HENCE FROM (8.3.53)

$$P_2(\alpha) = Q\left(\sqrt{\frac{2\alpha^2 \mathcal{E}_b}{N_0}}\right)$$

BUT $\quad P_2 = \int_0^\infty P_2(\alpha) f(\alpha)\, d\alpha$

SINCE $\quad P_2(\alpha) = P(\text{ERROR} \mid \alpha)$

$$P_2 = \int_0^\infty Q\left(\sqrt{\frac{2\alpha^2 \mathcal{E}_b}{N_0}}\right) \frac{\alpha}{\sigma^2} e^{-\frac{1}{2}\alpha^2/\sigma^2}\, d\alpha$$

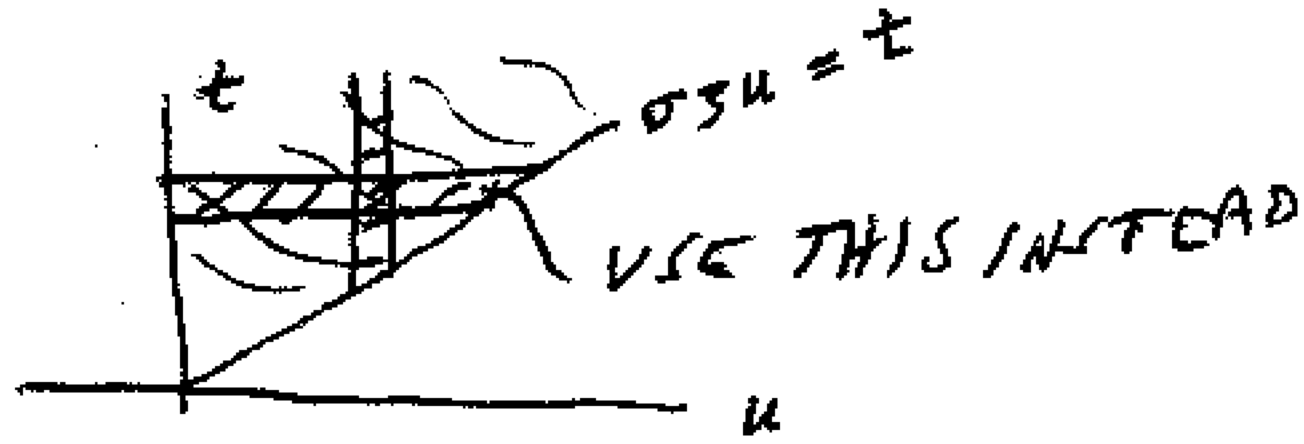
TO EVALUATE

$$P_2 = \int_0^\infty \int_{\alpha\zeta}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt\, \frac{\alpha}{\sigma^2} e^{-\frac{1}{2}\alpha^2/\sigma^2}\, d\alpha$$

WHERE $\zeta = \sqrt{2\mathcal{E}_b/N_0}$

LET $\quad u = \alpha/\sigma$

$$P_2 = \int_0^\infty \int_{\sigma\zeta u}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt\, u e^{-\frac{1}{2}u^2}\, du$$



$$= \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \Big[\underbrace{\int_0^{t/\sigma\zeta} u e^{-\frac{1}{2}u^2} du}_{1 - e^{-\frac{1}{2}t^2/\sigma^2\zeta^2}} \Big] dt$$

$$= \frac{1}{2} \int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \left(1 - e^{-\frac{1}{2}t^2/\sigma^2\zeta^2}\right) dt$$

$$= \frac{1}{2} - \frac{1}{2}\int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2(1 + 1/\sigma^2\zeta^2)}\, dt$$

$$= \frac{1}{2} - \frac{\eta}{2}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\underbrace{\left(1+\frac{1}{\sigma^2\rho^2}\right)^{-1}}_{\eta^2}}}\, e^{-\frac{1}{2}t^2\left(1+\frac{1}{\sigma^2\rho^2}\right)}\,dt$$

$$= \frac{1}{2} - \frac{\eta}{2} = \frac{1}{2} - \frac{1}{2}\sqrt{\left(1+\frac{1}{\sigma^2\rho^2}\right)^{-1}} = \frac{1}{2}\left(1 - \sqrt{\frac{\sigma^2\rho^2}{1+\sigma^2\rho^2}}\right)$$

$$P_2 = \frac{1}{2}\left(1 - \sqrt{\frac{\sigma^2\, 2\mathcal{E}_b/N_0}{1+\sigma^2\, 2\mathcal{E}_b/N_0}}\right)$$

