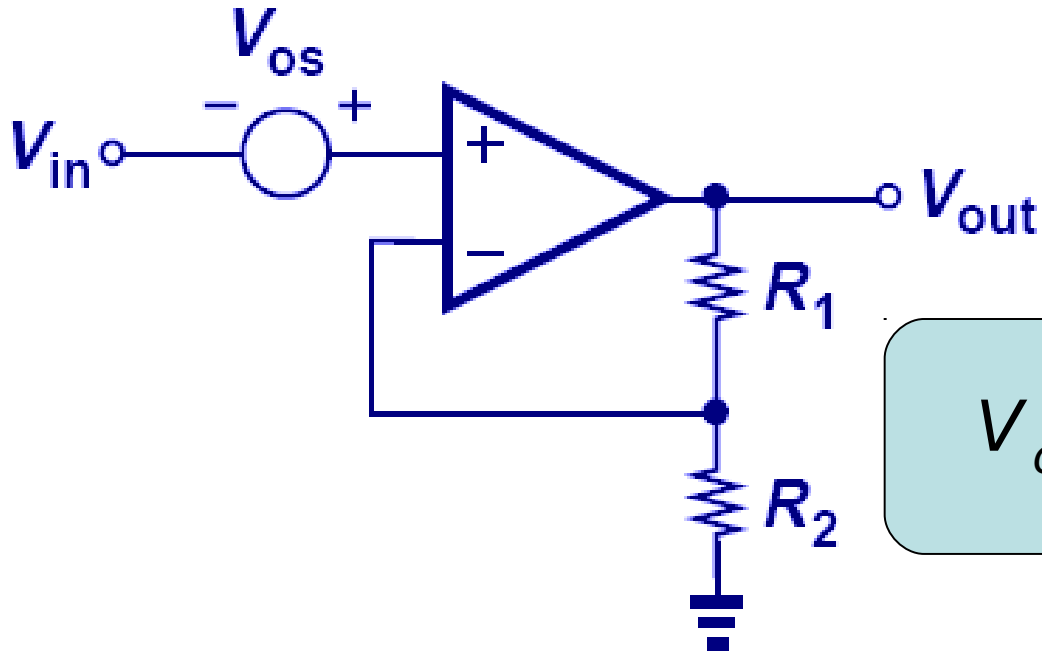
- By replacing the bipolar transistor with a MOSFET, an amplifier with a square-root characteristic can be built.
- This is because the current to voltage conversion of a MOSFET is square-root.

# Op Amp Nonidealities: DC Offsets



- **Offsets in an op amp that arise from input stage mismatch cause the input-output characteristic to shift in either the positive or negative direction (the plot displays positive direction).**

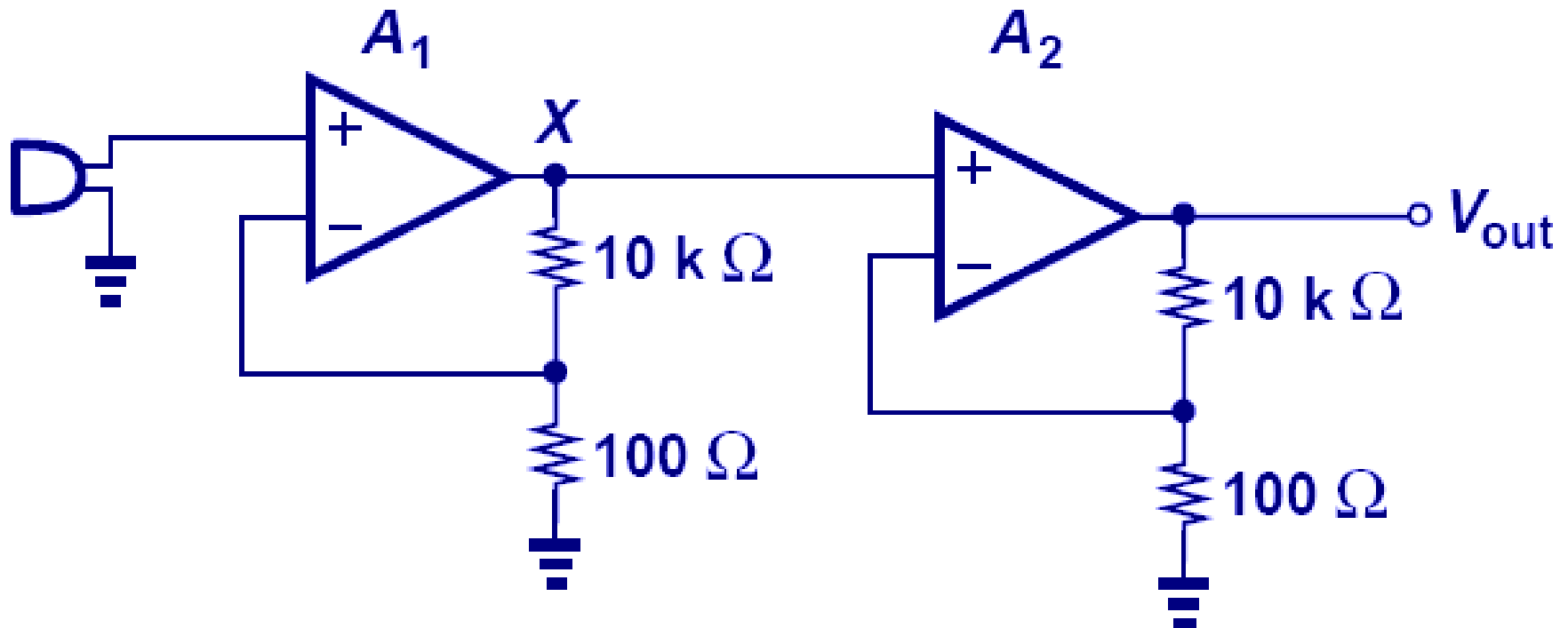
## Effects of DC Offsets



$$V_{out} = \left( 1 + \frac{R_1}{R_2} \right) (V_{in} + V_{os})$$

- As it can be seen, the op amp amplifies the input as well as the offset, thus creating errors.

