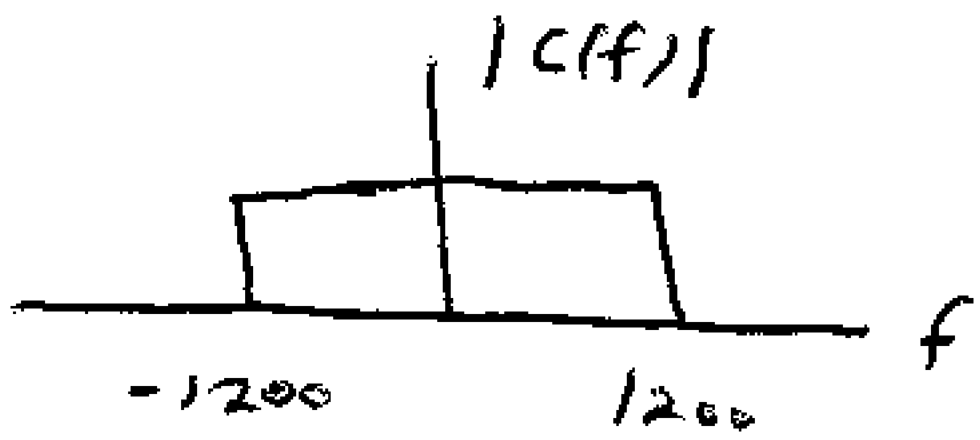
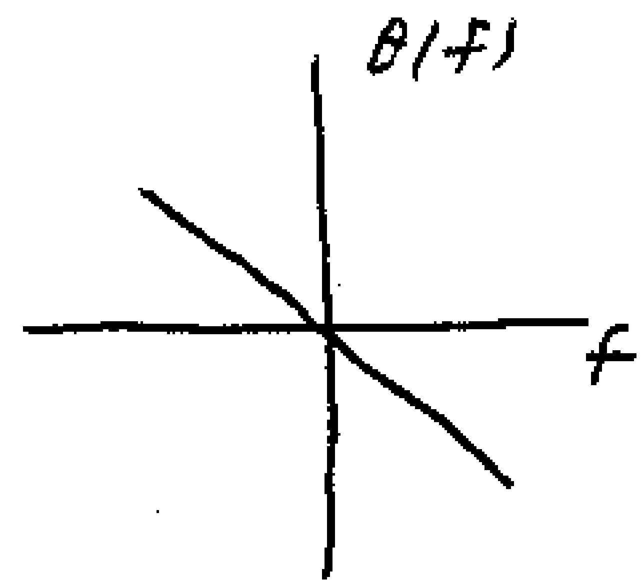
SET $X(f) = X_{RC}(f) \Rightarrow X_{RC}(f) = |G_T(f)|^2$

$$\therefore \quad G_T(f) = \sqrt{|X_{RC}(f)|}\; e^{-j2\pi f t_0}$$

WHERE t_0 CHOSEN TO MAKE $G_T(f)$ REALIZABLE (APPROXIMATELY)

EXAMPLE :





$$C(f) = |C(f)|\, e^{j\theta(f)}$$

$$\theta(f) = -2\pi f t_0$$

MAXIMUM SIGNALING RATE IS $1/T = 2W = 2400$ SYMBOLS/SEC

A MORE PRACTICAL RATE IS $\frac{1}{2T} + \frac{\alpha}{2T} = W$

WITH $\alpha = \frac{1}{2} \Rightarrow 1/T = \frac{2W}{1+\alpha} = \frac{2W}{3/2} = \frac{4}{3}W = 1600$.

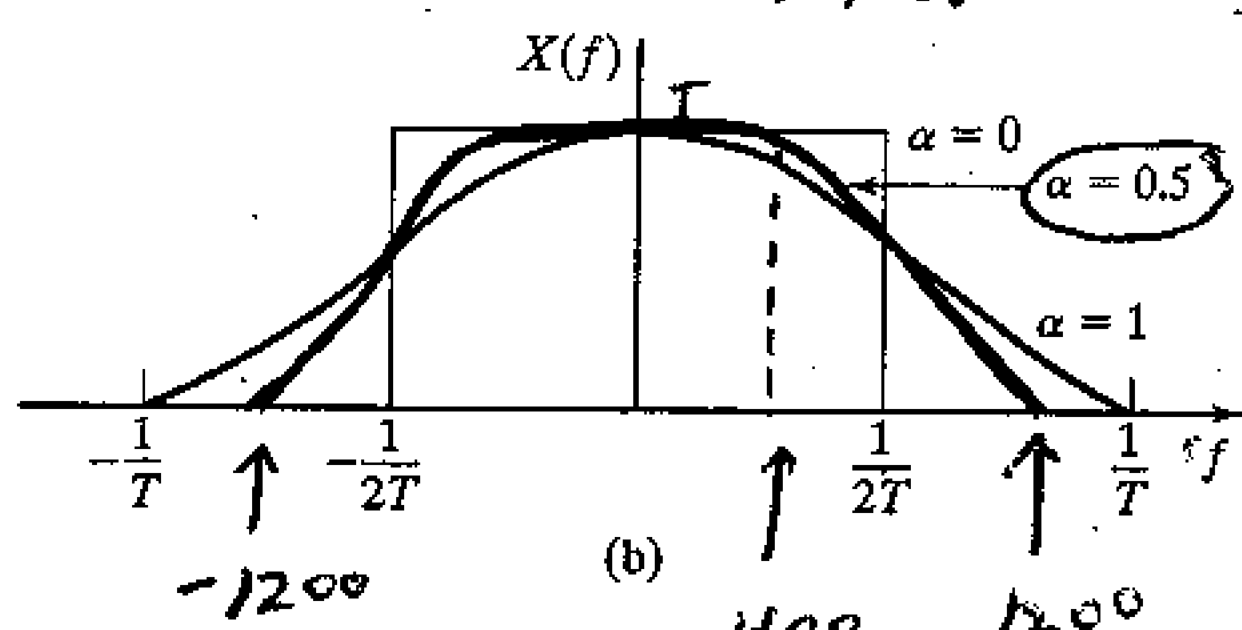


Figure 9.9 Pulses having a raised cosine spectrum.

$$X_{rc}(f) = \begin{cases} T, & 0 \le |f| \le 400 \\ \frac{T}{2}\left[1 + \cos\left(\frac{\pi}{800}(|f| - 400)\right)\right], & 400 \le |f| \le 1200 \\ 0, & |f| \ge 1200 \end{cases}.$$

SYSTEM DESIGN - CHANNEL DISTORTION

FOR NO ISI $\quad G_T(f)\, C(f)\, G_R(f) = X_{rc}(f)$,

BUT NOW ASSUME CHANNEL IS NOT IDEAL.

$$C(f) = |C(f)|\, e^{j\theta_c(f)}$$

$|C(f)|$: NOT A CONSTANT WITH FREQ.