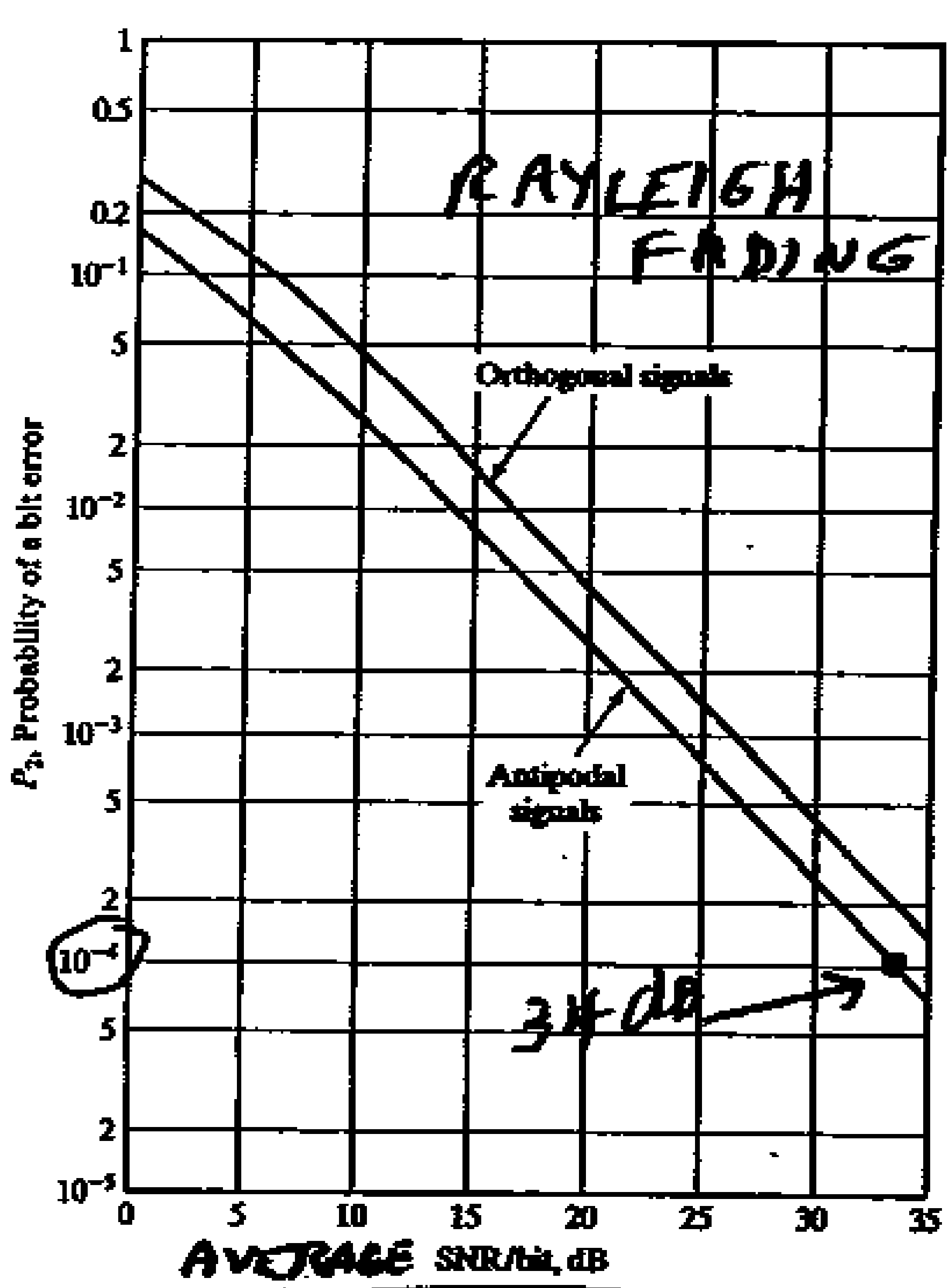
BUT $E(\alpha^2) = 2\sigma^2$

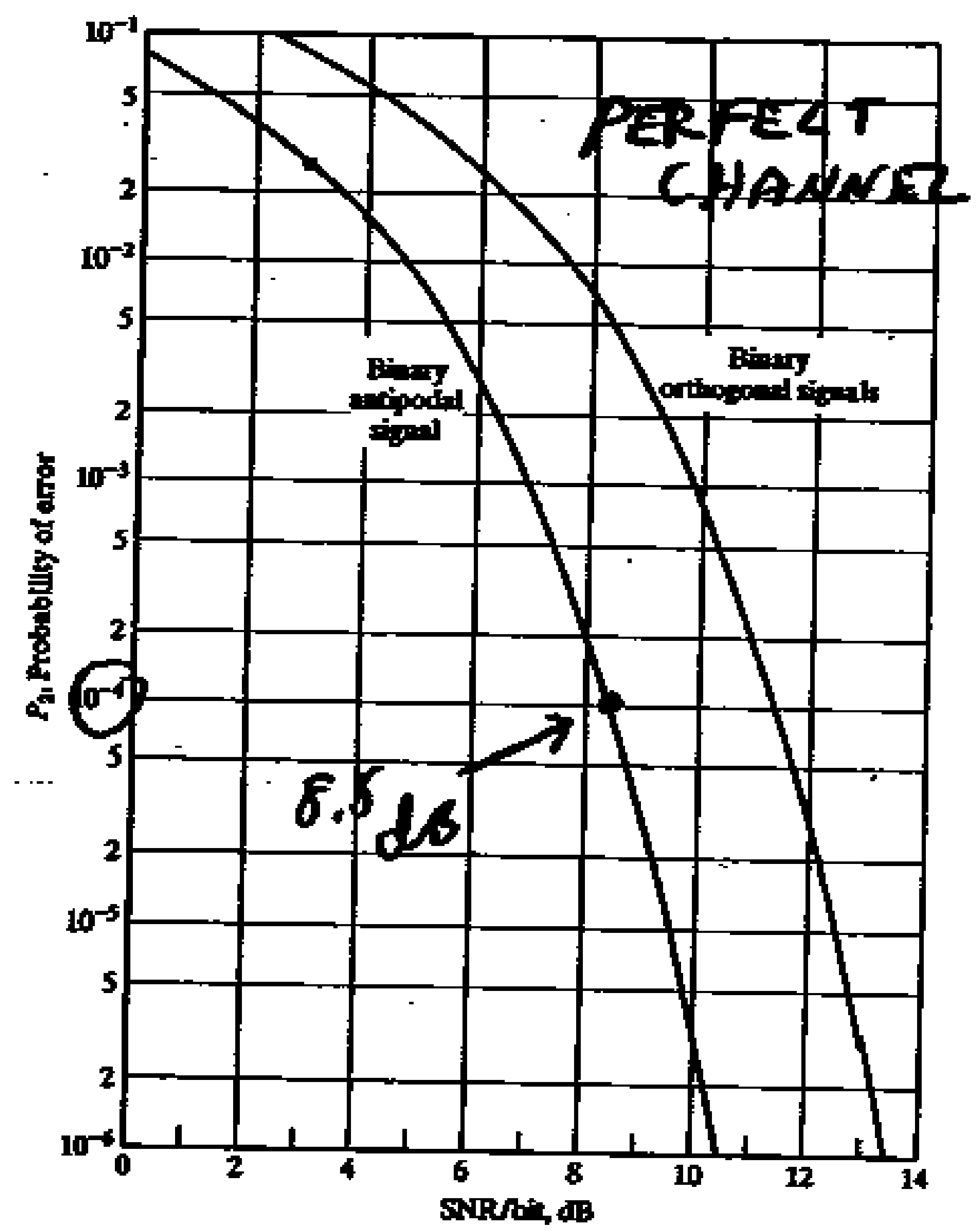
$$\Rightarrow \quad P_2 = \frac{1}{2}\left(1 - \sqrt{\frac{\mathcal{E}_b/N_0\, E(\alpha^2)}{1+\frac{\mathcal{E}_b}{N_0}E(\alpha^2)}}\right)$$

