ELE/MCE 504 Homework #1 Spring '09

Due Wednesday, February 4

1. This problem is taken from the book, problem 2.5. Consider the following state-space model:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \mathbf{x}(t).$$

   (a) Find the poles and zeros (use Matlab, `eig` and `tzero`).

   (b) Plot the principal gains of the system (use Matlab, `sigma`).

2. This problem is taken from the book, problem 2.6. Consider the following state-space model:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \mathbf{u}(t)$$
$$\mathbf{y}(t) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{x}(t).$$

   (a) Find the poles and zeros (use Matlab, `eig` and `tzero`).

   (b) Plot the principal gains of the system (use Matlab, `sigma`).

   (c) Simulate the output when the input is

$$\mathbf{u}(t) = \begin{bmatrix} 0 \\ \cos(t) \end{bmatrix}.$$

   Estimate the gain from $\mathbf{u}(t)$ to $\mathbf{y}(t)$. How does this number compare to the principal gains?

   (d) Simulate the second output signal $y_2(t)$ when the input is

$$\mathbf{u}(t) = \begin{bmatrix} \cos(t) \\ 0 \end{bmatrix}.$$

   Estimate the gain from $\mathbf{u}(t)$ to $y_2(t)$. Compare this number with the principal gains at $\omega = 1$ rad/sec and explain how the result makes sense.

3. This problem is taken from the book, Computer Exercise 2.3. An autopilot for a ship is tasked with maintaining the ship's heading. A very simple mathematical model of the ship is given by the following differential equations:

$$M\ddot{\theta}(t) = -d\dot{\theta}(t) - c\alpha(t)$$
$$\dot{\alpha}(t) = -0.1\alpha(t) + 0.1\alpha_c(t)$$

   where $\theta(t)$ is the heading error (the angle between the ship's true heading and the desired heading, $\alpha(t)$ is the rudder angle, and $\alpha_c(t)$ is the commanded rudder angle (this is the system input, $u(t)$). In what follows, use an initial heading-rate error of $\dot{\theta}(0) = 0.1$, $M = 10^7\ kg\text{-}m^2$, $d = 10^6\ N\text{-}m\text{-}sec$, and $c = 5000\ N\text{-}m/rad$.

(a) Obtain a state-space model using the following vector of state variables:

$$\mathbf{x}(t) = \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \\ \alpha(t) \end{bmatrix}.$$

(b) Let the desired settling time be 200 seconds. Use Matlab (`fbg`) to calculate a feedback gain vector $\mathbf{K}$ to place the closed-loop poles at: `[-0.1 s2/Ts]`, where `s2` is a vector containing the roots of the normalized second-order Bessel polynomial. Use `areg` to simulate and `aregp` to obtain plots of $\theta(t)$ and $\alpha(t)$ on two different graphs.

(c) Suppose now that only $\theta(t)$ is measured; that is, $y(t) = [1 \quad 0 \quad 0]\,\mathbf{x}(t)$. Use `fbg` to calculate an observer gain vector `G` to place the observer poles at $s3/T_{so}$, where $T_{so} = Ts/5$.

(d) Use `aregob` and `aregobp` to simulate and plot the response of the observer-based regulator using the feedback gain vector from part (c). Show the estimated state variables on the same plot as the corresponding actual plant state variables (for $\theta(t)$ and $\alpha(t)$ only).

4. The poles of an observer-based analog regulator are the eigenvalues of

$$\mathbf{M} = \begin{bmatrix} \mathbf{A} & -\mathbf{BK} \\ \mathbf{GC} & \mathbf{A} - \mathbf{GC} - \mathbf{BK} \end{bmatrix}.$$

It is know that the eigenvalues of a matrix $\mathbf{M}$ and the matrix $\mathbf{TMT}^{-1}$ are identical, for any nonsingular matrix $\mathbf{T}$. Consider the following matrix:

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{I} & -\mathbf{I} \end{bmatrix}.$$

(a) Show that $\mathbf{T}^{-1} = \mathbf{T}$. [Hint: show that $\mathbf{T} * \mathbf{T} = \mathbf{I}$.]

(b) Show that $\mathbf{T} * \mathbf{M} * \mathbf{T}^{-1}$ is a block upper triangular matrix whose $2n$ eigenvalues are the n eigenvalues of $\mathbf{A} - \mathbf{BK}$ and the n eigenvalues of $\mathbf{A} - \mathbf{GC}$.

5. We showed in class that the maximum and minimum values of

$$\frac{\|\mathbf{G}(j\omega)\boldsymbol{\alpha}\|}{\|\boldsymbol{\alpha}\|}$$

are σ_1 and σ_p, respectively. If we take the singular value decomposition of $G(j\omega)$ for any frequency ω:

$$G(j\omega) = \mathbf{U}(\omega)\boldsymbol{\Sigma}(\omega)V^H(\omega),$$

then we know that the maximum value of $\|\mathbf{G}(j\omega)\boldsymbol{\alpha}\|/\|\boldsymbol{\alpha}\|$ is obtained by setting $\boldsymbol{\alpha}$ equal to the first column of $\mathbf{V}(\omega)$, and the minimum value is obtained by setting $\boldsymbol{\alpha}$ equal to the last column. Use the system from Problem 2 and verify this numerically in Matlab at frequency $\omega = 1$ rad/sec. Compare the results of this problem with the principal gains plotted in Problem 2.