1. Listed below are state-space models for state-feedback regulators. For each system, obtain a Nyquist plot and zoom in to see clearly the critical region near the point at -1. Put shading on the plot and label the points that are used to get the gain and phase margins. Give the numerical values of these margins.

   (a)

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ \mathbf{K} = [\, 21.9024 \quad 7.1060 \,]$$

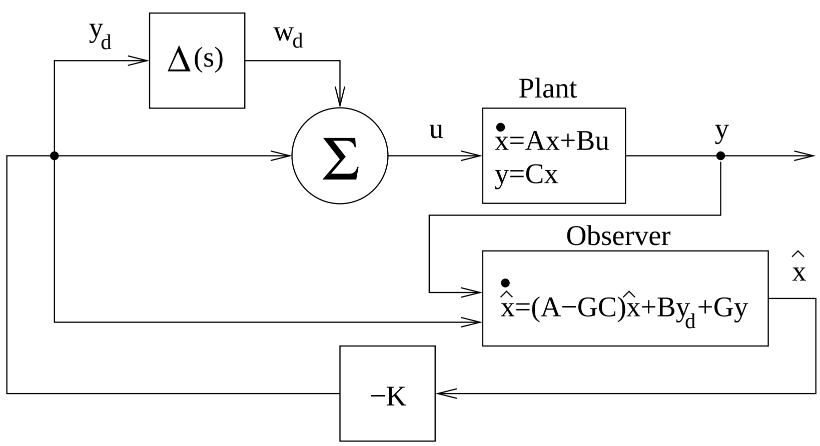
   (b)

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ \mathbf{K} = [\, 21.9024 \quad 9.1060 \,]$$

   (c)

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 23.1 & 0 & 0 & -0.1189 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -25 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ 12.5 \\ 0 \\ 26.33 \end{bmatrix}, \ \mathbf{K} = [\, 3.1582 \quad 0.6720 \quad -2.2914 \quad -0.5436 \,]$$

2. The figure below shows an observer-based regulator with input-multiplicative plant uncertainty.



Notice that the inputs to the observer are the measured plant output, $\mathbf{y}$, and the signal that is sent *to* the plant, $\mathbf{y}_d$. The actual plant input $\mathbf{u}$ is different from $\mathbf{y}_d$ unless $\Delta(s) = 0$. The stability robustness of this control system is assessed by finding the ∞ norm of the system with input $\mathbf{w}_d$ and output $\mathbf{y}_d$. Derive a state-space model for this system. Use the state vector

$$\begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix}.$$

3. Consider a MIMO plant with the following state-space model:

$$\dot{\mathbf{x}} = \begin{bmatrix} -0.8000 & -0.0006 & -13.2000 & 0 \\ 0 & -0.0140 & -16.6400 & -32.2000 \\ 1.0000 & -0.0001 & -1.6500 & 0 \\ 1.0000 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -19.0000 & -2.5000 \\ -0.6600 & -0.5000 \\ -0.1600 & -0.6000 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \mathbf{x}$$

A state-feedback gain matrix has been calculated to be:

$$\mathbf{K} = \begin{bmatrix} -0.2526 & 0.0024 & -0.2376 & -0.5288 \\ -0.0377 & -0.0002 & -0.3578 & 0.1039 \end{bmatrix}$$

Two different observer gain matrices have been calculated:

$$\mathbf{G}_1 = \begin{bmatrix} -0.0720 & 6.6444 \\ -46.1888 & 15.2563 \\ 3.5735 & -2.8600 \\ 5.2155 & -0.7052 \end{bmatrix}, \quad \mathbf{G}_2 = \begin{bmatrix} 31.6333 & -1.1155 \\ 132.6187 & 131.7369 \\ 10.3710 & -6.8511 \\ 5.1477 & -4.6285 \end{bmatrix}$$

(a) Find the stability robustness norm for the full-state feedback regulator.

(b) Verify that the observer poles corresponding to both observer gain matrices are the same.

(c) Find the stability robustness norms for the two different observer-based regulators.

4. This problem explores the relationships between classical stability margins and the system infinity norm approach.

(a) If $\delta_{\text{max}} = 1$ for a single-input control system, what are the classical stability margins (UGM, LGM, PM) for that system?

(b) If $\delta_{\text{max}} = 1$ for a multiple-input control system, how much gain and phase variation can occur simultaneously on each input channel before the system goes unstable? [Hint: consider the matrix transfer function $\Delta(s)$ to be a diagonal matrix with complex numbers on the main diagonal. The maximum singular value of such a matrix is the largest magnitude of the diagonal elements.]

5. For Problem 1 (c), compute (in Matlab) the eigenvalues of $\mathbf{A} - \mathbf{B}q\mathbf{K}$ for different values of the number q and perform a search to determine the following.

(a) Estimate the largest value of a real number q such that the eigenvalues all have negative real parts.

(b) Estimate the smallest value of a real number $q > 0$ such that the eigenvalues all have negative real parts.

(c) Let $q = e^{-j\phi}$. Find the largest value of ϕ such that the eigenvalues all have negative real parts.

(d) How do the results of (a)-(c) relate to the information given by a Nyquist plot for Problem 1 (c)?