AND $\bar{\rho}_b = \frac{\mathcal{E}_b}{N_0}E(\alpha^2)$ IS AN AVERAGE

SNR PER BIT (RECALL $\mathcal{E} = \alpha^2\mathcal{E}_b \Rightarrow$
$E(\mathcal{E}) = E(\alpha^2)\mathcal{E}_b = 2\sigma^2\mathcal{E}_b$)

$$P_2 = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\rho}_b}{1+\bar{\rho}_b}}\right] \to 0 \text{ AS } \bar{\rho}_b \to \infty$$





REQUIRE 25.5 dB MORE SNR DUE TO ENERGY LOSSES BECAUSE OF FADING CHANNEL!

SIGNAL DIVERSITY

DUE TO FADING THE SIGNAL POWER CAN (WITH HIGH PROBABILITY) BE CLOSE TO ZERO. TO MAKE SURE THIS DOESN'T HAPPEN WE CAN TRANSMIT MULTIPLE COPIES OF SIGNAL VIA DIFFERENT "PATHS" TO ENSURE THAT SOME COPIES HAVE HIGH SNR AT RECEIVER.
$\Rightarrow$ SIGNAL DIVERSITY

EXAMPLES:
1) FREQ. DIVERSITY - USE SAME SIGNAL BUT DIFFERENT CARRIER FREQS.
2) TIME DIVERSITY - USE SAME SIGNAL BUT DIFFERENT TIME SLOTS

WANT FADES TO BE INDEPENDENT

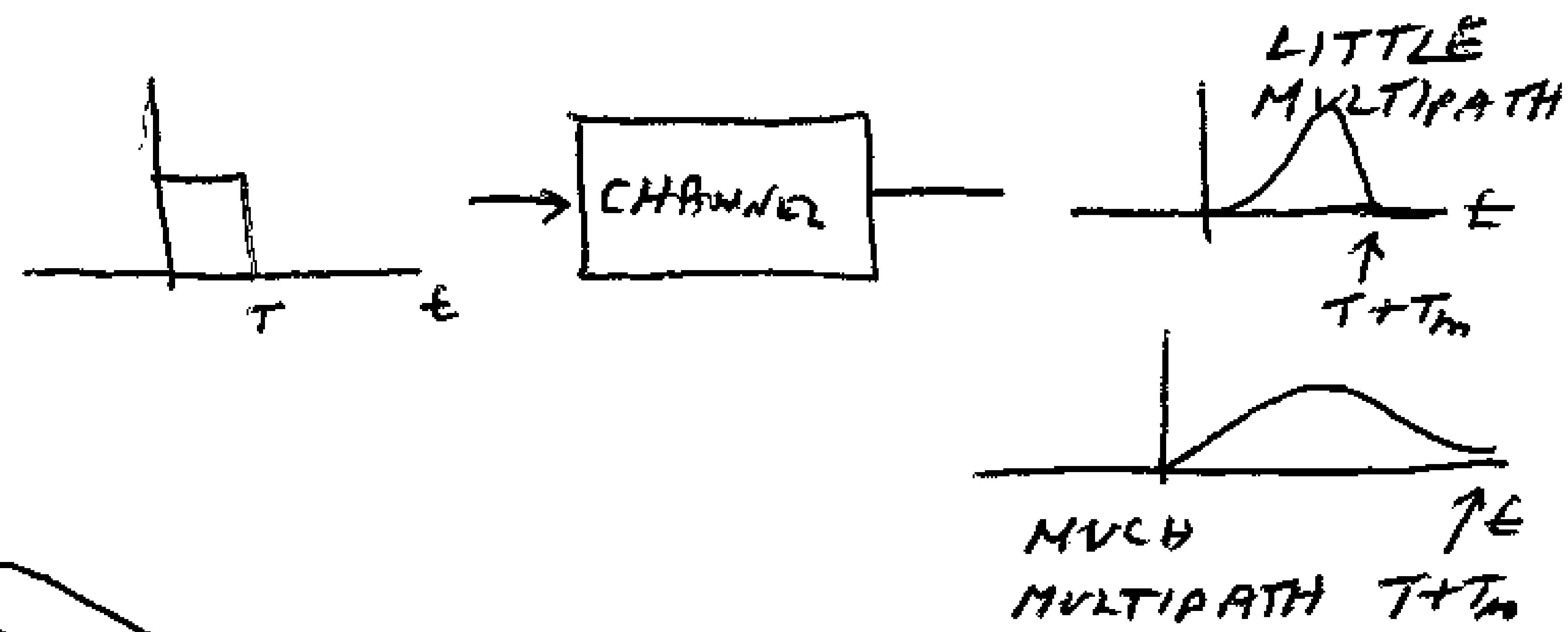
IF p = PROB. OF A SIGNAL FADING AND WE TRANSMIT D SIGNALS, THEN PROB. OF ALL SIGNALS FADING