$\theta_c(f)$: NOT LINEAR $(\neq -2\pi f t_0)$

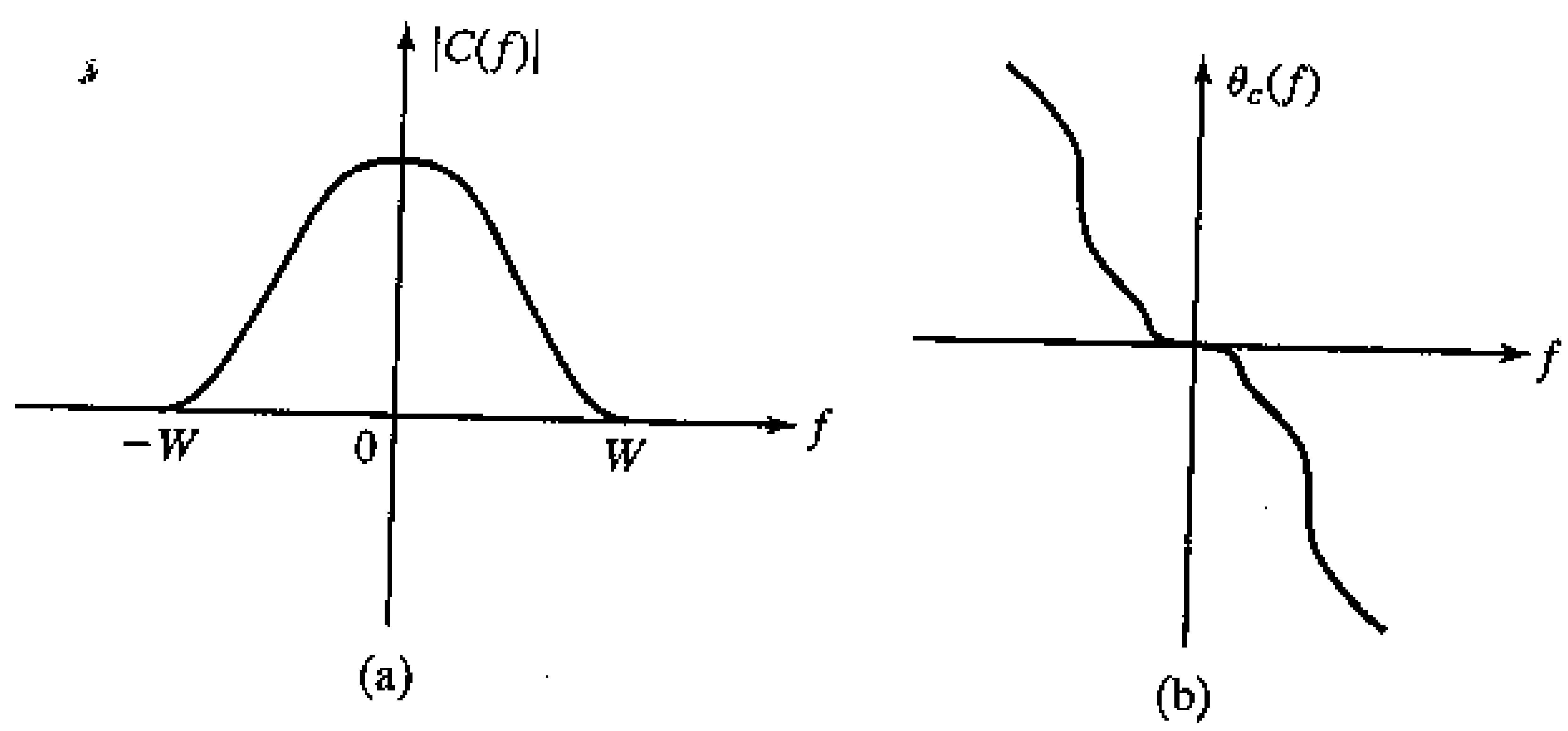
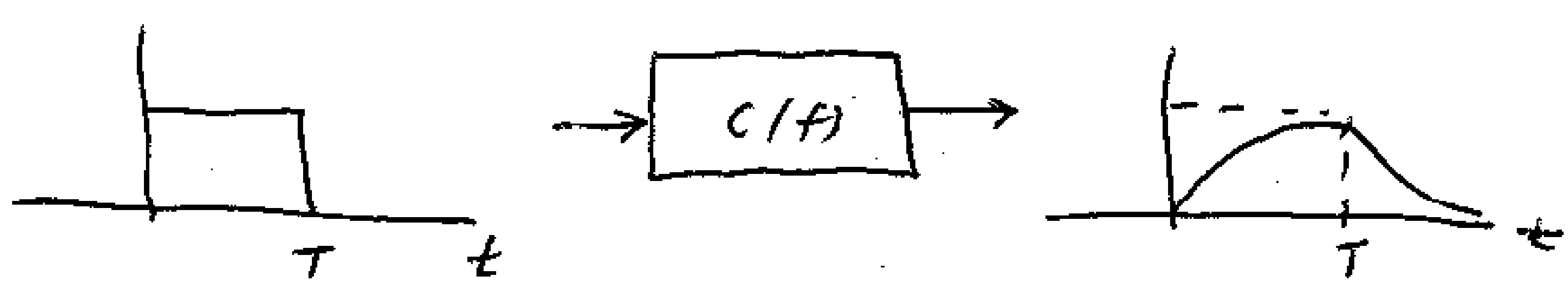


Figure 9.15 Channel characteristics illustrating (a) amplitude distortion and (b) phase distortion.

AMPLITUDE DISTORTION - HIGHER FREQUENCY COMPONENTS, AS AN EXAMPLE, ARE ATTENUATED

$\Rightarrow$ SIGNAL IS SMOOTHED



PHASE DISTORTION - CAN BE VIEWED MORE EASILY BY CONSIDERING TIME DELAY VS FREQUENCY OF CHANNEL

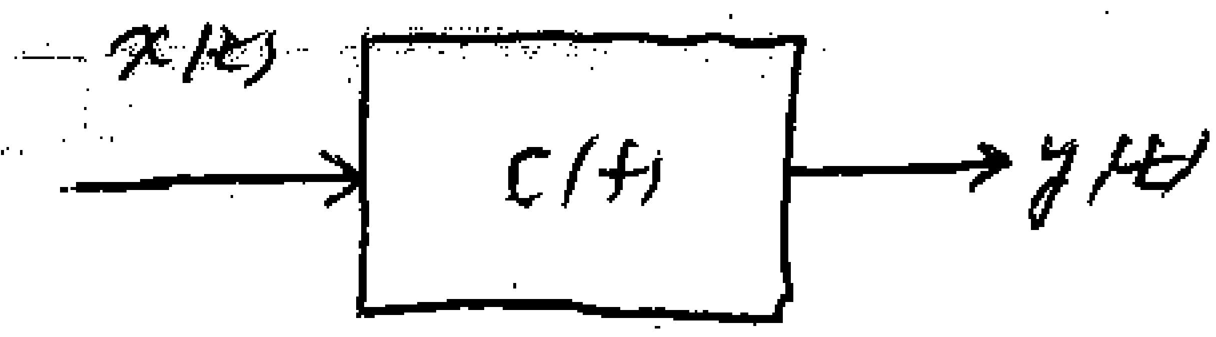
$$\tau(f) = -\frac{1}{2\pi}\frac{d\,\theta_c(f)}{df}$$

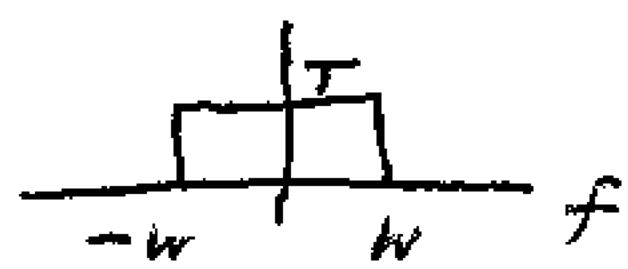
$$= -\frac{1}{2\pi}\frac{d}{df}(-2\pi f t_0) = t_0 \qquad |f| \le W$$

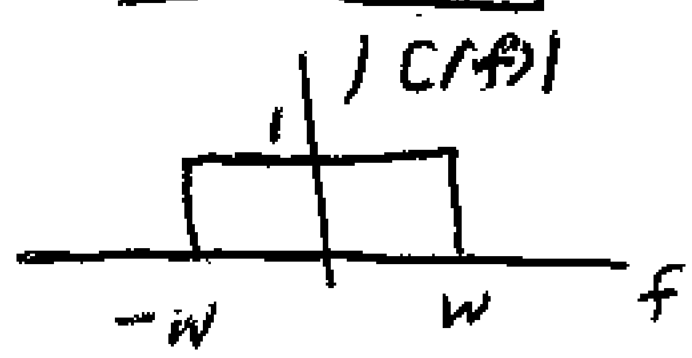
FOR IDEAL CHANNEL.

FOR NONIDEAL CHANNEL, $\theta_c(f)$ IS NONLINEAR $\Rightarrow$ $\tau(f)$ DEPENDS ON f $\Rightarrow$ FREQ. COMPONENTS "ARRIVE" AT DIFFERENT TIMES. ALSO, CALLED DELAY DISTORTION

