LINEAR TRANSFORMATION PROPERTY

2) $E\{n(t)\} = \underbrace{E(n_c(t))}_{=0} \cos 2\pi f_c t - \underbrace{E(n_s(t))}_{=0} \sin 2\pi f_c t$

$= 0$

3) $R_n(\tau) = E\{n(t)\,n(t+\tau)\}$

$= E\big[(n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t)$

$(n_c(t+\tau)\cos(2\pi f_c(t+\tau))$

$- n_s(t+\tau)\sin(2\pi f_c(t+\tau)))\big]$

$= \overbrace{E\{n_c(t)\,n_c(t+\tau)\}}^{R_{n_c}(\tau)} \cos 2\pi f_c t \cos(2\pi f_c(t+\tau))$

$+ \underbrace{E\{n_s(t)\,n_s(t+\tau)\}}_{R_{n_s}(\tau) = R_{n_c}(\tau)} \sin 2\pi f_c t \sin(2\pi f_c(t+\tau))$

INDEPENDENCE

SINCE $E(n_c(t)\,n_s(u)) = E(n_c(t))\,E(n_s(u))$

$= 0$ ALL t AND u

$$R_n(\tau) = R_{n_c}(\tau)\cos(2\pi f_c \tau)$$

$$\Rightarrow S_n(f) = \tfrac{1}{2}\left(S_{n_c}(f-f_c) + S_{n_c}(f+f_c)\right)$$

HENCE, RECEIVED WAVEFORM IS

BANDPASS "AWGN"

$$r(t) = s_m(t)\cos 2\pi f_c t + n(t)$$

TO DEMODULATE LET

$$\psi_{c,i}(t) = \sqrt{2}\,\psi_i(t)\cos 2\pi f_c t$$

EXAMPLE: MARY PAM

$$s_m(t) = s_m \psi(t)$$

$$s_m = \pm A, \pm 3A, \ldots, \pm(M-1)A$$

$$y(T) = \int_0^T r(t)\,\psi_c(t)\,dt$$

$$= \int_0^T (s_m(t)\cos 2\pi f_c t + n(t))\,\psi_c(t)\,dt$$

$$= \int_0^T \frac{s_m}{\sqrt{2}} \underbrace{\sqrt{2}\,\psi(t)\cos 2\pi f_c t}_{\psi_c(t)}\,\psi_c(t)\,dt$$

$$+ \int_0^T n(t)\,\psi_c(t)\,dt$$

$$= \underbrace{\frac{s_m}{\sqrt{2}}}_{s_{c,m}} + \underbrace{\int_0^T n(t)\,\psi_c(t)\,dt}_{n}$$

IN GENERAL, $y(T) = s_{c,m} + n$ ← GAUSSIAN AND ZERO MEAN

OPTIMAL DETECTOR DECIDES SIGNAL SENT THAT MINIMIZES

$$D(y, s_m) = (y - s_m)^2 \qquad m = 1, 2, \ldots, M$$

PHASE-MODULATED SIGNALS

BANDPASS SIGNALS ARE

$$u_m(t) = g_T(t)\cos(2\pi f_c t + \phi_m)$$

$$m = 0, 1, 2, \ldots, M-1$$

WHERE

AND FOR $M=4$

$$\phi_m = 0,\ \pi/2,\ \pi,\ 3\pi/2$$

ϕ_m

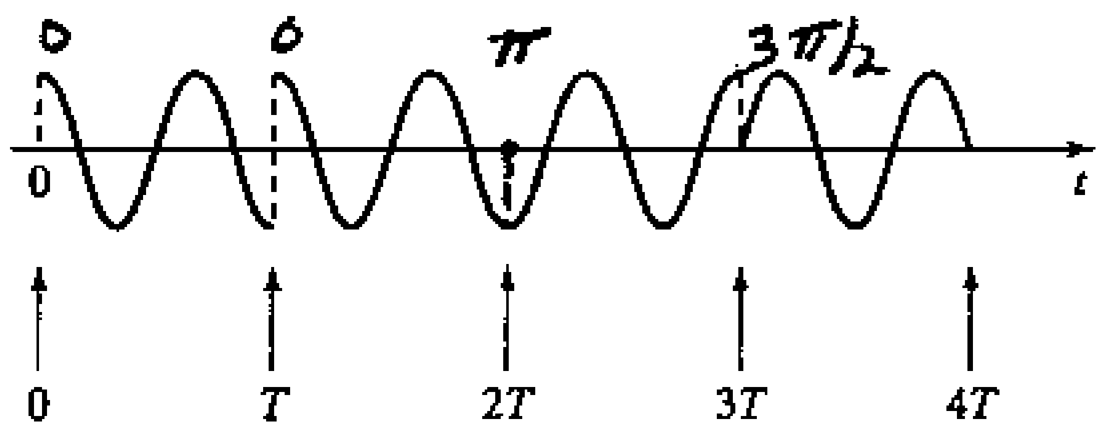


Figure 10.8 Example of a four-phase PSK signal. (QPSK)

PHASE SHIFT KEYING

IN GENERAL

$$\phi_m = \frac{2\pi m}{M}$$

$$u_m(t) = \sqrt{\frac{2\mathcal{E}_s}{T}} \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right)$$

$$\text{WHERE } \mathcal{E}_s = \int_0^T u_m^2(t)\,dt$$

FOR $M=2$ BINARY PSK

$M=4$ QUADRATURE PSK

EASIEST WAY TO GENERATE PSK

LET $g_T(t) = \sqrt{\frac{2\mathcal{E}_s}{T}}$ $\quad 0 \le t \le T$

$$u_m(t) = g_T(t) \overbrace{\cos\phi_m}^{A_{m,c}} \cos 2\pi f_c t - g_T(t) \underbrace{\sin\phi_m}_{A_{m,s}} \sin 2\pi f_c t$$

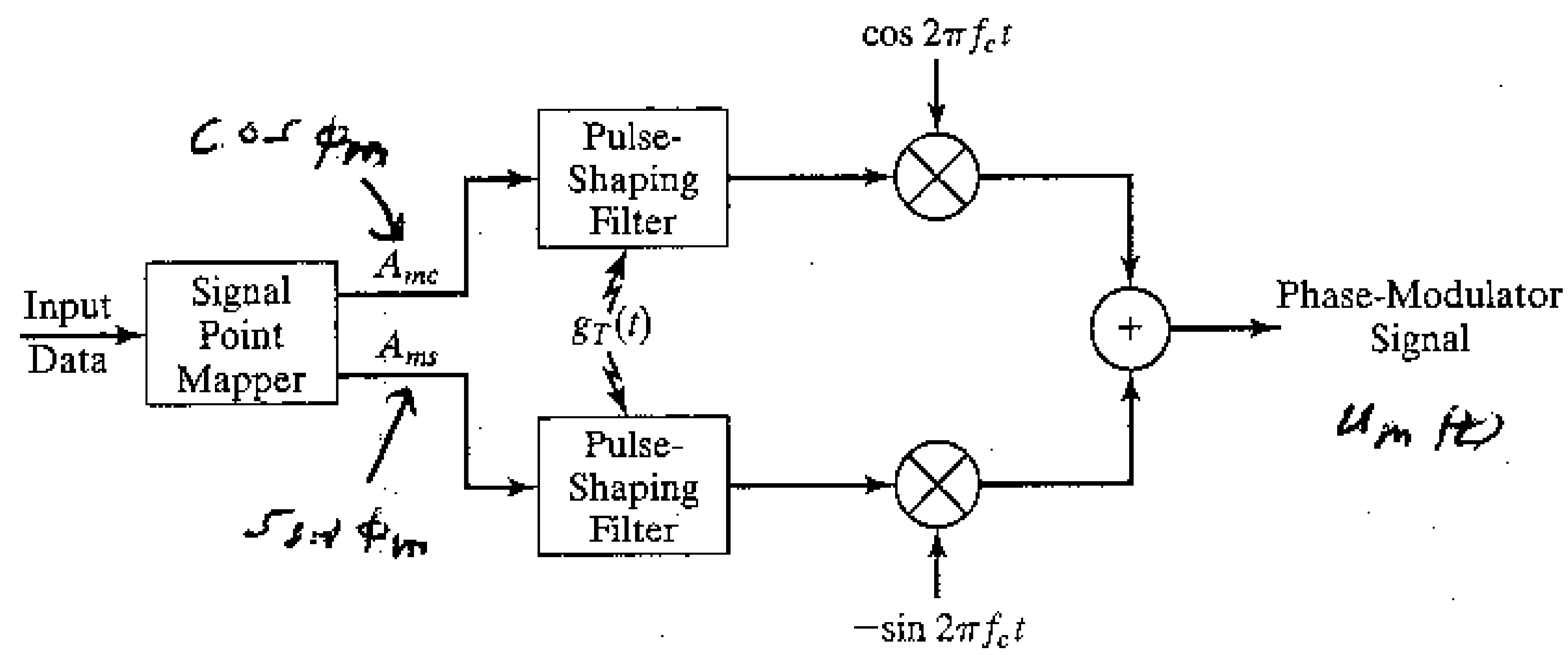


Figure 10.9 Block diagram of a digital-phase modulator.