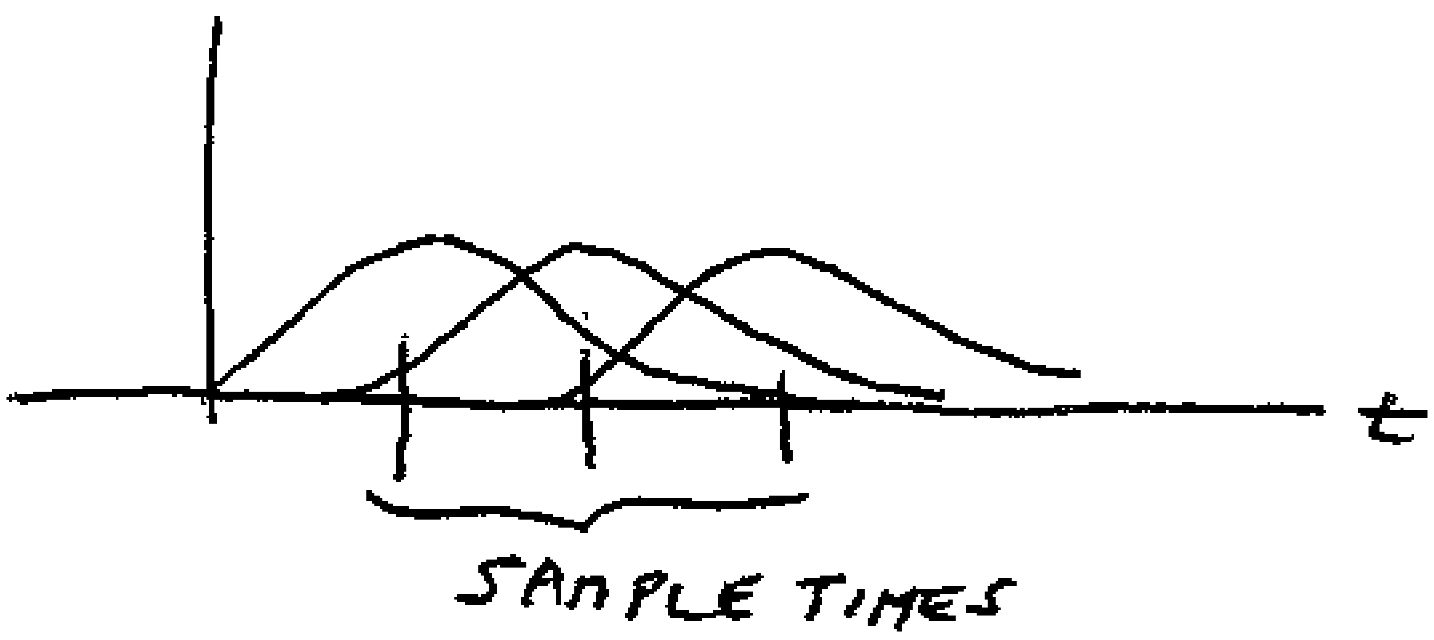
$= p^D \to 0$ FOR LARGE D.