CONSIDER A SINC PULSE INPUT, $x(t) = \frac{\sin 2\pi W t}{2\pi W t}$

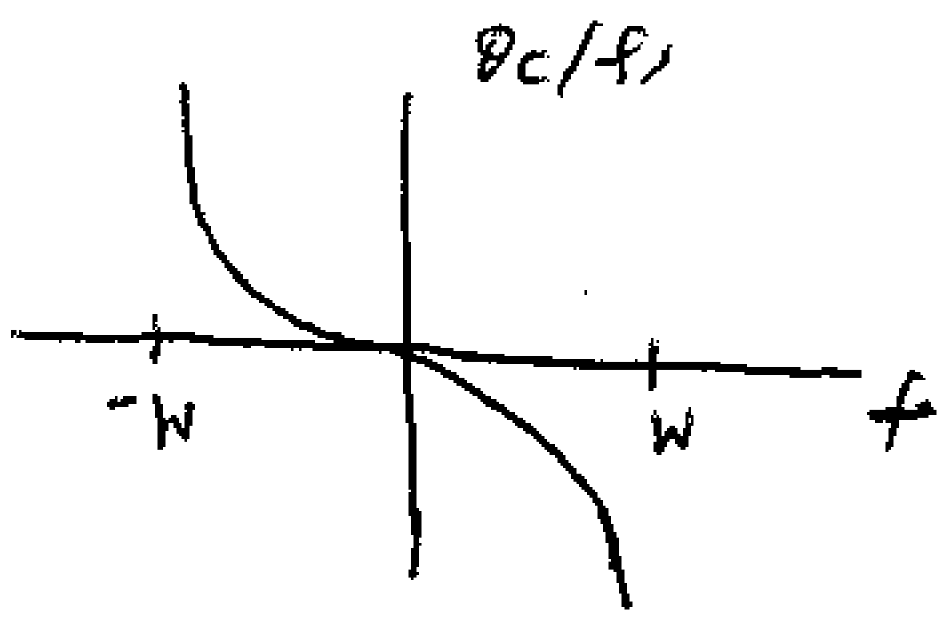




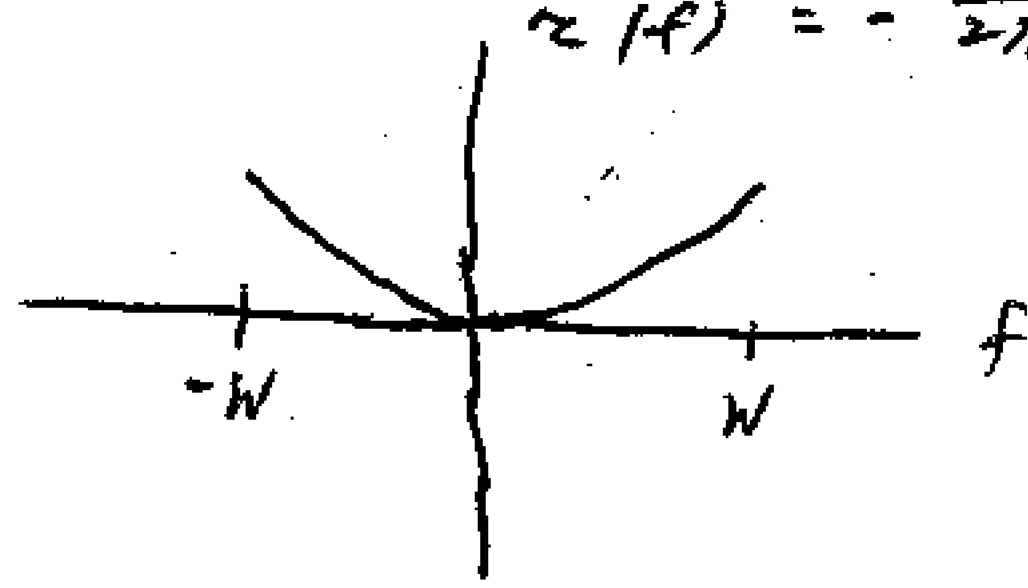


NO AMPLITUDE DISTORTION

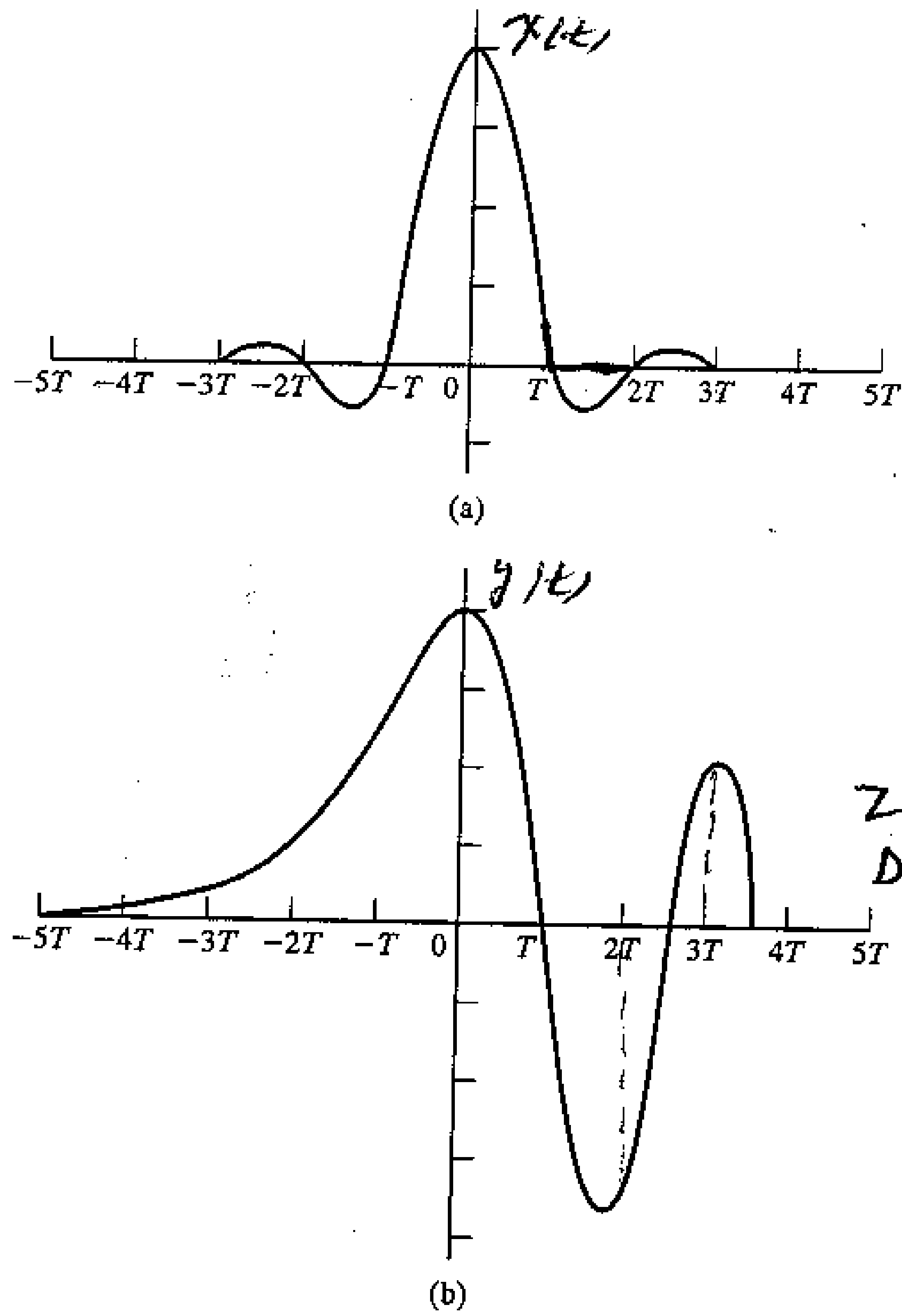
$$\tau(f) = -\frac{1}{2\pi}\frac{d}{df}(-2\pi f^3)$$
$$= 3f^2$$



PHASE DISTORTION



DELAY DISTORTION



$T = \frac{1}{2W}$

ZEROS ARE DISPLACED.

Figure 9.16 Effect of channel distortion in (a) channel input and (b) channel output.

DESIGN OF $G_R(f)$, $G_T(f)$

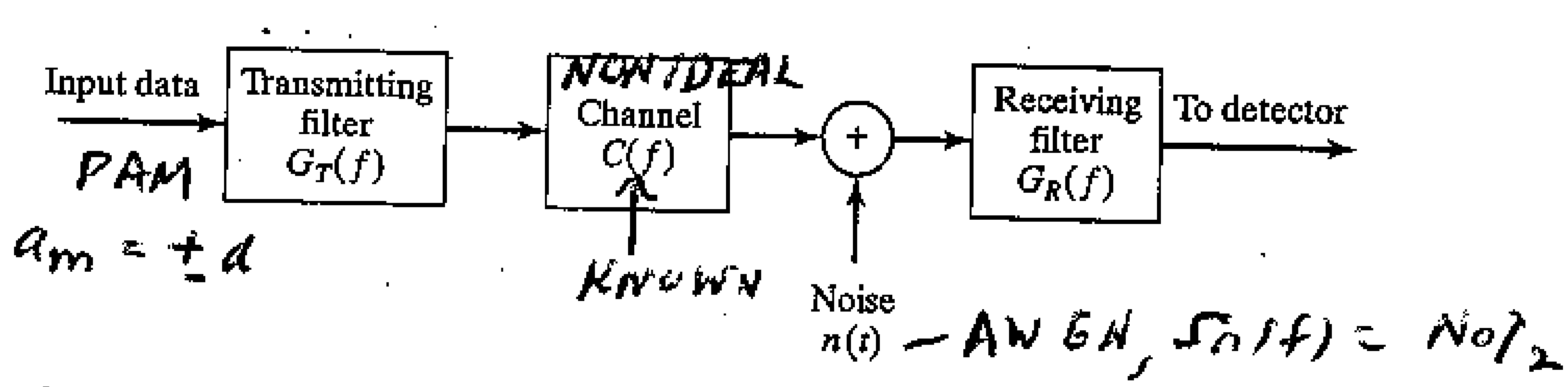


Figure 9.17 System configuration for the design of $G_T(f)$ and $G_R(f)$.

HOW DO WE CHOOSE $G_T(f)$, $G_R(f)$ IF CHANNEL IS NONIDEAL BUT KNOWN ?

WHAT IS LOSS DUE TO CHANNEL ?