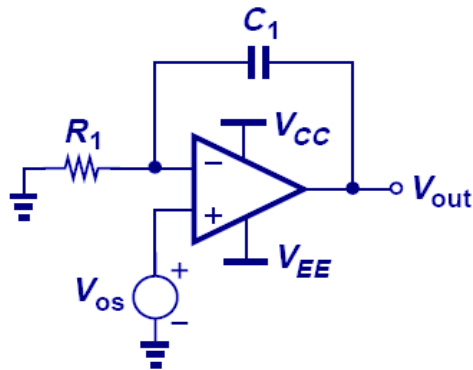
## Saturation Due to DC Offsets



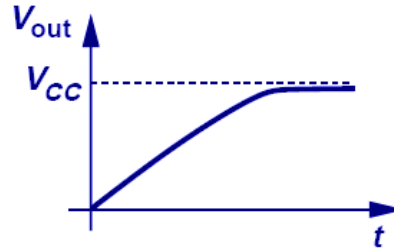
- Since the offset will be amplified just like the input signal, output of the first stage may drive the second stage into saturation.



# Offset in Integrator

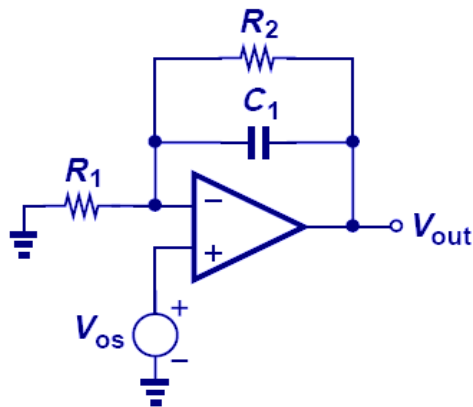


(a)

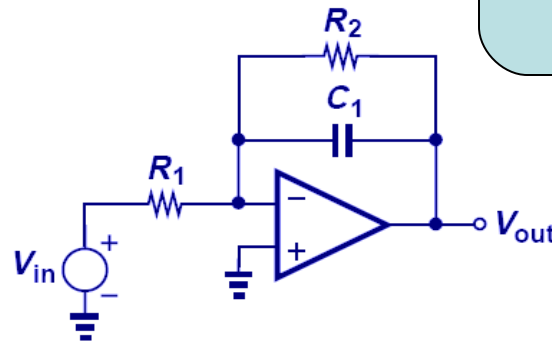


(b)

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \frac{1}{R_2 C_1 s + 1}$$



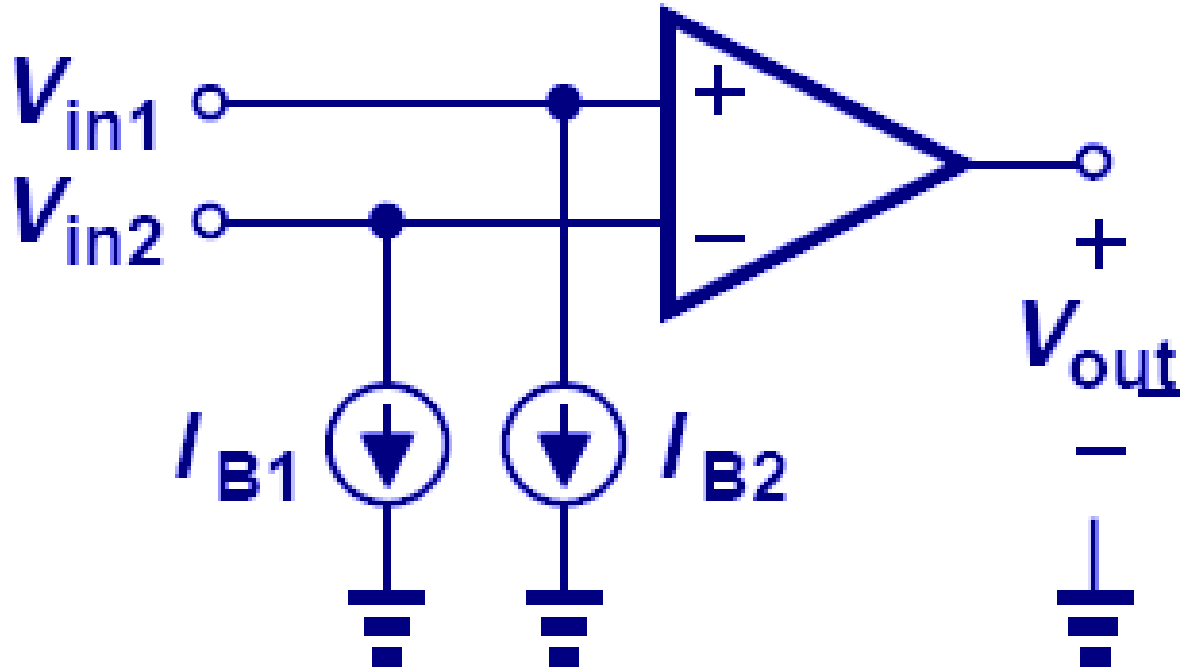
(c)



(d)