TO FIND SIGNAL CONSTELLATION:

$$u_m(t) = g_T(t)\cos\phi_m \cos 2\pi f_c t$$

$$- g_T(t)\sin\phi_m \sin 2\pi f_c t$$

LET $\psi_1(t) = \sqrt{2/T}\cos 2\pi f_c t$

$\psi_2(t) = -\sqrt{2/T}\sin 2\pi f_c t$

$$\underbrace{\psi_1\cdot\psi_2 = 0 \qquad \psi_1\cdot\psi_1 = \psi_2\cdot\psi_2 = 1}_{\text{APPROXIMATE}}$$

APPROXIMATE UNLESS $f_c = \frac{k}{2T}$

k AN INTEGER

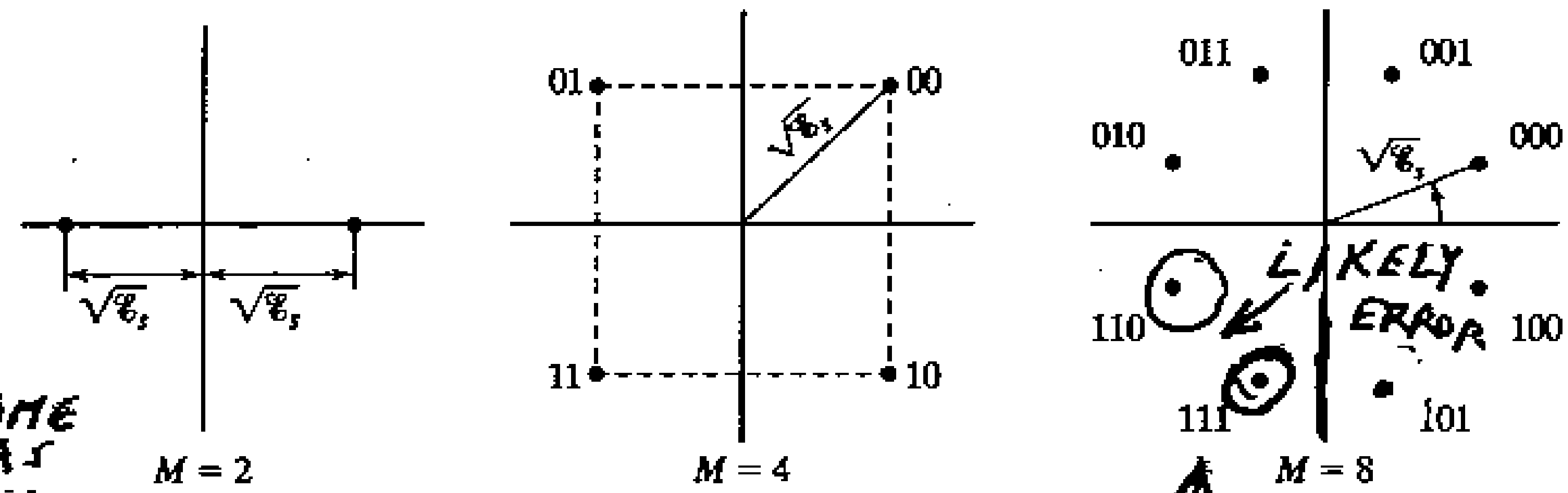
$$u_m(t) = \sqrt{\frac{2E_s}{T}}\cos\phi_m\sqrt{T/2}\,\psi_1(t)$$

$$+ \sqrt{\frac{2E_s}{T}}\sin\phi_m\sqrt{T/2}\,\psi_2(t)$$

$$= s_{mc}\psi_1(t) + s_{ms}\psi_2(t)$$

$$\underline{s}_m = (s_{mc}, s_{ms}) = (\sqrt{E_s}\cos\phi_m, \sqrt{E_s}\sin\phi_m)$$

NOTE $\underline{s}_m\cdot\underline{s}_m = E_s$ (ALL POINTS ARE ON CIRCLE OF RADIUS $\sqrt{E_s}$)



SAME AS PAM

Figure 10.10 PSK signal constellations.

GRAY CODE ⇒ SINGLE BIT ERROR

$$\phi_m = \frac{2\pi m}{M}$$

PROB. OF ERROR DECREASES AS $\mathcal{E}_s$ INCREASES, INCREASES AS M INCREASES SINCE SIGNAL POINTS BECOME CLOSER

DEMOD. AND DETECTION OF PHASE-MODULATED SIGNALS

BANDPASS "AWGN"

$$r(t) = u_m(t) + n(t)$$

$$u_m(t) = \underbrace{A_{mc}}\, g_T(t) \cos 2\pi f_c t - A_{ms}\, g_T(t) \sin 2\pi f_c t$$

$$\sqrt{2\mathcal{E}_s/T}$$

$\psi_1(t)$

$$= A_{mc}\sqrt{\mathcal{E}_s}\left(\sqrt{\tfrac{2}{T}} \cos 2\pi f_c t\right)$$

$$+ A_{ms}\sqrt{\mathcal{E}_s}\left(-\sqrt{2/T}\, \sin 2\pi f_c t\right)$$

$\psi_2(t)$

AT CORRELATOR OUTPUTS WE HAVE

$$y_1 = A_{mc}\sqrt{\mathcal{E}_s} + \int_0^T n(t)\,\psi_1(t)\,dt$$

$$y_2 = A_{mc}\sqrt{\mathcal{E}_s} + \int_0^T n(t)\,\psi_2(t)\,dt$$

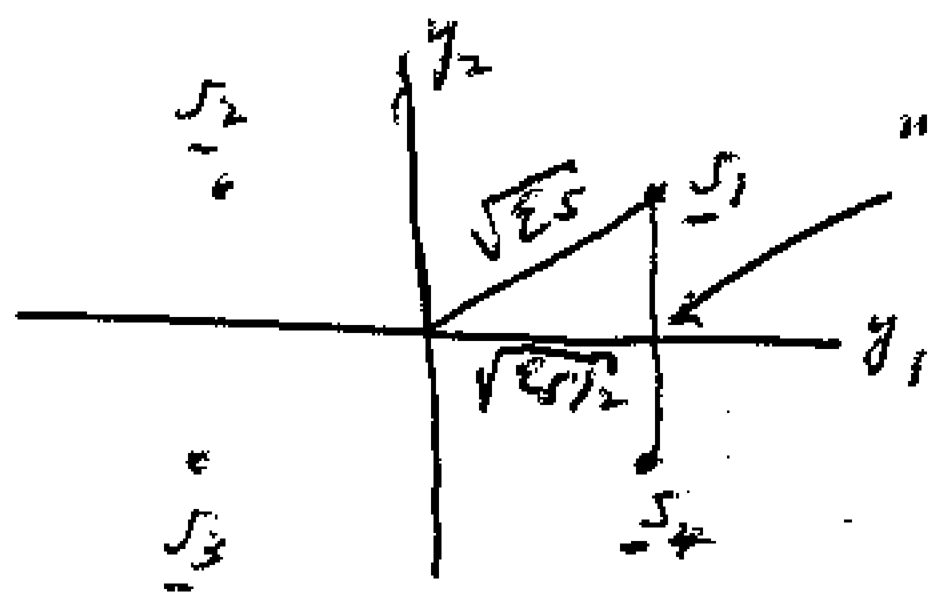
$$\Rightarrow \quad \underline{y} = \underline{s}_m + \underline{n}$$

WHERE $\underline{s}_m = \begin{bmatrix} \sqrt{E_s}\, A_{mc} \\ \sqrt{E_s}\, A_{ms} \end{bmatrix}$, $\underline{n} \sim N(\underline{0}, \frac{N_0}{2} \underline{I})$

FOR $M = 2$ WE HAVE ANTIPODAL SIGNALS $\Rightarrow$ $P_2 = Q\left(\sqrt{2E_s/N_0}\right) = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$

FOR $M = 4$ WE DERIVE P_4 NEXT USING AN APPROXIMATION.