APPROACH:

1) GUARANTEE NO ISI

$$\Rightarrow \quad G_T(f)\,C(f)\,G_R(f) = X_{RC}(f)\,e^{-j2\pi f t_0} \qquad |f| \le W$$

(t_0 CHOSEN FOR REALIZABILITY)

2) CHOOSE

$$G_T(f)\,C(f) = \sqrt{X_{RC}(f)}\;e^{-j2\pi f t_0} \qquad \text{WHY?}$$

3) CHOOSE

$$G_R(f) = \sqrt{X_{RC}(f)}\;e^{-j2\pi f t_R} \qquad \text{WHY?}$$

(NOTE: THIS IS A MATCHED FILTER, MATCHED TO CHANNEL <u>OUTPUT</u>)

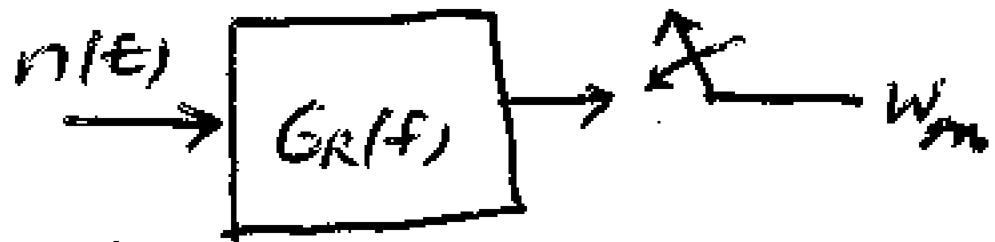
TO DETERMINE LOSS DUE TO NONIDEAL CHANNEL, WITH NO ISI,

$$y_m = a_m + w_m = \pm d + w_m$$

DECIDE $+d$ IF $y_m > 0$ (EQUIPROBABLE SIGNALS)
$-d$ IF $y_m < 0$

$$y_m \sim N(\pm d, \sigma_W^2)$$

WHERE

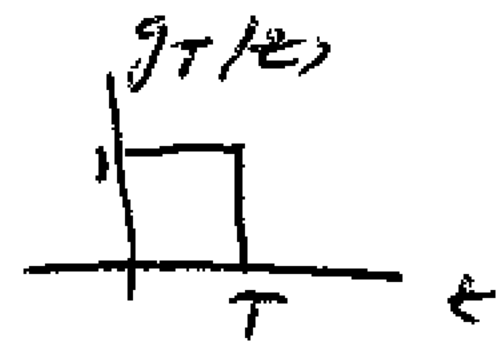


$$\sigma_W^2 = \int_{-\infty}^{\infty} |G_R(f)|^2 \frac{N_0}{2}\, df = \frac{N_0}{2} \int_{-W}^{W} X_{RC}(f)\, df$$

$$P_2 = Q\left(\frac{d-0}{\sigma_W}\right) = Q\left(\sqrt{d^2/\sigma_W^2}\right)$$

PERFORMANCE IMPROVES WITH INCREASING d^2/σ_W^2. HOLDING AVERAGE TRANSMIT POWER FIXED, WE CAN FIND DEGRADATION.

RECALL THAT $s(t) = \underset{\pm d}{a_m}\, g_T(t)$



$$\text{AVERAGE POWER} = \underset{P_{AV} =}{} E[a_m^2]\ \frac{1}{T}\int_{-\infty}^{\infty} g_T^2(t)\, dt$$

$$= \frac{d^2}{T} \int_{-W}^{W} |G_T(f)|^2\, df = \frac{d^2}{T} \int_{-W}^{W} \frac{X_{RC}(f)}{|C(f)|^2}\, df$$

$$\Rightarrow \quad d^2 = \frac{P_{AV}\, T}{\int_{-W}^{W} \frac{X_{RC}(f)}{|C(f)|^2}\, df}$$

ALSO, $\sigma_W^2 = \frac{N_0}{2} \int_{-W}^{W} X_{RC}(f)\, df$

$$= \frac{N_0}{2} \underbrace{\int_{-W}^{W} X_{RC}(f)\, e^{j2\pi f t}\, df \Big|_{t=0}}_{x_{RC}(0) = 1}$$

$$= N_0/2$$

$$\frac{d^2}{\sigma_W^2} = \frac{2 P_{AV} T / N_0}{\int_{-W}^{W} \frac{X_{RC}(f)}{|C(f)|^2}\, df}$$

FOR IDEAL CHANNEL $|C(f)| = 1 \Rightarrow \frac{d^2}{\sigma_W^2} = \frac{2 P_{AV} T}{N_0}$

THUS DEGRADATION IN "SNR" d^2/σ_W^2 IS

$$10 \log_{10} \int_{-W}^{W} \frac{X_{RC}(f)}{|C(f)|^2}\, df \quad dB$$

NOTE THAT THERE IS NO LOSS DUE TO PHASE DISTORTION SINCE THE TRANSMIT FILTER IS

$$G_T(f) = \frac{\sqrt{X_{RC}(f)}}{C(f)} e^{-j 2\pi f t_0}$$

$$= \frac{\sqrt{X_{RC}(f)}}{|C(f)|\, e^{j\theta(f)}} = \frac{\sqrt{X_{RC}(f)}}{|C(f)|} e^{-j2\pi f t_0 - j\theta(f)}$$

PHASE COMPENSATION

WHERE DOES LOSS COME FROM?
ASSUMING $|C(f)| < 1$ (NO AMPLIFIERS IN CHANNEL) AND $X_{RC}(f)$ IS FIXED

$$P_{AV} = \text{CONSTANT} = \frac{d^2}{T} \int_{-W}^{W} \frac{X_{RC}(f)}{|C(f)|^2}\, df$$

AS $|C(f)|$ DECREASES, $\int_{-W}^{W} \frac{X_{rc}(f)}{|C(f)|^2}\,df$

INCREASES AND SO d^2 MUST BE DECREASED TO MAINTAIN AVERAGE TRANSMIT POWER.

NOTE: WOULD BE BETTER TO OPTIMIZE OVER CHOICE OF $G_T(f)$ (WAS JUST SELECTED WITHOUT JUSTIFICATION)

CHANNEL EQUALIZATION

WHAT DO WE DO IF $C(f)$ IS UNKNOWN? COULD EVEN BE TIME VARYING, FOR EXAMPLE, TELEPHONE CALL SETUP - CONNECTIONS SWITCHED AS NEEDED, OR CELL PHONES FROM CARS THAT ARE MOVING.

ONE APPROACH IS TO USE EQUALIZING FILTER AT RECEIVER, WHICH MAY BE ADAPTIVE (TO ACCOMMODATE TIME-VARYING CHANNELS)

WITHOUT EQUALIZER WE HAVE

$$y_m = x_0 a_m + \sum_{\substack{n=-\infty \\ n \neq m}}^{\infty} a_n x_{m-n} + w_m$$

OR

$$y_m = \sum_{n=-\infty}^{\infty} a_n x_{m-n} + w_m$$

$$= \sum_{n=-\infty}^{\infty} x_n a_{m-n} + w_m$$

$$= x_0 a_m + \underbrace{\sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} x_n a_{m-n}}_{ISI} + w_m$$

THIS IS WHAT IS OBTAINED IF WE IGNORE EFFECT OF NONIDEAL CHANNEL BY LETTING $G_T(f) G_R(f) = X_{RC}(f) e^{-j2\pi f t_0}$.

USUALLY ISI ONLY EXTENDS INTO A FEW ADJACENT SYMBOL INTERVALS OR

$$y_m = x_0 a_m + \sum_{\substack{n=-L_1 \\ n \neq 0}}^{L_2} x_n a_{m-n} + w_m$$

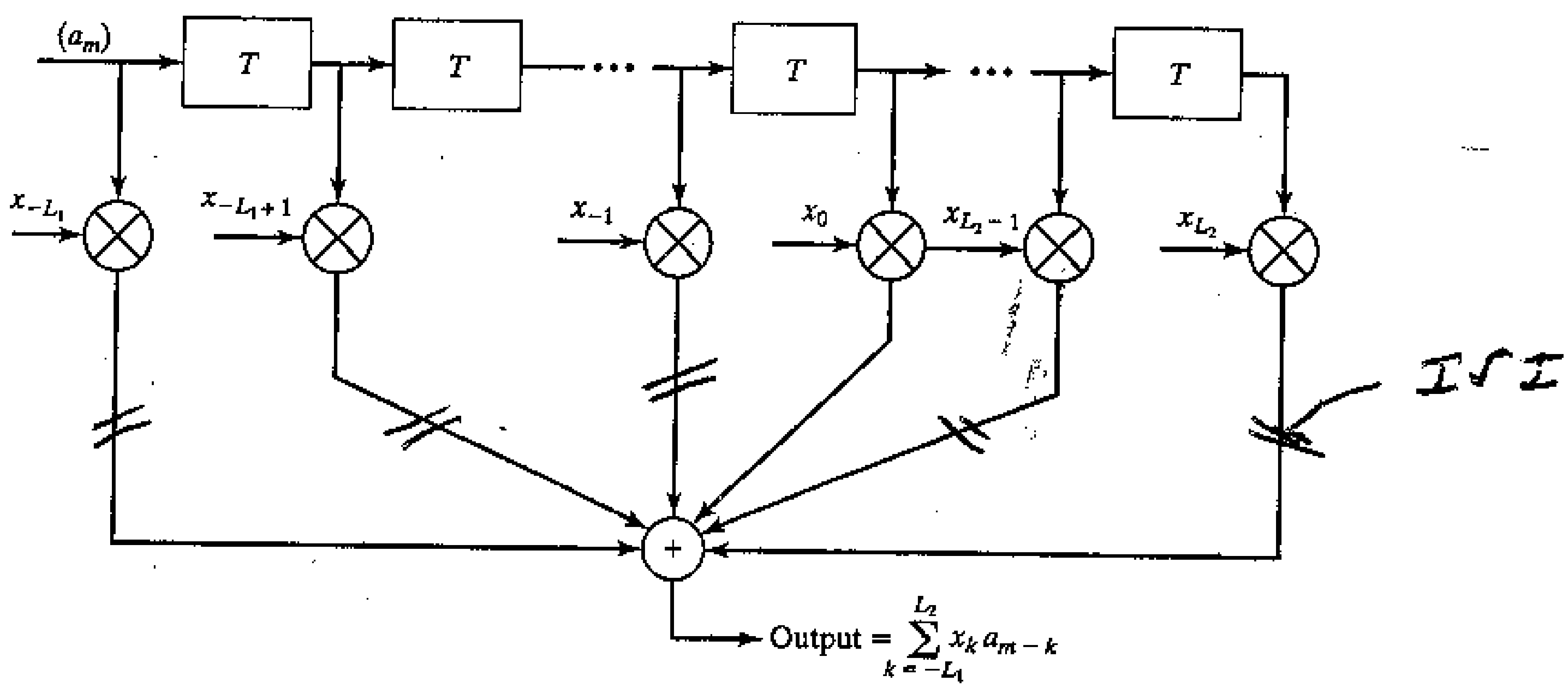


Figure 9.18 Equivalent discrete-time channel filter (JUST AN FIR FILTER)

$$\mathcal{H}(z) = \sum_{k=-L_1}^{L_2} x_k z^{-k}$$

WANT $\mathcal{H}(z) = x_0$ OR $x_k = 0$ FOR $k \neq 0$.