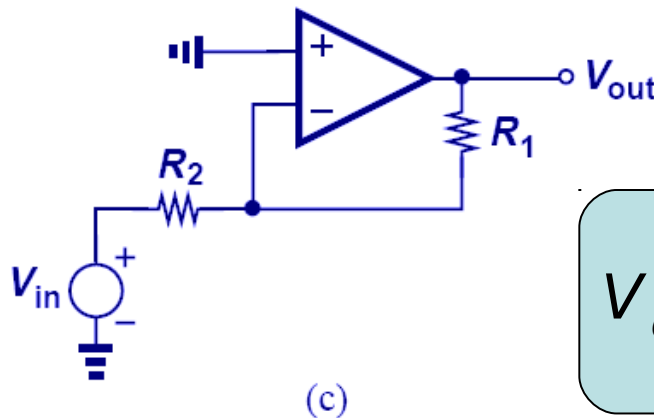
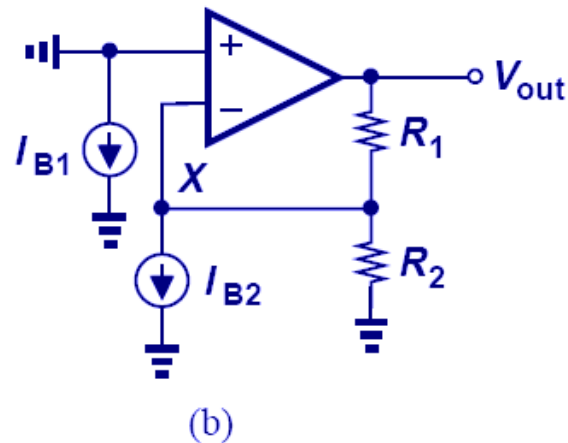
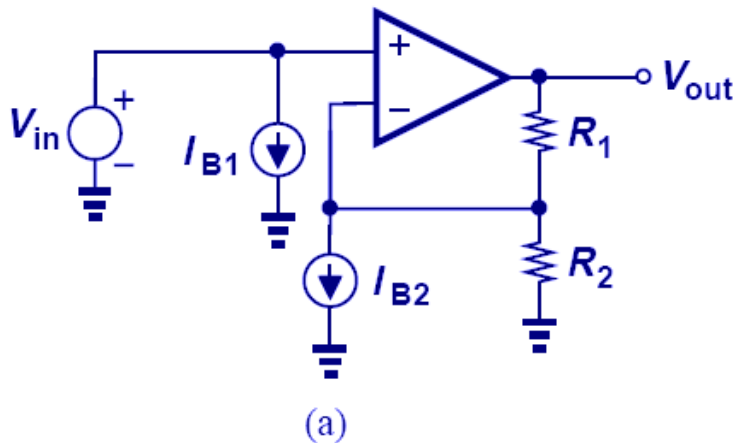
- A resistor can be placed in parallel with the capacitor to “absorb” the offset. However, this means the closed-loop transfer function no longer has a pole at origin.

# Input Bias Current



➤ The effect of bipolar base currents can be modeled as current sources tied from the input to ground.

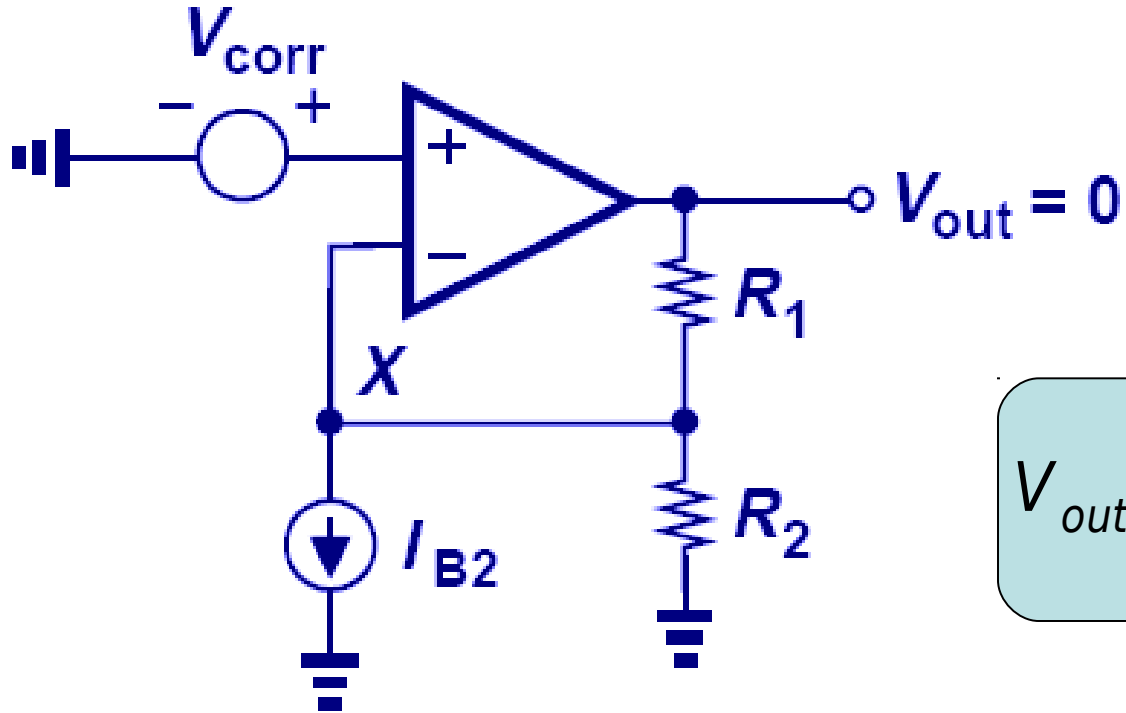
# Effects of Input Bias Current on Noninverting Amplifier



$$V_{out} = -R_2 I_{B2} \left( -\frac{R_1}{R_2} \right) = R_1 I_{B2}$$

➤ It turns out that  $I_{B1}$  has no effect on the output and  $I_{B2}$  affects the output by producing a voltage drop across  $R_1$ .