"ORIGIN" FOR ANTIPODAL INTERPRETATION



ASSUME $\underline{s}_1$ TRANSMITTED

BY SYMMETRY

$P_4 = P(\text{ERROR} / \underline{s}_1)$

BUT IF AN ERROR OCCURS AND SNR IS HIGH WE WILL CHOOSE $\underline{s}_2$ OR $\underline{s}_4$

$$\Rightarrow P_4 = 2\, P(\text{CHOOSE } \underline{s}_4 / \underline{s}_1)$$

$\underline{s}_1, \underline{s}_4$ "ANTIPODAL"

$$= 2\, Q\left(\sqrt{\frac{\varepsilon}{N_0/2}}\right)$$

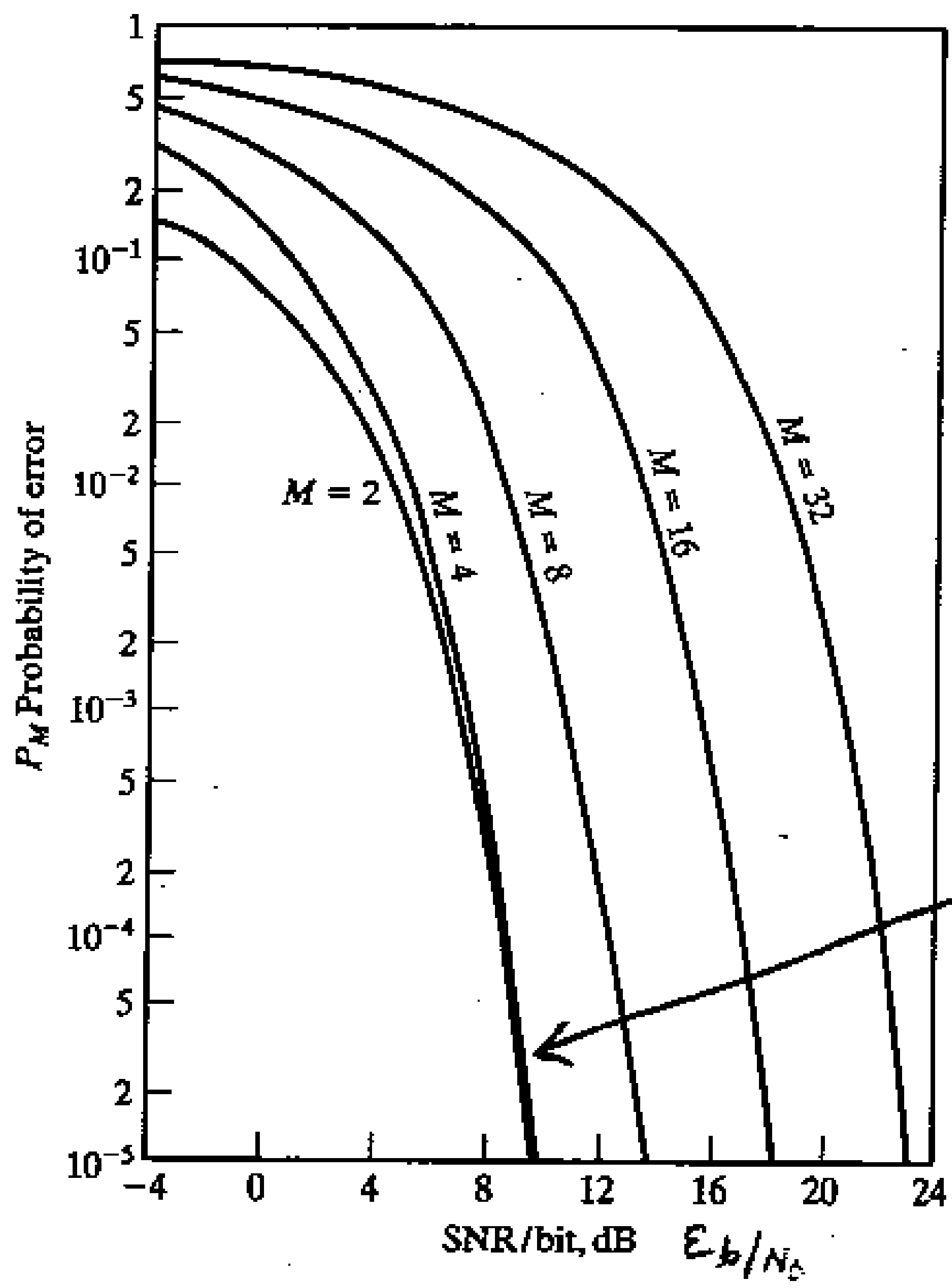
ε = DISTANCE FROM "ORIGIN" $= \sqrt{E_s/2}$

$$= 2\, Q\left(\sqrt{\frac{2}{N_0} E_s/2}\right)$$

$$= 2\, Q\left(\sqrt{E_s/N_0}\right)$$

$$= 2\, Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = 2P_2$$

ALMOST EQUAL $\Rightarrow$ $M = 4$ USED MUCH IN PRACTICE (QPSK)



FOR SAME SNR GET TWICE BIT RATE USING QPSK

Figure 10.16 Probability of a symbol error for PSK signals.

QUADRATURE AMPLITUDE MODULATION (QAM)

SIMILAR TO PHASE MODULATION BUT NOW A_{mc}, A_{ms} ARE NOT CONSTRAINED SO THAT $\begin{bmatrix} A_{mc} \\ A_{ms} \end{bmatrix}$ DO NOT LIE ON CIRCLE.

$$u_m(t) = A_{mc}\, g_T(t) \cos 2\pi f_c t + A_{ms}\, g_T(t) \sin 2\pi f_c t$$

SINCE
$$\alpha \cos 2\pi f_c t + \beta \sin 2\pi f_c t = \sqrt{\alpha^2+\beta^2} \cos(2\pi f_c t - \arctan \beta/\alpha)$$

$$u_m(t) = \underbrace{\sqrt{A_{mc}^2 + A_{ms}^2}}_{A_m}\, g_T(t) \cos\Big[2\pi f_c t \underbrace{- \arctan \frac{A_{ms}}{A_{mc}}}_{\theta_m}\Big]$$

AMPLITUDE AND PHASE MODULATION.

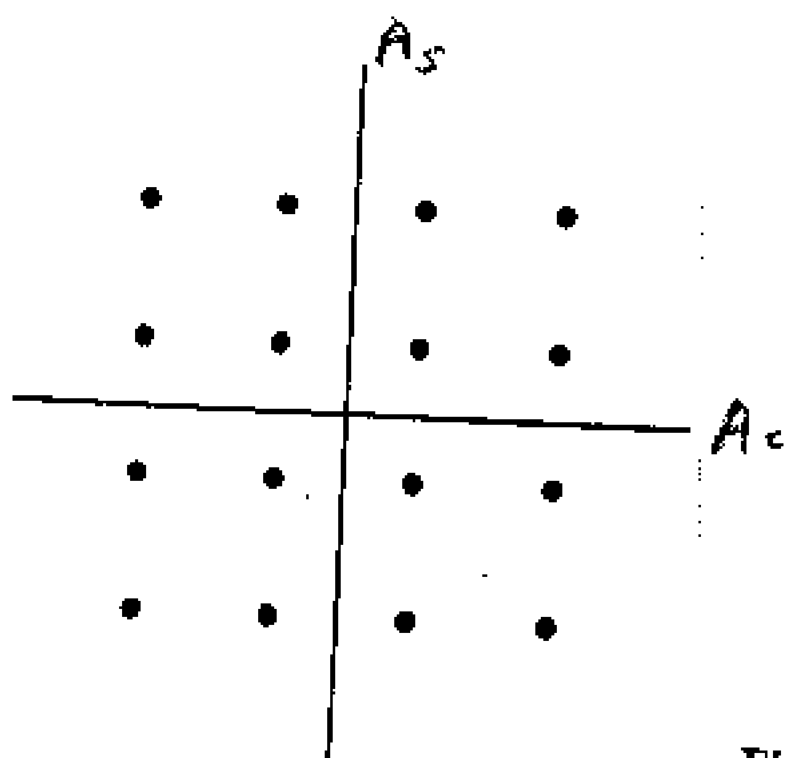
CAN USE DIFFERENT AMPLITUDES FOR SAME PHASE AND VICEVERSA.

EXAMPLE : RECTANGULAR QAM

$$u_{mn}(t) = A_m g_T(t) \cos(2\pi f_c t + \theta_n)$$

$$m = 1, 2, \ldots, M_1$$

$$n = 1, 2, \ldots, M_2$$



$M_1 = 4$

$M_2 = 4$

$M = M_1 M_2 = 16$

$\Rightarrow k = \log_2 M = 4$ BITS ENCODED

Figure 10.18 $M = 16$-QAM signal constellation.