ONE POSSIBLE APPROACH IS TO INSERT EQUALIZING FILTER AT RECEIVER,

$$y_m \longrightarrow \boxed{\frac{1}{\mathcal{H}(z)}} \longrightarrow$$

$y_m = \sum_{n=-L_1}^{L_2} x_n a_{m-n} + w_n$

IIR FILTER NOT PRACTICAL, ALSO WILL ENHANCE NOISE

$$Y(z) = \underbrace{\mathcal{H}(z)}_{\uparrow \text{ UNDO THIS}} A(z) + W(z)$$

BETTER PRACTICAL SOLUTION IS FIR ANALOG EQUALIZER, $G_E(z) = \sum_{n=-N}^{N} c_n z^{-n}$ OR

$$g_E(t) = \sum_{n=-N}^{N} c_n \delta(t - nT)$$

SO THAT AT THE OUTPUT OF EQUALIZER

$$q(t) = x(t) * g_E(t)$$

$$= x(t) * \sum_{n=-N}^{N} c_n \delta(t - nT)$$

$$= \sum_{n=-N}^{N} c_n x(t - nT)$$

AND WHEN SAMPLED

$$q(mT) = \sum_{n=-N}^{N} c_n \, x(mT - nT)$$

$$\stackrel{?}{=} \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}$$

$y(t) = x(t) + w(t)$

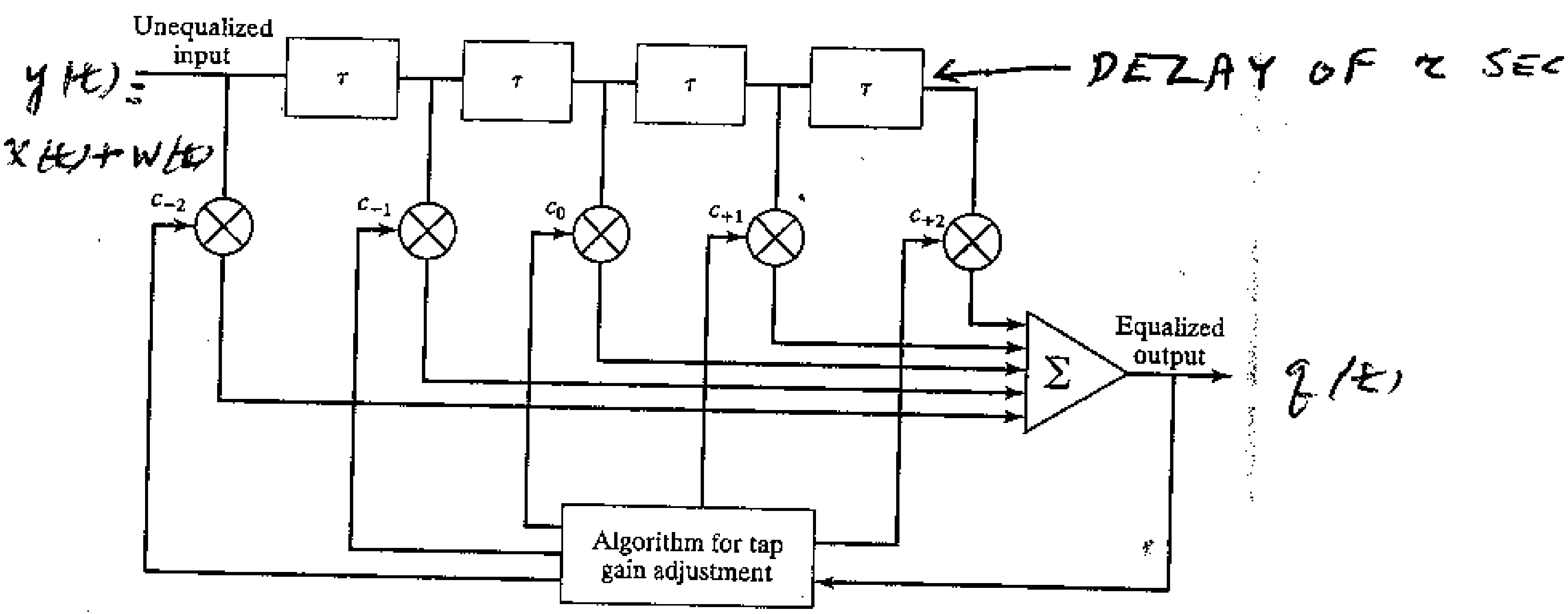


Figure 9.20 Linear transversal filter.

$\tau = T$ OR $T/2$ (FRACTIONALLY SPACED TAPS)

NOT POSSIBLE TO HAVE $q(mT) = 0$ FOR ALL $m \neq 0$. INSTEAD, FORCE $q(mT) = 0$ FOR $m = \pm 1, \pm 2, \dots, \pm N$ AND $q(0) = 1$
$\Rightarrow$ CALLED ZERO-FORCING EQUALIZER.

HAVE $2N+1$ COEFFICIENTS AND $2N+1$ CONSTRAINTS $\Rightarrow$ CAN NOW SOLVE FOR c_n's.

EXAMPLE: $\quad q(mT) = \sum_{n=-2}^{2} c_n x(mT - nT)$ $\qquad -2 \le m \le 2$

$$
\begin{array}{c|c}
m \backslash n & \begin{array}{ccccc} -2 & -1 & 0 & 1 & 2 \end{array} \\
\begin{array}{c} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{array} &
\underbrace{\begin{bmatrix}
x(0) & x(-T) & x(-2T) & x(-3T) & x(-4T) \\
x(T) & x(0) & x(-T) & x(-2T) & x(-3T) \\
x(2T) & x(T) & x(0) & x(-T) & x(-2T) \\
x(3T) & x(2T) & x(T) & x(0) & x(-T) \\
x(4T) & x(3T) & x(2T) & x(T) & x(0)
\end{bmatrix}}_{\underline{X}}
\underbrace{\begin{bmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{bmatrix}}_{\underline{c}}
=
\underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{\underline{q}}
\end{array}
$$

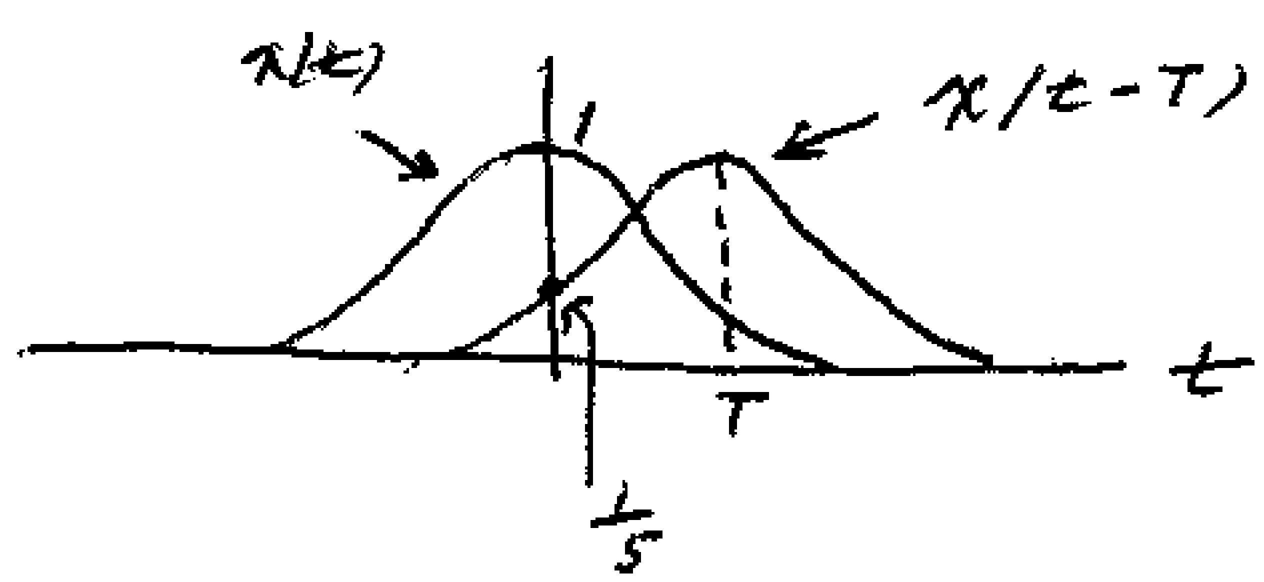
CAN EASILY SOLVE FOR C_n's.