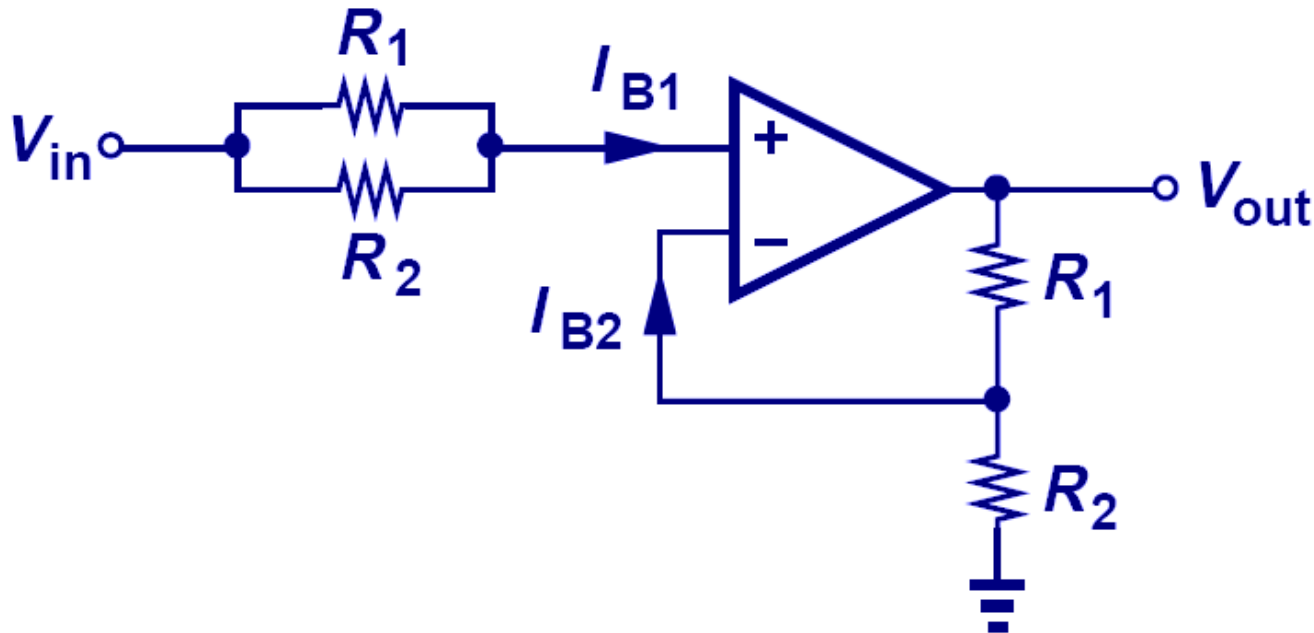
# Input Bias Current Cancellation



$$V_{out} = V_{corr} \left( 1 + \frac{R_1}{R_2} \right) + I_{B2} R_1$$

- We can cancel the effect of input bias current by inserting a correction voltage in series with the positive terminal.
- In order to produce a zero output,  $V_{corr} = -I_{B2}(R_1 || R_2)$ .

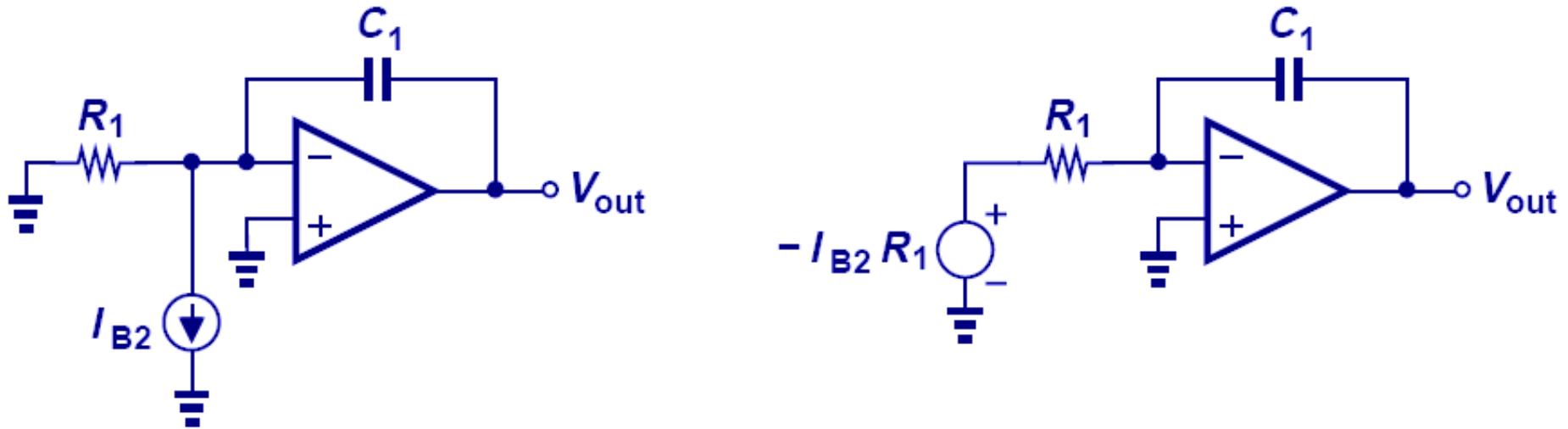
## Correction for $\beta$ Variation



$$I_{B1} = I_{B2}$$

- Since the correction voltage is dependent upon  $\beta$ , and  $\beta$  varies with process, we insert a parallel resistor combination in series with the positive input. As long as  $I_{B1} = I_{B2}$ , the correction voltage can track the  $\beta$  variation.

