1 Introduction

The purpose of the two-part lab is to design digital state feedback regulators for a motor position control system, and for the inverted pendulum-on-a-cart system. You will perform MATLAB simulations as well as hardware experiments. Different designs will be compared by simulation, and the stability margins will also be compared.

2 Motor Position Control - Week 1

The hardware for this system is the same motor/cart that was used for Lab 1. You have already obtained a transfer function from volts into the power amplifier to the output of motor position in position-volts. Section 3.6.3 of the book shows a state-space model for this system where the first state variable is motor position, and the second state variable is motor velocity. Use the rule of thumb given in class to choose an appropriate sampling interval. Use the ZOH equivalent plant model to calculate state feedback gain vectors $L$ corresponding to different values of $T_S$, the desired settling time. In the control system shown below, a step input for motor position is introduced into the regulator structure. Because the state-space block should send the state vector (both state variables) as outputs, the Simulink block for the Motor System should have $C: \text{eye}(2)$ (a $2 \times 2$ identity matrix) and $D: [0;0]$. The feedback gain vector $L$ can be implemented in Simulink using a gain block (found in Math Operations). However, you must change the Multiplication field from its default value of Elementwise to Matrix($K*u$).

In the second control system, shown on the next page, shaped inputs for position ($\text{ref}$) and velocity ($\text{rvel}$) are introduced into the regulator structure using From Workspace blocks as described in Section 3.2 of the Lab 3 handout. The signals $\text{ref}$ and $\text{rvel}$ are computed in Matlab using sinusoidal signals as described in the Lab 3 handout, with the following changes:

- Set $H = 3$ to get a position command of 3 position volts.
- Choose $D$ equal to the desired duration of the step response. Then solve equation (4) in the Lab 3 handout for $A$, the maximum acceleration parameter. The value of $D$ should be varied to get the smallest value for which the plant input does not exceed 5 volts and for which the plant output does not have overshoot.
- To calculate the closed-loop regulator poles, choose $T_s = D/3$. By making the regulator poles “3-times faster” than the duration of the reference input, the regulator response should closely follow the reference signal and end up at the destination in about $D$ seconds.
- Note that a new value of sampling interval $T$, different from that used for the step-input regulator, will have to be computed based on $T_s$. This value of $T$ should also be used for computing $\text{ref}$ and $\text{rvel}$. 

![Figure 1: State-feedback regulator with step input for motor position](image_url)
3 Inverted Pendulum-on-a-Cart - Week 2

A linearized model for this system is given in equation (3.138) of the book, and the parameter values are shown in Table 3.9. Use this model but replace the values of $C$ and $D$ as follows. Assume that the transfer function of the motor system found in Lab 1 is

$$\frac{\beta}{s + \alpha}.$$  

Replace the values of $D$ with 20 times $\beta$, and replace $C$ with $\alpha$. The reason that we multiply $\beta$ by 20 is to make motor position have units of radians in the cart/pendulum model. The pendulum angle is also measured in radians. For the hardware station closest to the window, use the value $A = 23.1$ in equation (3.138). (Note, call this variable something else in Matlab, because $A$ is reserved for the state-space matrix of the continuous time plant model.) For the other hardware station, use $A = 41.5$. (The second hardware station uses a shorter pendulum than the first station, and the value of $A$ is the square of the radian natural frequency of the pendulum.) The four state variables of the plant are:

- $x_1 =$ pendulum position (rad)
- $x_2 =$ pendulum velocity (rad/sec)
- $x_3 =$ motor position (rad)
- $x_4 =$ motor velocity (rad/sec)

Use a sampling interval of $T = 0.01$ seconds. We would like to compare two different regulators, each designed to achieve a different set of closed-loop poles. The regulators are implemented as shown in Fig. 3. The first set of closed-loop poles are the roots of a 4th-order Bessel polynomial, scaled by $T_S$ to meet the constraints given below. The second set of desired closed-loop poles is defined as follows: the plant pole that is furthest into the left half-plane and the roots of a 3rd-order Bessel polynomial scaled by $T_S$.

1. The roots of the Bessel polynomials should be scaled to achieve the following constraints. An initial condition of 0.17 radians ($10^\circ$) (with all other state variables zero) results in a control signal less than 5 volts in amplitude.
2. Obtain plots of motor and pendulum positions as well as the input to the plant in response to the two initial conditions given above.

3. Now design a regulator with the second set of desired closed-loop poles: the plant pole that is furthest into the left half-plane and the roots of a 3rd-order Bessel polynomial scaled by $T_S$. You will have to find a value of $T_S$ to satisfy the constraint on the plant input with the initial condition given in (1) above. Obtain plots as in (2) for this regulator.

4. Compare the stability margins of the two different regulators.

4 Write Up

A single lab report is due after the completion of Week 2. The report should contain results from both Week 1 and Week 2.

Week 1

In the first week of this lab, two control systems were designed. The results for each control system include

- The value of the sampling interval, $T$, and how it was obtained.
- The choice of closed-loop pole locations (in the $s$-plane).
- A graph of the simulated step response (plant input and output).
- The settling time achieved by the simulated control system. (Note that the value of $T_S$ used to calculate the desired closed-loop poles is not equal to the achieved settling time for these systems.)
- For the second control system, a graph of the response of the hardware system (plant input and output).

Week 2

The second week of the lab dealt with the pendulum on a cart. The results for week 2 include

- Two different sets closed-loop pole locations and a $T_S$ parameter to scale the speed of response without violating the input constraint. Describe how each set of closed-loop poles, along with the appropriate value of $T_S$, was obtained.
- The stability margins of the two regulators (compare these).
- Simulation of the cart/pendulum system that can be compared with the measured data. The appropriate initial condition vector to simulate a tap on a balanced pendulum (as was done in the lab) has a nonzero initial velocity, as follows:

$$\mathbf{x}_0 = \begin{bmatrix} 0 \\ 0.17 \\ 0 \\ 0 \end{bmatrix}.$$  

Compare this simulation result with a plot from the hardware system.
- Explain how the control system was implemented on the lab computer, what the difficulties were, and how the problems were solved. In particular, describe the problems with noisy state-variable measurements and how this affected the final design.