TO GENERATE QAM SIGNAL USE QUADRATURE REPRESENTATION.
EASILY GENERATED IN PRACTICE.

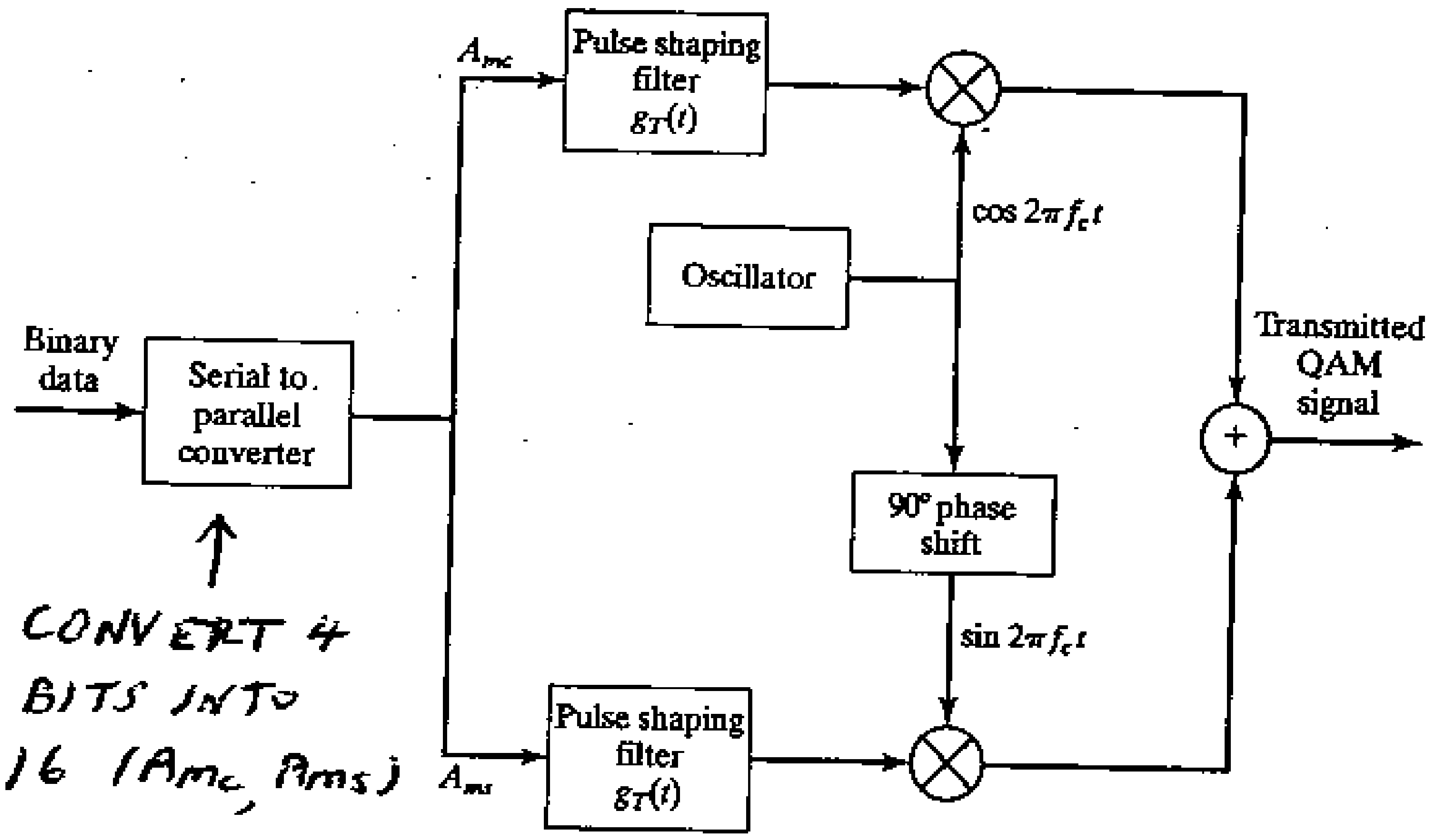


Figure 10.19 [redacted] for QAM.

SEE BOOK FOR PERFORMANCE ANALYSIS.

FREQUENCY MODULATED DIGITAL SIGNALS

SIMPLEST EXAMPLE IS BINARY FREQUENCY-SHIFT KEYING (FSK)

$$u_0(t) = \sqrt{\frac{2\mathcal{E}_b}{T_b}} \cos 2\pi f_0 t \qquad 0 \le t \le T_b$$

$$u_1(t) = \sqrt{\frac{2\mathcal{E}_b}{T_b}} \cos 2\pi f_1 t \qquad 0 \le t \le T_b$$

WHERE $f_1 = f_0 + \Delta f$

FOR M-ARY

$$u_m(t) = \sqrt{\frac{2\mathcal{E}_s}{T}} \cos\left[2\pi (f_c + m\Delta f) t\right]$$

$m = 0, 1, \ldots, M-1$

$\mathcal{E}_s = k\mathcal{E}_b$

k

$0 \le t \le T = (\log_2 M) T_b$

THE FREQUENCY SEPARATION Δf CHOSEN TO MAKE SIGNALS ORTHOGONAL (MORE EASILY DEMODULATED).

THE SIGNAL CORRELATION COEFFICIENT IS

$$\gamma_{mn} = \frac{\underline{u}_m \cdot \underline{u}_n}{\sqrt{\|\underline{u}_m\|^2 \|\underline{u}_n\|^2}}$$ EUCLIDEAN VECTOR ANALOGY

HERE $\|u_m\|^2 = \|u_n\|^2 = \mathcal{E}_S$

$$\Rightarrow \quad \gamma_{mn} = \frac{\int_0^T u_m(t)\, u_n(t)\, dt}{\mathcal{E}_S}$$

NOTE THAT $-1 \le \gamma_{mn} \le 1$ (CAUCHY-SCHWARTZ)

$$\gamma_{mn} = \frac{1}{\mathcal{E}_S} \int_0^T \frac{2\mathcal{E}_S}{T} \cos[2\pi(f_c + m\Delta f)t] \cdot \cos[2\pi(f_c + n\Delta f)t]\, dt$$

$$\approx \frac{1}{T} \int_0^T \cos[2\pi(m-n)\Delta f\, t]\, dt$$ (IGNORE $4\pi f_c t$ TERM)

$$= \frac{1}{T} \left. \frac{\sin 2\pi(m-n)\Delta f t}{2\pi(m-n)\Delta f} \right|_0^T$$