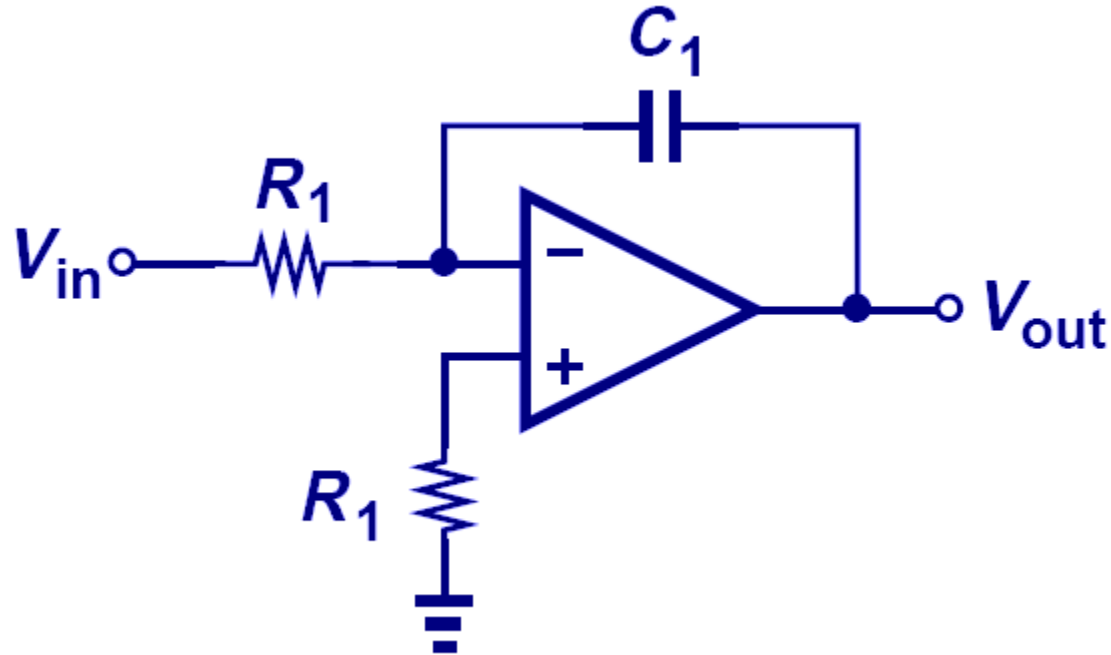
## Effects of Input Bias Currents on Integrator



$$V_{out} = -\frac{1}{R_1 C_1} \int (-I_{B2} R_1) dt$$

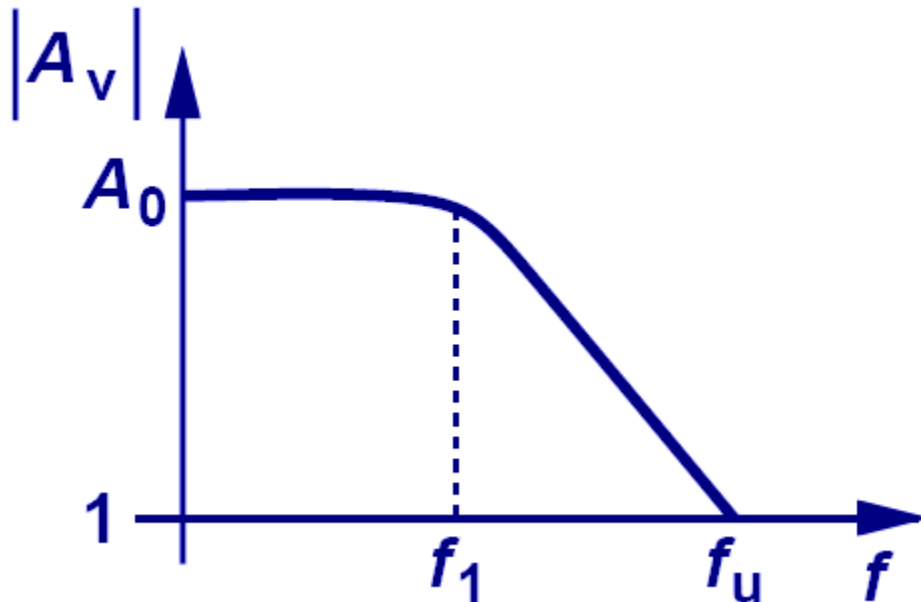
- Input bias current will be integrated by the integrator and eventually saturate the amplifier.

# Integrator's Input Bias Current Cancellation



- By placing a resistor in series with the positive input, integrator input bias current can be cancelled.
- However, the output still saturates due to other effects such as input mismatch, etc.