EXAMPLE: FRACTIONALLY SPACED EQUALIZER $\tau = T/2$. SAMPLING RATE AT EQUALIZER OUTPUT IS $1/\tau = 2 \times \frac{1}{T}$ OR TWICE AS LARGE - REDUCES ALIASING FREQUENCIES IF $\alpha = 1$ USED FOR RAISED COSINE PULSE.

$$x(t) = \frac{1}{1 + (2t/T)^2}$$

$\frac{1}{T}$ = SYMBOL RATE

$2/T$ = OUTPUT RATE OF EQUALIZER



$$q(mT) = \sum_{n=-2}^{2} c_n x(mT - n\overbrace{T/2}^{\tau}) \qquad \tau = T/2$$

$$= 1 \qquad m = 0$$

$$ 0 \qquad m = \pm 1, \pm 2$$

NEED $x((m-n/2)T) = \dfrac{1}{1+\left[\dfrac{2(m-n/2)T}{T}\right]^2}$

$$= \frac{1}{1+(2m-n)^2} = [\underline{X}]_{mn}$$

$m = -2, \ldots, 2$

$n = -2, \ldots, 2$

FOR EXAMPLE $m=-2$, $n=-2$

$$[\underline{X}]_{-2\,-2} = \frac{1}{1+(-4+2)^2} = \frac{1}{5}$$

$$[\underline{X}]_{-2\,0} = \frac{1}{1+(-4-0)^2} = \frac{1}{17}$$

SEE BOOK FOR $\underline{X}$. SOLUTION IS

$$\underline{c}_{OPT} = \begin{bmatrix} -2.2 \\ 4.9 \\ -3 \\ 4.9 \\ -2.2 \end{bmatrix}$$

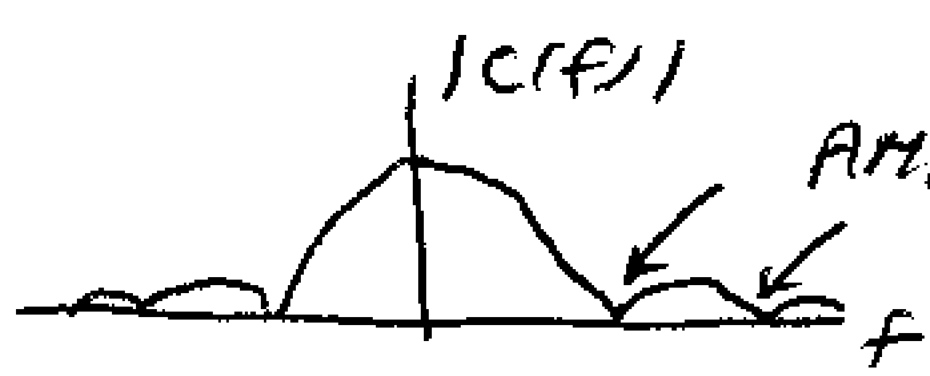
FREQ. RESPONSE?

MMSE EQUALIZERS

PROBLEM WITH PREVIOUS EQUALIZER - NO ACCOUNTING FOR NOISE. ATTEMPTS TO IMPLEMENT

$$G_E(f) = \frac{1}{C(f)}$$

IF



AMPLIFYS NOISE IN THESE BANDS

BETTER TO INCLUDE NOISE EFFECTS BY USING MEAN SQUARE ERROR AS DESIGN CRITERION.

OUTPUT OF EQUALIZER IS -

$$z(t) = \sum_{n=-N}^{N} c_n\, y(t - n\tau)$$

$$\Rightarrow \quad z(mT) = \sum_{n=-N}^{N} c_n\, y(mT - n\tau)$$

DESIRED OUTPUT IS TRANSMITTED SYMBOL a_m. HENCE, CHOOSE c_n's TO MINIMIZE MSE

$$MSE = E\left[(z(mT) - a_m)^2\right]$$

$$= E\left[\left(\sum_{n=-N}^{N} c_n\, y(mT - n\tau) - a_m\right)^2\right]$$

$$= E\Big[\sum_n \sum_k c_n c_k\, y(mT - n\tau)\, y(mT - k\tau)$$

$$- 2\sum_n c_n\, y(mT - n\tau)\, a_m + a_m^2\Big]$$

$$= \sum_n \sum_k c_n c_k\, E\left[y(mT - n\tau)\, y(mT - k\tau)\right]$$

$$- 2\sum_n c_n\, E\left[y(mT - n\tau)\, a_m\right]$$

$$+ E\left[a_m^2\right]$$

SINCE a_m IS IID SEQUENCE AND NOISE IS AWGN, CAN SHOW a_m, $y(t)$ ARE JOINTLY STATIONARY.

$$MSE = \sum_n \sum_k c_n c_k R_y(n-k) - 2 \sum_k c_k R_{AY}(k) + E(a_m^2)$$

WHERE $R_y(n-k) = E[y(mT-n\tau)\, y(mT-k\tau)]$
$R_{AY}(k) = E[y(mT-k\tau)\, a_m]$

TO FIND C_n'S DIFFERENTIATE

$$\frac{\partial MSE}{\partial c_\ell} = \sum_n \sum_k (c_n \delta_{k\ell} + \delta_{n\ell} c_k) R_y(n-k)$$

$$- 2 R_{AY}(\ell) + 0$$

$$= \sum_n c_n R_y(n-\ell) + \sum_k c_k R_y(\ell-k)$$

$$- 2 R_{AY}(\ell)$$

$$= 2 \left(\sum_n c_n R_y(n-\ell) - R_{AY}(\ell) \right) = 0$$

$$\ell = -N, \dots, N$$

$$\sum_{n=-N}^{N} c_n R_y(\ell-n) = R_{AY}(\ell) \qquad \ell = -N, \dots, N$$

$2N+1$ LINEAR EQUATIONS IN $2N+1$ UNKNOWNS $\Rightarrow$ SOLVE FOR C_n'S

TO FIND $R_Y(n)$, $R_{AY}(n)$ WE USE TEST SIGNALS (BEFORE DATA IS SENT) AND USE ESTIMATES

$$\hat{R}_Y(n) = \frac{1}{K} \sum_{k=1}^{K} y(kT - nw)\, y(kT)$$

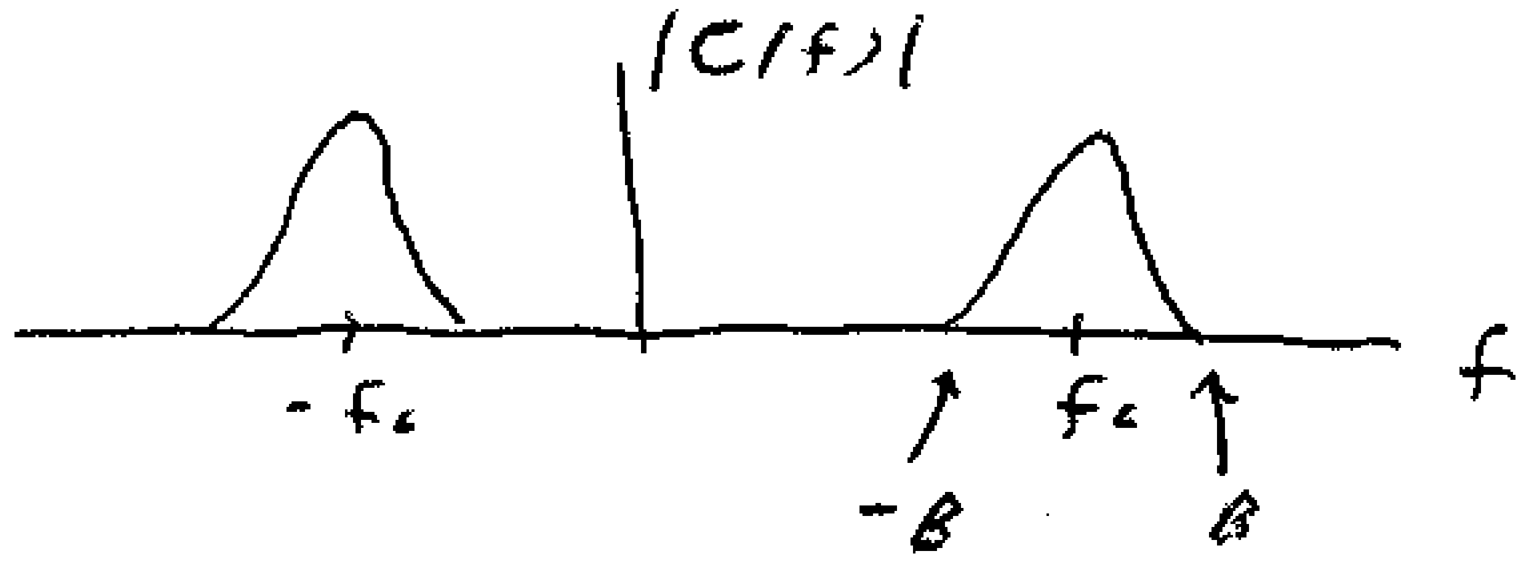
$$\hat{R}_{AY}(n) = \frac{1}{K} \sum_{k=1}^{K} y(kT - nw)\, a_k$$

KNOWN TO RECEIVER

SEE BOOK FOR ADAPTIVE VERSION OF THIS EQUALIZER.