$$= \frac{\sin 2\pi(m-n)\Delta f T}{2\pi(m-n)\Delta f T}$$

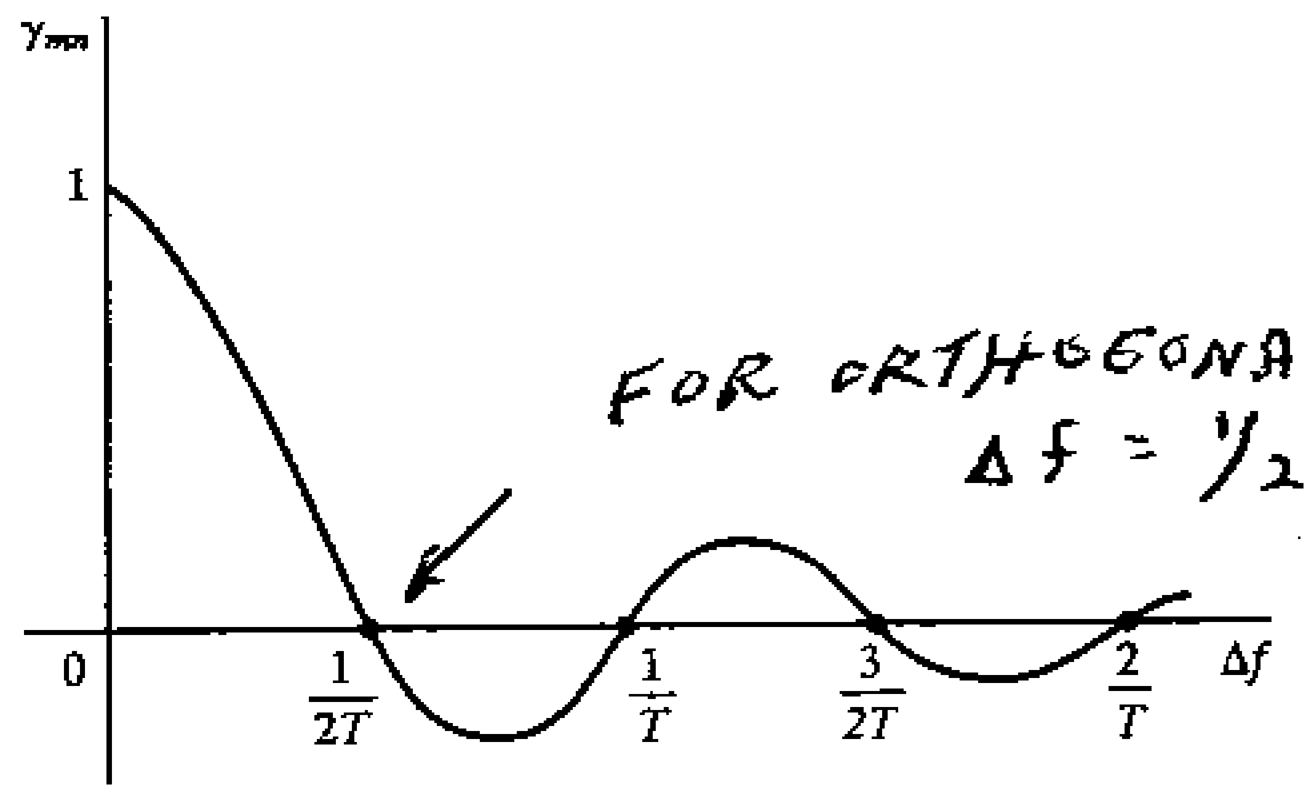
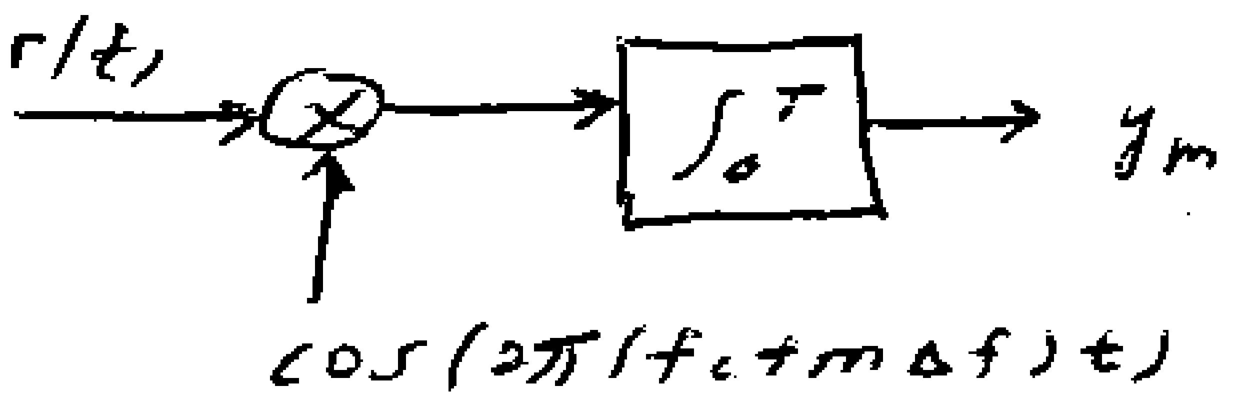


Figure 10.27 Cross-correlation coefficient as a function of frequency separation for FSK signals.

IF $\Delta f = 1/2T$, M-ARY FSK SIGNALS ARE ALL ORTHOGONAL.

DEMODULATION AND DETECTION FOR M-ARY FSK

WILL NEED M CORRELATORS



$m = 0, 1, \ldots, M-1$

CALLED COHERENT DEMODULATION.
REQUIRES OSCILLATOR AT TRANSMITTER TO BE IN SYNCHRONIZATION WITH THAT AT RECEIVER. ALSO, TRANSMISSION PHASE SHIFTS RESULT IN

$$r(t) = \sqrt{\frac{2E_s}{T}} \cos(2\pi(f_c+m\Delta f)t + \phi_m) + n(t)$$

IN PRACTICE ϕ_m IS UNKNOWN AND IS EITHER ESTIMATED OR RECEIVER IS MODIFIED.

CONSIDER NOW A RECEIVER THAT DOES <u>NOT</u> NEED TO KNOW ϕ_m. $\Rightarrow$ <u>NONCOHERENT DETECTION</u>

$$y_{m_c} = \int_0^T r(t) \sqrt{2/T} \cos(2\pi(f_c + m\Delta f)t)\, dt \qquad m = 0, 1, \ldots, M-1$$

$$= \int_0^T r(t)\, \psi_{m_c}(t)\, dt$$

$u_k(t)$ TRANSMITTED

$$= \int_0^T \left[\sqrt{\frac{2E_s}{T}} \cos(2\pi(f_c + k\Delta f)t + \phi_k) + n(t) \right] \psi_{m_c}(t)\, dt$$

$$= \sqrt{E_s} \int_0^T \sqrt{\frac{2}{T}} \Big[\cos(2\pi(f_c + k\Delta f)t) \cos\phi_k - \sin(2\pi(f_c + k\Delta f)t) \sin\phi_k \Big] \psi_{m_c}(t)\, dt$$

$$+ \underbrace{\int_0^T n(t)\, \psi_{m_c}(t)\, dt}_{n_{m_c}}$$

$$= \sqrt{E_s} \int_0^T \psi_{k_c}(t)\, \psi_{m_c}(t) \cos\phi_k\, dt$$

$$- \sqrt{E_s} \int_0^T \sqrt{2/T} \cos(2\pi(f_c + m\Delta f)t) \sin(2\pi(f_c + k\Delta f)t) \cdot \sin\phi_k\, dt$$

$$+ n_{m_c}$$

BUT $\int_0^T \cos(2\pi(f_c + m\Delta f)t) \sin(2\pi(f_c + k\Delta f)t)\, dt$

$$= \frac{1}{2}\int_0^T \sin\left[2\pi(2f_c+(m+k)\Delta f)t\right]$$

$$+\sin\left[2\pi(k-m)\Delta f\, t\right]\, dt$$

USING $\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$

$$\approx \frac{1}{2}\int_0^T \sin\left[2\pi(k-m)\Delta f\, t\right] dt \underset{\uparrow}{=} 0 \quad \text{ALL } k, m$$

LET $\Delta f = 1/T$

$$\Rightarrow y_{mc} = \sqrt{E_s}\cos\phi_k \overset{\delta_{mk}}{\wedge} + n_{mc}$$

IF $\phi_k = \pi/2 \Rightarrow y_m = n_{mc}$ NOISE ONLY

TO AVOID THIS WE NEED A SECOND CORRELATOR

$$y_{ms} = \int_0^T r(t) \underbrace{\sqrt{2/T}\sin(2\pi(f_c+m\Delta f)t)}_{\psi_{ms}(t)} dt$$

YIELDS $y_{ms} = \sqrt{E_s}\sin\phi_m + n_{ms}$

WHERE $n_{ms} = \int_0^T n(t)\psi_{ms}(t)\,dt$

EITHER y_{mc} OR y_{ms} OR BOTH WILL CONTAIN THE SIGNAL IF $m=k$.

$$\Rightarrow \begin{pmatrix} y_{mc} \\ y_{ms} \end{pmatrix} = \begin{bmatrix} \sqrt{E_s}\cos\phi_m \\ \sqrt{E_s}\sin\phi_m \end{bmatrix}\delta_{mk} + \underbrace{\begin{pmatrix} n_{mc} \\ n_{ms} \end{pmatrix}}_{\sim N(0,\frac{N_0}{2}I)}$$

$m = 0, 1, \ldots, M-1$

NOTE THAT

$$\cos(2\pi(f_c + m\Delta f)t + \phi_m)$$

$$= \cos(2\pi(f_c + m\Delta f)t) \qquad \phi_m = 0$$

$$= \sin(2\pi(f_c + m\Delta f)t) \qquad \phi_m = -\pi/2$$

MUST CORRELATE WITH SIN( ) AND COS( ).

CALLED <u>NONCOHERENT DEMODULATION AND DETECTION</u>.