# Speed Limitation

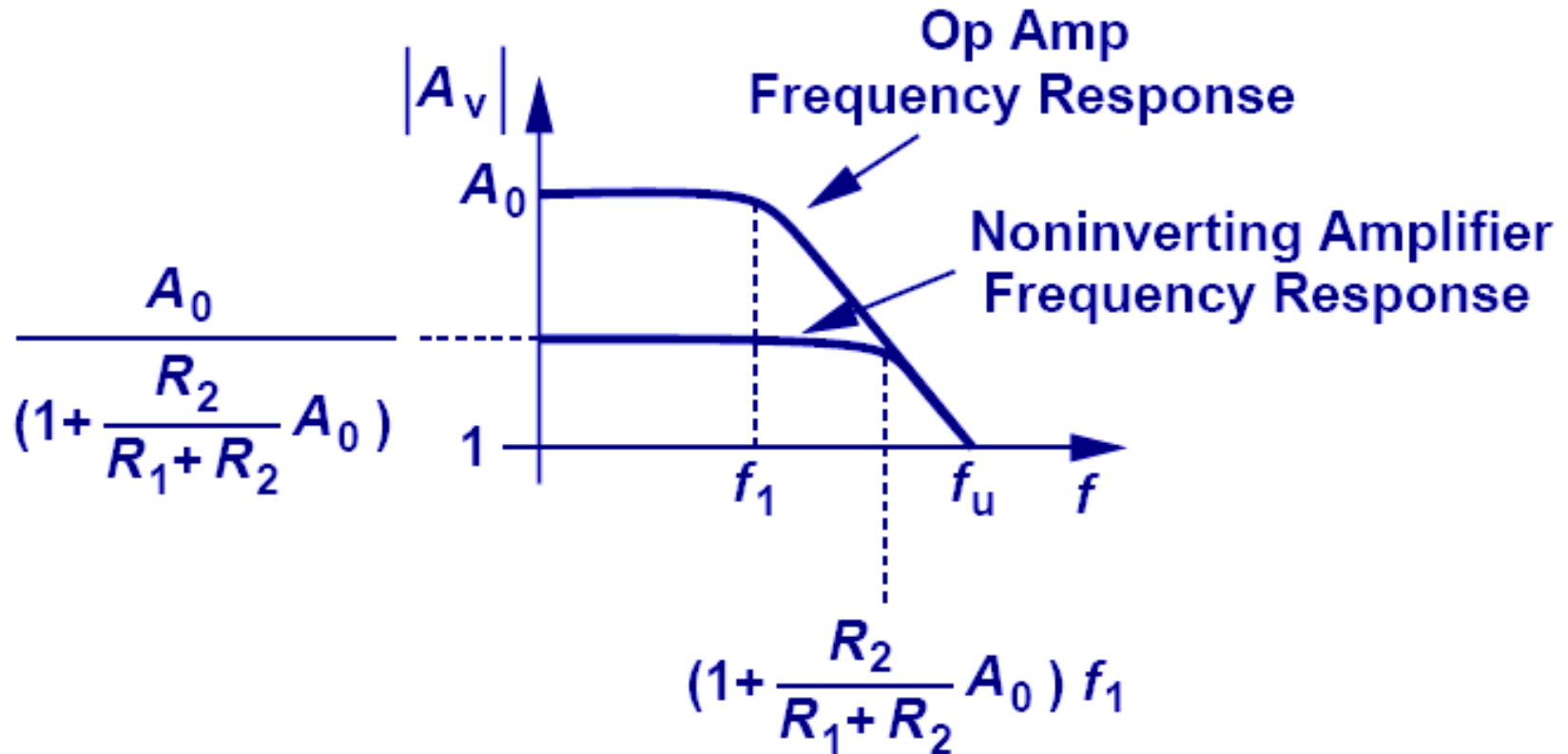


$$\frac{V_{out}}{V_{in1} - V_{in2}}(s) = \frac{A_0}{1 + \frac{s}{\omega_1}}$$

- Due to internal capacitances, the gain of op amps begins to roll off.