CHAPTER 10 – CARRIER MODULATION
(10.1 – 10.5 - SELECTED TOPICS)

CONSIDERED BASEBAND CHANNELS - PASSBANDS WERE NEAR $f = 0$, E.G., WIRELINE. NOW CONSIDER <u>BANDPASS</u> CHANNELS, E.G., WIRELESS COMMUNICATIONS AT 2.4 Ghz ($G = 10^9$ = "GIGA")



f_c = 2.4 Ghz

B = ? Mhz

$f_c / B \approx 1000$

TO TRANSMIT THROUGH CHANNEL A BASEBAND SIGNAL:

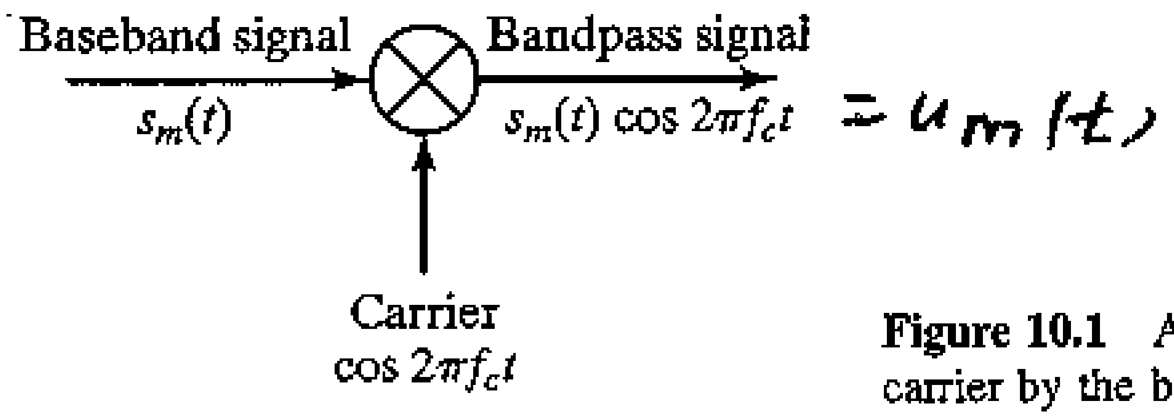


Figure 10.1 Amplitude modulation of the sinusoidal carrier by the baseband signal.

$$u_m(t) = s_m(t)\cos 2\pi f_c t \qquad f_c = \text{CARRIER FREQ.}$$

$$= s_m(t)\left(\tfrac{1}{2}e^{j2\pi f_c t} + \tfrac{1}{2}e^{-j2\pi f_c t}\right)$$

$$\Rightarrow \quad U_m(f) = \tfrac{1}{2}S_m(f-f_c) + \tfrac{1}{2}S_m(f+f_c)$$

THIS IS A DOUBLE SIDEBAND SUPPRESSED CARRIER AM SIGNAL.

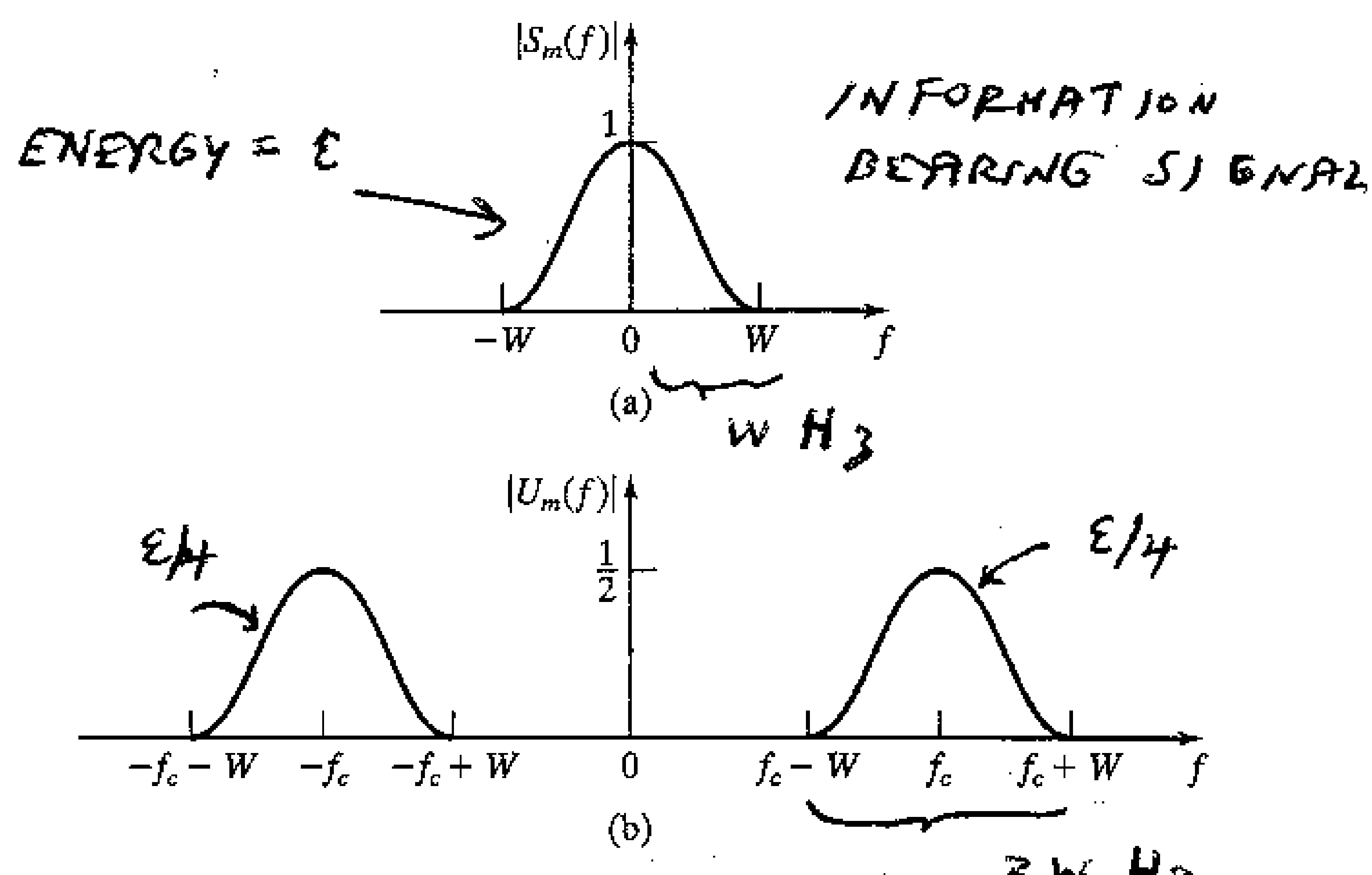
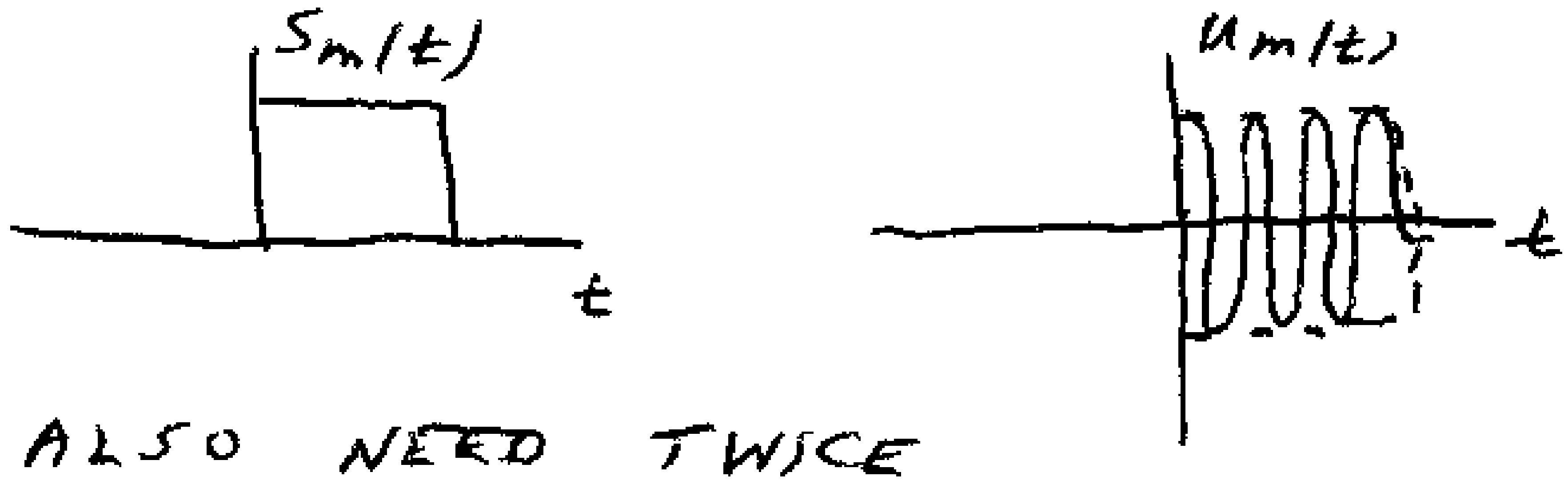


Figure 10.2 Spectra of (a) baseband and (b) amplitude-modulated signals.

$$\mathcal{E} = \int_{-\infty}^{\infty} |S_m(f)|^2\,df \left(= \int_{-\infty}^{\infty} s_m^2(t)\,dt\right)$$

$\frac{1}{2}$ OF ENERGY LOST DUE TO CARRIER



ALSO NEED TWICE THE BANDWIDTH.