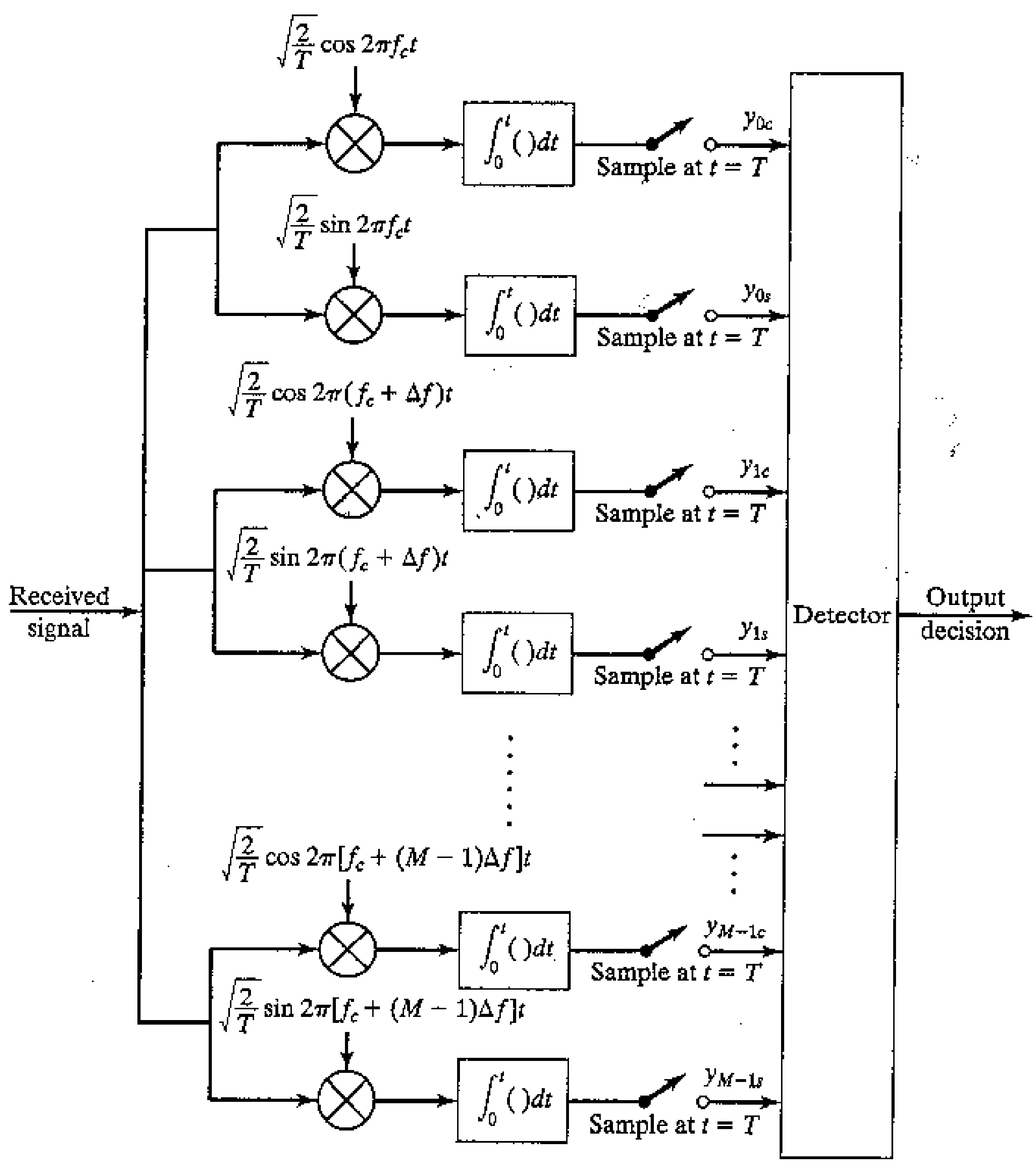


Figure 10.29 Demodulation of M-ary FSK signals for noncoherent detection.

OPTIMAL DETECTOR FOR NONCOHERENT BINARY FSK

ASSUME ϕ_m IS A RANDOM VARIABLE UNIFORM ON $(0, 2\pi)$. ALSO, ASSUME EQUIPROBABLE $\underline{s}_0$, $\underline{s}_1$. USE ML RULE

$\Rightarrow$ CHOOSE $\underline{s}_0$ IF

$$f(\underline{y} \mid \underline{s}_0) > f(\underline{y} \mid \underline{s}_1) \qquad \underline{y} = \begin{bmatrix} y_{0c} \\ y_{0s} \\ y_{1c} \\ y_{1s} \end{bmatrix} = \begin{pmatrix} \underline{y}_0 \\ \underline{y}_1 \end{pmatrix}$$

GIVEN $\underline{s}_0$ WE HAVE

$$\underline{y}_0 = \begin{bmatrix} \sqrt{E_s}\cos\phi_0 \\ \sqrt{E_s}\sin\phi_0 \end{bmatrix} + \underline{n}_0$$

$$\underline{y}_1 = \underline{n}_1$$

GIVEN $\underline{s}_1$ WE HAVE

$$\underline{y}_0 = \underline{n}_0$$

$$\underline{y}_1 = \begin{bmatrix} \sqrt{E_s}\cos\phi_1 \\ \sqrt{E_s}\sin\phi_1 \end{bmatrix} + \underline{n}_1$$

IT CAN BE SHOWN THAT $\underline{n}_0 \sim N(\underline{0}, \frac{N_0}{2} \underline{I}_2)$

$\underline{n}_1 \sim N(\underline{0}, \frac{N_0}{2} \underline{I}_2)$

AND $\underline{n}_0$, $\underline{n}_1$ ARE INDEPENDENT. $\Rightarrow$

$$f(\underline{y} \mid \underline{s}_m) = f(\underline{y}_0, \underline{y}_1 \mid \underline{s}_m)$$

$$= f(\underline{y}_0 \mid \underline{s}_m)\, f(\underline{y}_1 \mid \underline{s}_m)$$

$$f(\underline{y} \mid \underline{s}_0) = f(\underline{y}_0 \mid \underline{s}_0)\, f(\underline{y}_1 \mid \underline{s}_0)$$

$$= \int_0^{2\pi} \underbrace{f(\underline{y}_0 \mid \underline{s}_0, \phi_0)}_{N\left(\sqrt{E_s}\begin{pmatrix}\cos\phi_0 \\ \sin\phi_0\end{pmatrix},\ \frac{N_0}{2}\underline{I}_2\right)} \underbrace{f(\phi_0)}_{1/2\pi}\, d\phi_0 \quad \underbrace{f(\underline{y}_1 \mid \underline{s}_0)}_{N(\underline{0},\ \frac{N_0}{2}\underline{I}_2)}$$

$$f(\underline{y}_1 \mid \underline{s}_0) = \frac{1}{\pi N_0}\, e^{-\frac{1}{N_0}\underline{y}_1^T\underline{y}_1}$$

$$f(\underline{y}_0 \mid \underline{s}_0) = \int_0^{2\pi} \frac{1}{2\pi(N_0/2)}\, e^{-\frac{1}{N_0}\left[(y_{0c} - \sqrt{E_s}\cos\phi_0)^2 + (y_{0s} - \sqrt{E_s}\sin\phi_0)^2\right]} \cdot \frac{1}{2\pi}\, d\phi_0$$

$$= \frac{1}{\pi N_0}\, e^{-\frac{1}{N_0}(y_{0c}^2 + y_{0s}^2 + E_s)} \cdot \underbrace{\frac{1}{2\pi}\int_0^{2\pi} e^{-\frac{1}{N_0}(-2\sqrt{E_s}\, y_{0c}\cos\phi_0 - 2\sqrt{E_s}\, y_{0s}\sin\phi_0)}\, d\phi_0}$$

$$\frac{1}{2\pi}\int_0^{2\pi} e^{2/N_0 \sqrt{E_s}\,(\underbrace{y_{0c}\cos\phi_0 + y_{0s}\sin\phi_0}_{\sqrt{y_{0c}^2 + y_{0s}^2}\,\cos(\phi_0 - \alpha)})}\, d\phi_0$$

$$= \frac{1}{2\pi}\int_0^{2\pi} e^{\frac{2}{N_0}\sqrt{E_s(\underline{y}_0^T\underline{y}_0)}\,\cos(\phi_0)}\, d\phi_0$$

INTEGRAND IS PERIODIC $\Rightarrow$ SAME RESULT FOR ANY α

$$= I_0\left(\frac{\sqrt{E_s\,\underline{y}_0^T\underline{y}_0}}{N_0/2}\right)$$

$$f(\underline{y} \mid \underline{s}_0) = \frac{1}{\pi N_0}\, e^{-\frac{1}{N_0}\underline{y}_1^T\underline{y}_1}\ \frac{1}{\pi N_0}\, e^{-\frac{1}{N_0}(\underline{y}_0^T\underline{y}_0 + E_s)} \cdot I_0\left(\sqrt{\frac{E_s\,\underline{y}_0^T\underline{y}_0}{N_0/2}}\right)$$

$I_0(x)$ IS CALLED THE MODIFIED BESSEL FUNCTION OF ORDER ZERO.