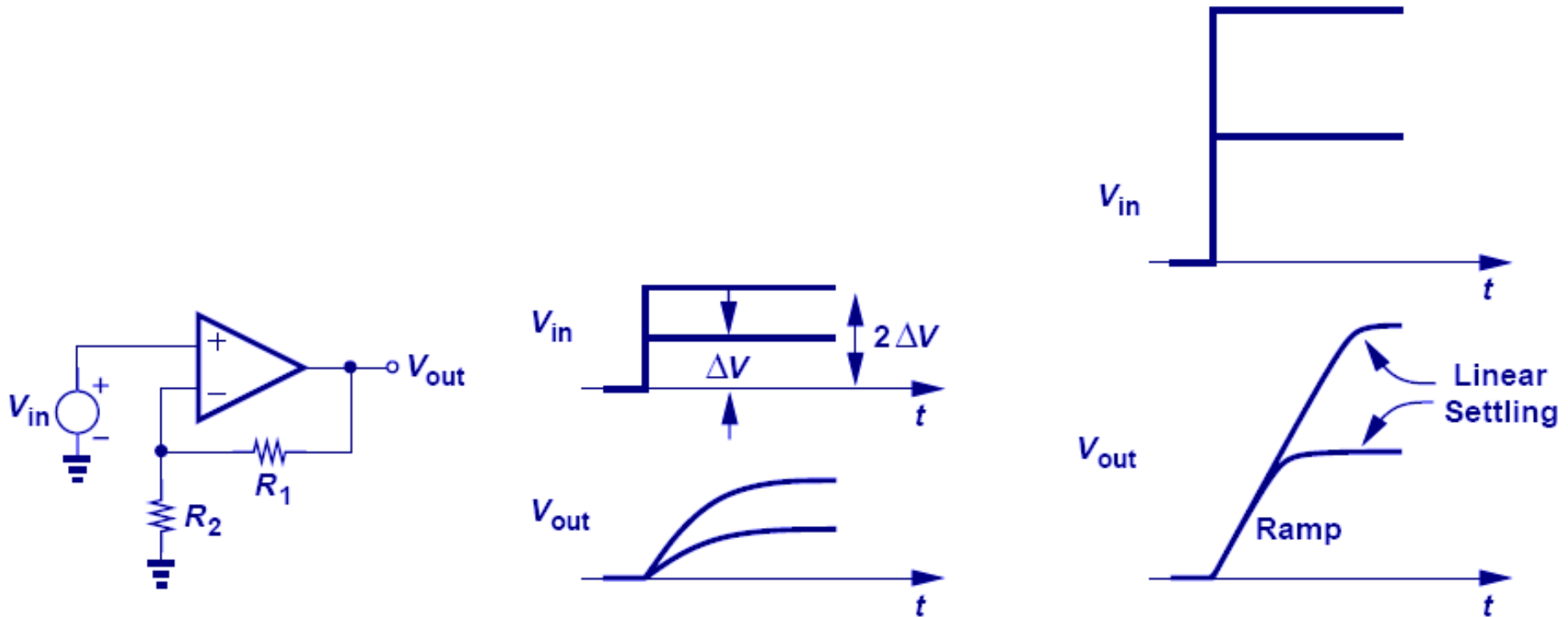


# Bandwidth and Gain Tradeoff



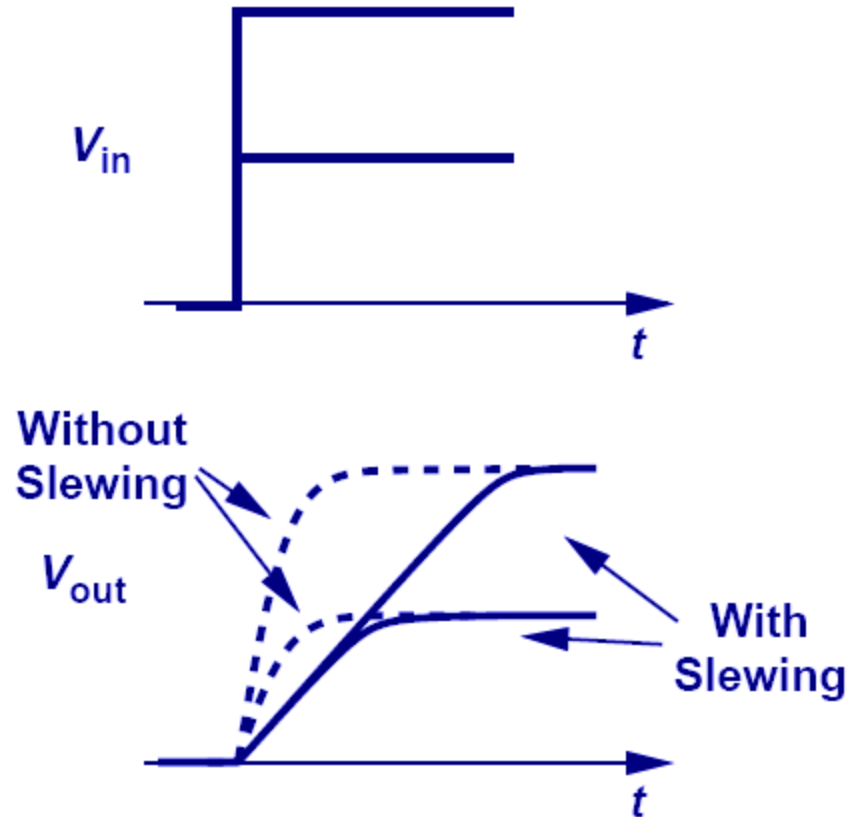
- Having a loop around the op amp (inverting, noninverting, etc) helps to increase its bandwidth. However, it also decreases the low frequency gain.

# Slew Rate of Op Amp



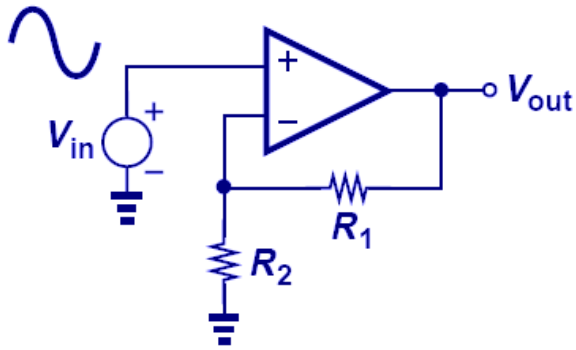
- In the linear region, when the input doubles, the output and the output slope also double. However, when the input is large, the op amp slews so the output slope is fixed by a constant current source charging a capacitor.
- This further limits the speed of the op amp.

# Comparison of Settling with and without Slew Rate

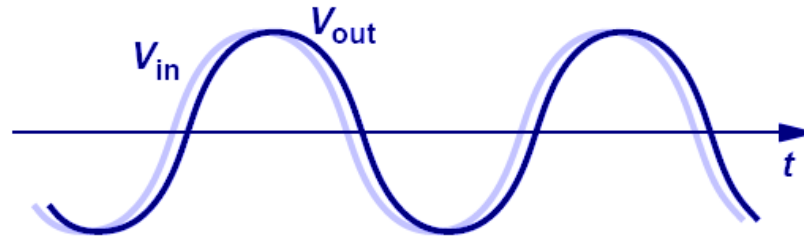


- As it can be seen, the settling speed is faster without slew rate (as determined by the closed-loop time constant).