GEOMETRIC REPRESENTATION

LET $s_m(t) = \sqrt{E_s}\, \psi_m(t) \qquad m = 1, 2, \ldots, M$

ORTHONORMAL BASEBAND

WANT SET OF ORTHONORMAL BANDPASS WAVEFORMS

$$\Rightarrow \quad u_m(t) = s_m(t) \cos 2\pi f_c t$$
$$= \sqrt{E_s}\, \psi_m(t) \cos 2\pi f_c t$$

ASIDE: $u_m(t)$'s STILL ORTHOGONAL SINCE

$$\int_{-\infty}^{\infty} \psi_i(t) \cos 2\pi f_c t \; \psi_j(t) \cos 2\pi f_c t =$$

$$\int_{-\infty}^{\infty} \psi_i(t)\, \psi_j(t) \left(\tfrac{1}{2} \cos 4\pi f_c t + \tfrac{1}{2} \right) dt$$

$$\approx \frac{1}{2} \int_{-\infty}^{\infty} \psi_i(t)\, \psi_j(t) = \frac{1}{2} \delta_{ij}$$

TO MAKE ORTHONORMAL, LET

$$\varphi_m(t) \longrightarrow \sqrt{2}\,\psi_m(t) \quad \text{OR}$$

$$s_m(t) = \sqrt{\frac{\mathcal{E}_s}{2}}\;\underbrace{\sqrt{2}\,\psi_m(t)}_{\psi_m'(t)}$$

$$\Rightarrow \quad u_m(t) = \sqrt{\mathcal{E}_s/2}\;\underbrace{\psi_m'(t)\cos 2\pi f_c t}_{\psi_{c,m}(t)}$$

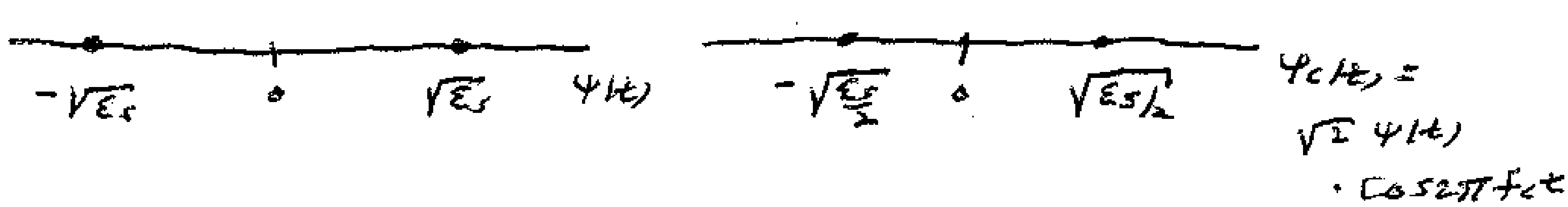
$\psi_{c,m}(t)$ IS BASIS SIGNAL FOR BANDPASS SIGNAL AND NOW

$$\int_{-\infty}^{\infty} \psi_{c,i}(t)\,\psi_{c,j}(t)\,dt = \delta_{ij}$$

AND $\mathcal{E}_s$ = ENERGY OF $s_m(t)$ - <u>BASEBAND</u> SIGNAL

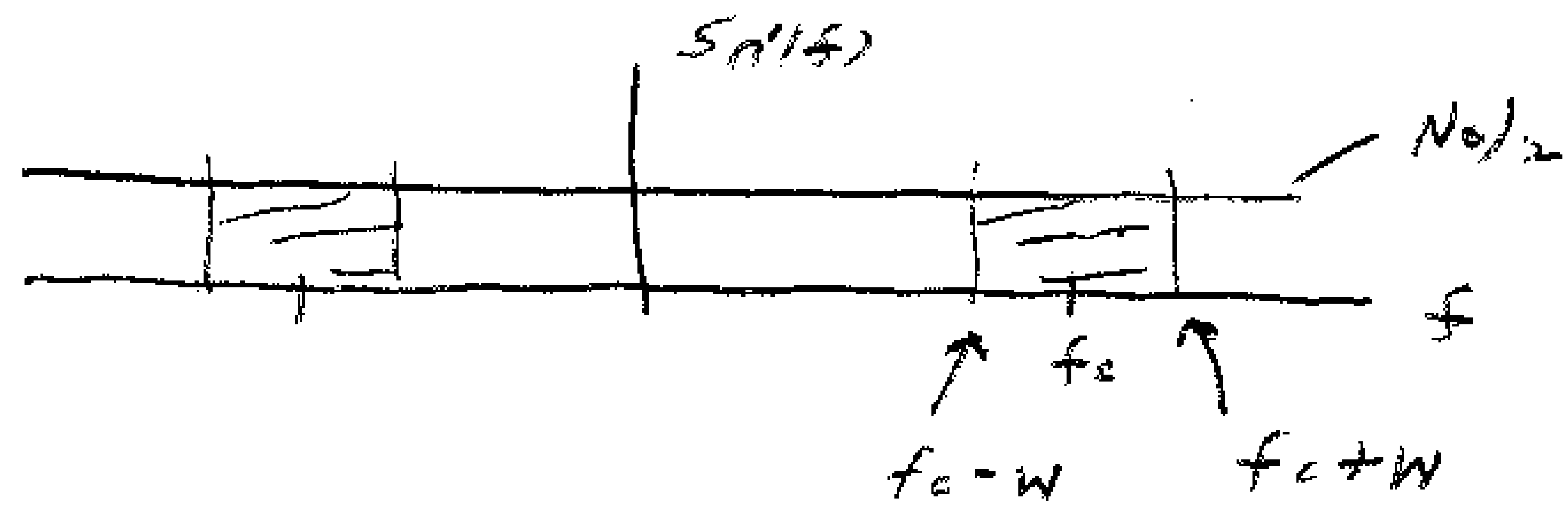
$\mathcal{E}_s/2$ = ENERGY OF <u>BANDPASS</u> SIGNAL

<u>EXAMPLE</u> : BINARY PAM



<u>DEMODULATION AND DETECTION</u>

NEED REPRESENTATION OF BANDPASS NOISE

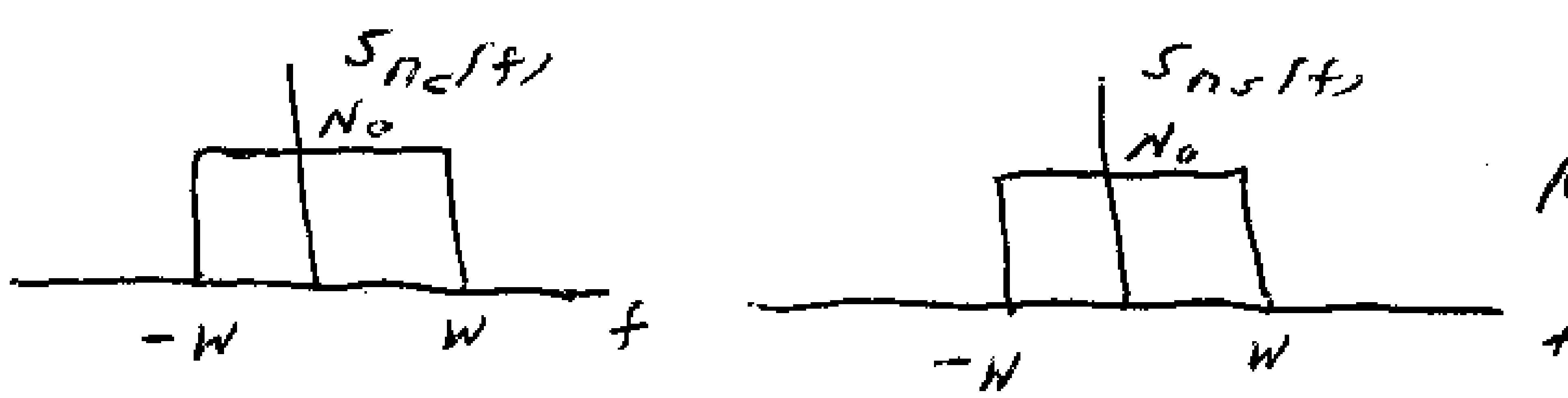


BANDPASS SIGNAL WILL BE FILTERED TO GET RID OF NOISE OUTSIDE OF $(f_c - W, f_c + W)$ BAND.

THIS RANDOM PROCESS CAN BE REPRESENTED AS

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

WHERE $n_c(t)$, $n_s(t)$ ARE EACH GAUSSIAN, ARE INDEPENDENT OF EACH OTHER, AND BOTH ARE ZERO MEAN AND WIDE SENSE STATIONARY WITH PSDs



$R_{n_c}(\tau) = R_{n_s}(\tau)$

NOTE THAT $n_c(t)$ (IN-PHASE COMPONENT) AND $n_s(t)$ (QUADRATURE COMPONENT) ARE <u>LOWPASS</u> RANDOM PROCESSES

1) SINCE $n_s(t)$, $n_c(t)$ ARE GAUSSIAN RANDOM PROCESS $\Rightarrow$ $n(t)$ IS GAUSSIAN