SIMILARLY,

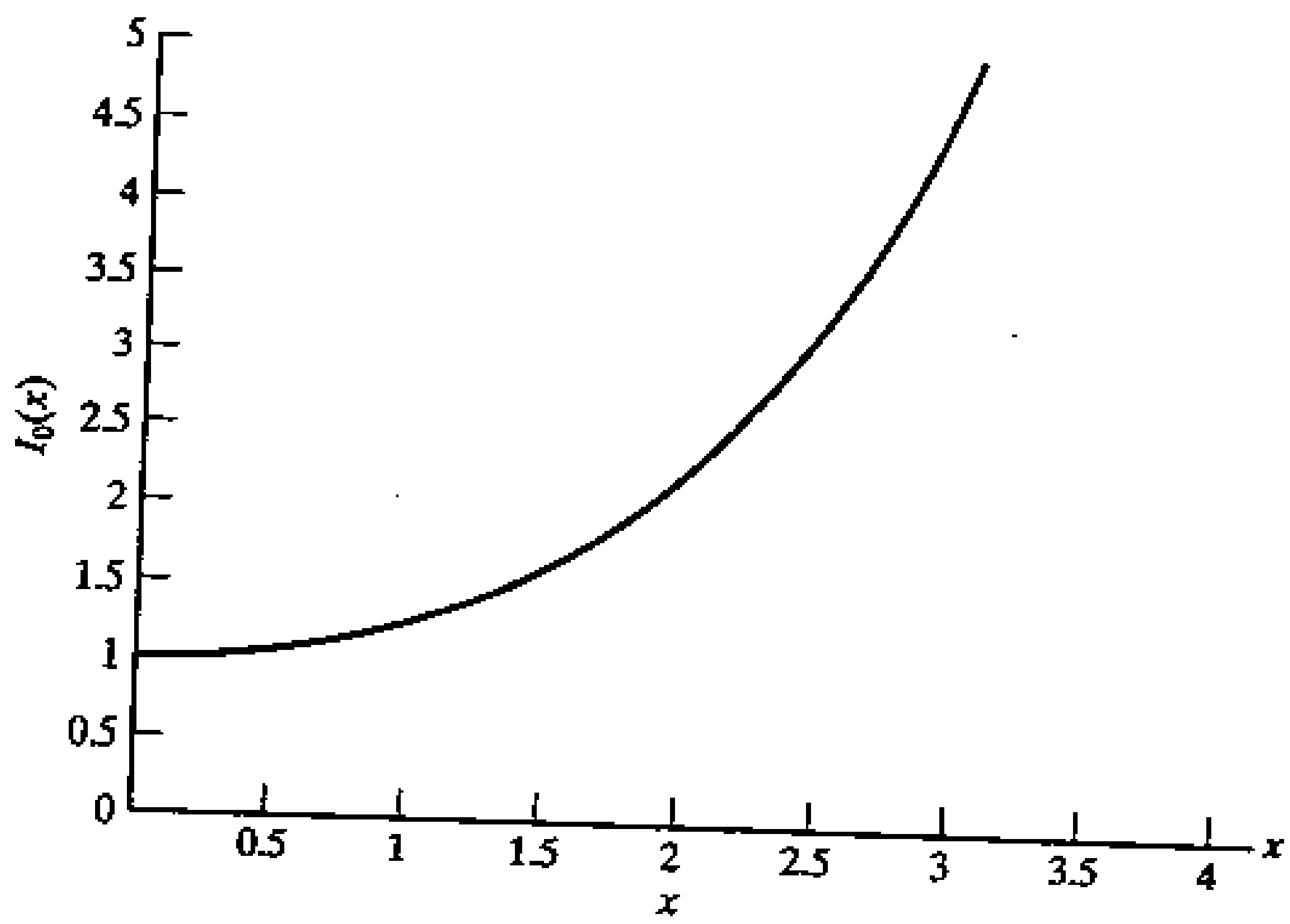
$$f(\underline{y}|\underline{s}_1) = \frac{1}{\pi N_0} e^{-\frac{1}{N_0}\underline{y}_0^T \underline{y}_0} \frac{1}{\pi N_0} e^{-\frac{1}{N_0}(\underline{y}_1^T \underline{y}_1 + E_s)}$$

$$\cdot\, I_0\left(\sqrt{\frac{E_s\, \underline{y}_1^T \underline{y}_1}{N_0/2}}\right)$$

DECIDE $\underline{s}_0$ IF

$$I_0\left(\sqrt{\frac{E_s}{N_0/2}}\sqrt{\underline{y}_0^T \underline{y}_0}\right) > I_0\left(\sqrt{\frac{E_s}{N_0/2}}\sqrt{\underline{y}_1^T \underline{y}_1}\right)$$

BUT $I_0(x)$ IS MONOTONE INCREASING WITH x.



$$I_0(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{2^{2k}(k!)^2}$$

Figure 10.30 Graph of $I_0(x)$.

DECIDE $\underline{s}_0$ IF $\sqrt{\underline{y}_0^T \underline{y}_0} > \sqrt{\underline{y}_1^T \underline{y}_1}$

CALLED AN <u>ENVELOPE DETECTOR</u> OR IF

$\underline{y}_0^T \underline{y}_0 > \underline{y}_1^T \underline{y}_1$ OR $y_{0c}^2 + y_{0s}^2 > y_{1c}^2 + y_{1s}^2$

CALLED A SQUARE LAW DETECTOR

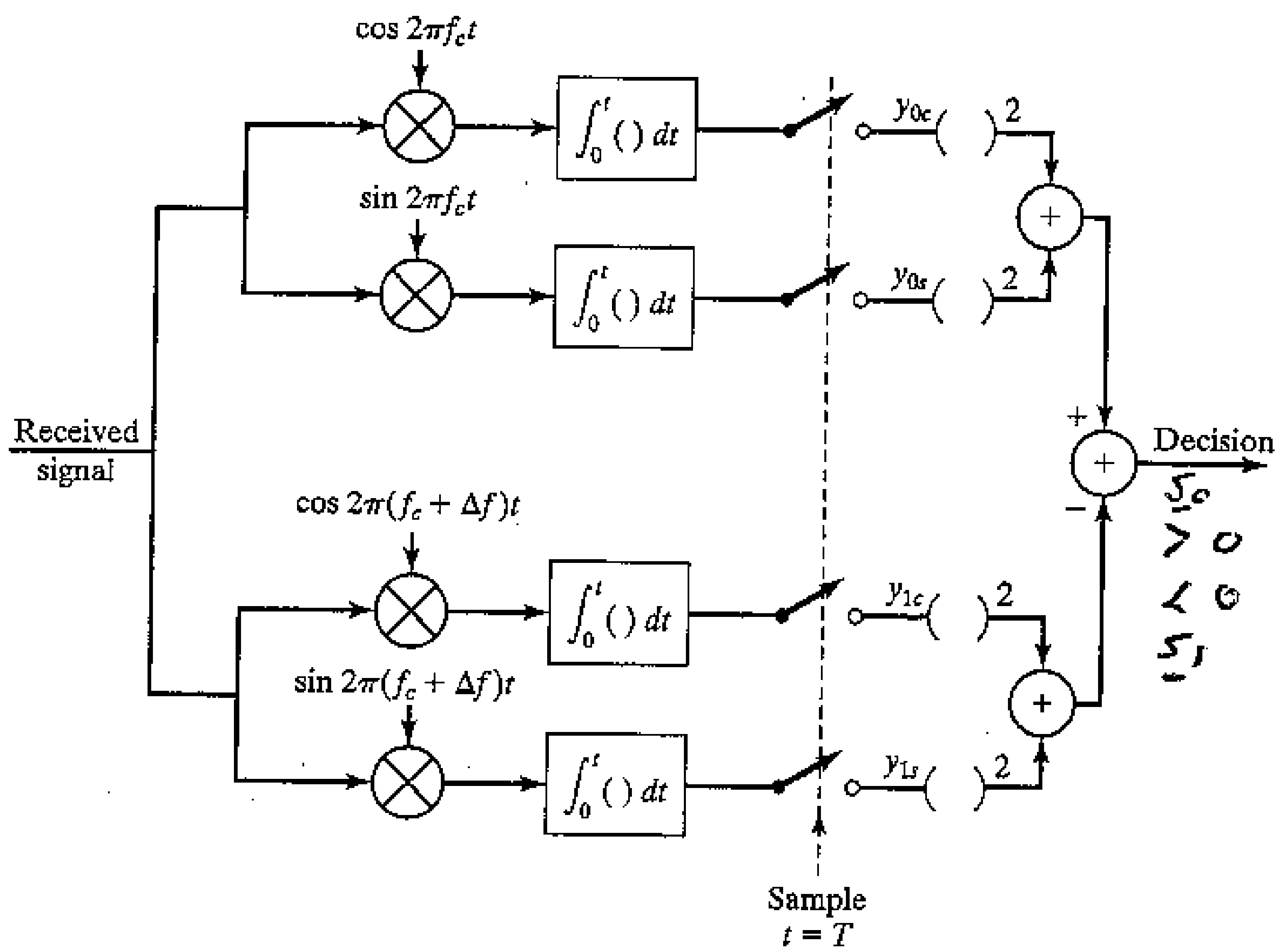


Figure 10.31 Demodulation and square-law detection of binary FSK signals.