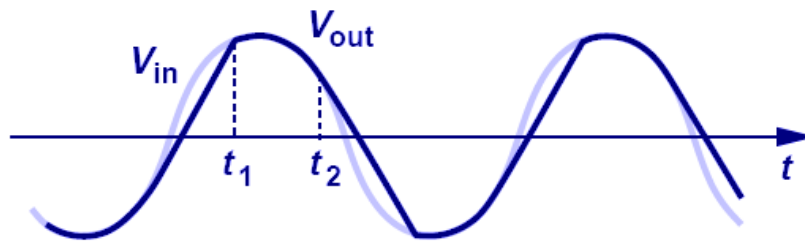
# Slew Rate Limit on Sinusoidal Signals



(a)



(b)

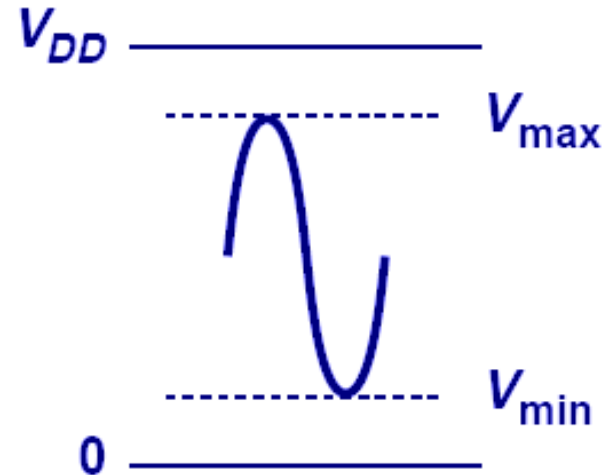
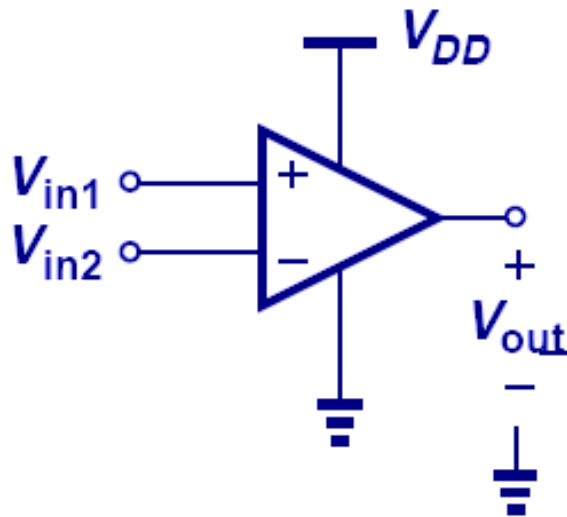


(c)

$$\frac{dV_{out}}{dt} = V_0 \left( 1 + \frac{R_1}{R_2} \right) \omega \cos \omega t$$

- As long as the output slope is less than the slew rate, the op amp can avoid slewing.
- However, as operating frequency and/or amplitude is increased, the slew rate becomes insufficient and the output becomes distorted.

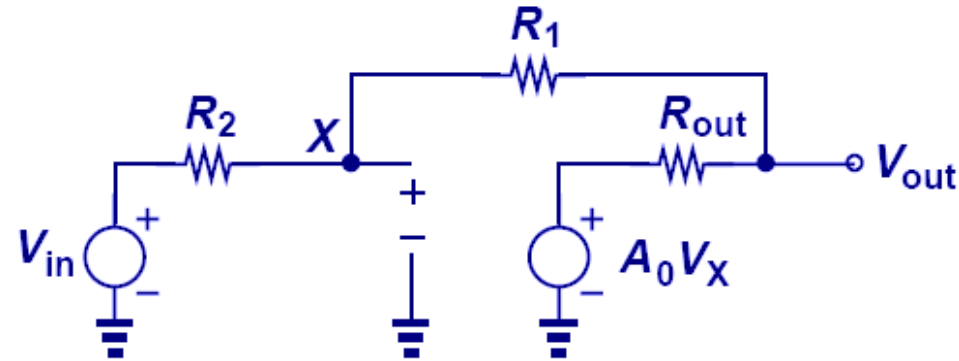
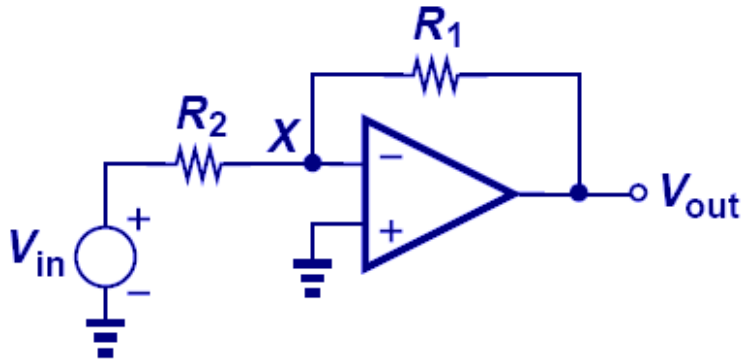
# Maximum Op Amp Swing



$$V_{out} = \frac{V_{max} - V_{min}}{2} \sin \omega t + \frac{V_{max} + V_{min}}{2} \quad \omega_{FP} = \frac{SR}{\frac{V_{max} - V_{min}}{2}}$$

- To determine the maximum frequency before op amp slews, first determine the maximum swing the op amp can have and divide the slew rate by it.

## Nonzero Output Resistance



$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{R_2} \frac{A_0 - \frac{R_{out}}{R_1}}{1 + \frac{R_{out}}{R_2} + A_0 + \frac{R_1}{R_2}}$$

- In practical op amps, the output resistance is not zero.
- It can be seen from the closed loop gain that the nonzero output resistance increases the gain error.

# Design Examples

- **Many design problems are presented at the end of the chapter to study the effects of finite loop gain, restrictions on peak to peak swing to avoid slewing, and how to design for a certain gain error.**