ALSO CALLED A <u>QUADRATURE MATCHED FILTER</u>.

FOR M-ARY JUST ADD BRANCHES FOR
$f = f_c + m\Delta f \quad m = 2, 3, \ldots, M-1$ AND
CHOOSE MAXIMUM $y_{mc}^2 + y_{ms}^2$ AS SIGNAL.

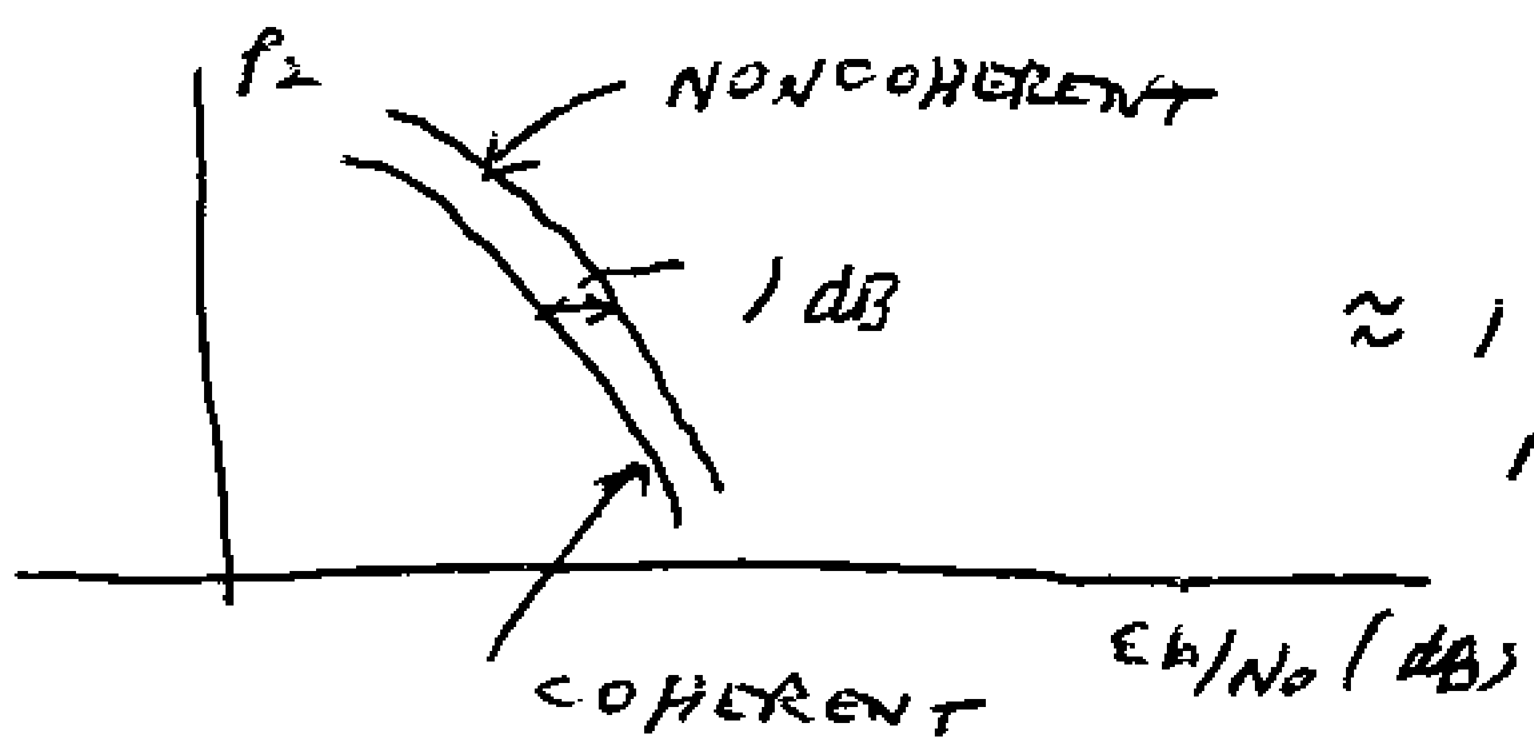
PROB. OF ERROR FOR NONCOHERENT DETECTION OF BINARY FSK

CAN BE SHOWN (SEE BOOK) TO BE

$$P_{2_{NC}} = \frac{1}{2} e^{-\mathcal{E}_b/N_0}$$

RECALL FOR COHERENT DETECTION OF BINARY ORTHOGONAL SIGNAL

$$P_{2_C} = Q\left(\sqrt{\mathcal{E}_b/N_0}\right) < P_{2_{NC}} \quad \text{WHY?}$$



$\approx$ 1 dB LOSS AT MOST

THIS IS GOOD TRADEOFF - ONLY LOSE 1 dB. DON'T NEED PHASE SYNCHRONIZATION.

IF WE HAVE PHASE SYNCHRONIZATION MIGHT AS WELL USE ANTIPODAL SIGNALS (3 dB GAIN OVER ORTHOGONAL)

CHAPTER 10 - INTRO. TO INFORMATION THEORY

PROVIDES FUNDAMENTAL LIMITS ON DATA COMPRESSION AND SPEED OF TRANSMISSION - THANKS TO C. SHANNON

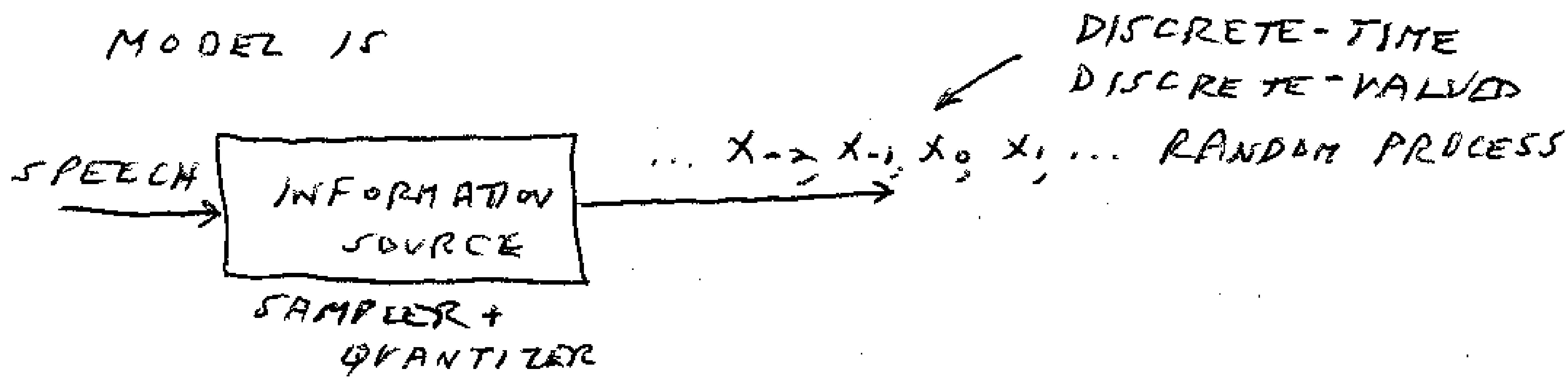
SHANNON THEOREMS

1) SOURCE COMPRESSION - MINIMUM NUMBER OF BITS PER SYMBOL REQUIRED
2) MAXIMUM TRANSMISSION RATE IN BITS PER SECOND WITH $P_e \to 0$.

INFORMATION SOURCES

SPEECH - CONVERTED INTO FINITE NUMBER OF LEVELS VIA SAMPLING AND QUANTIZATION

MODEL IS



TO SIMPLIFY DISCUSSION WE WILL ASSUME X_i'S ARE INDEPENDENT AND IDENTICALLY DISTRIBUTED (IID RANDOM PROCESS)