MULTICARRIER MODULATION AND OFDM

IF $T << T_m$, WHERE T_m IS DURATION OF MULTIPATH, THERE WILL BE ISI.

EXAMPLE :





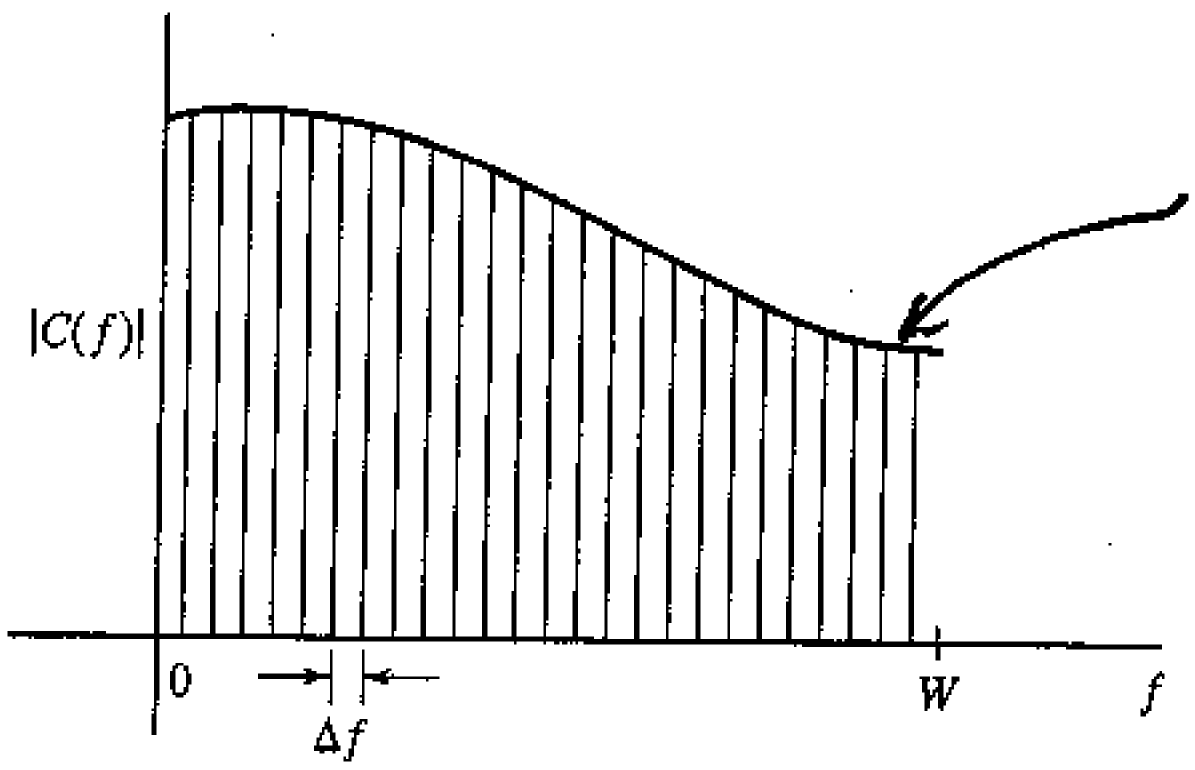
WILL NEED CAREFUL DESIGN OF ADAPTIVE EQUALIZER SINCE IN GENERAL MULTIPATH CHANGES WITH TIME. ALTERNATIVELY, MAYBE WE CAN AVOID MULTIPATH EFFECTS.

WANT T (SYMBOL INTERVAL, FOR EXAMPLE QAM) LONGER SO THAT $T >> T_m$. TO MAINTAIN DATA RATE USE MULTIPLE CARRIERS.

NOW IF WE HAVE W Hz BANDWIDTH

AND $\Delta f = 1/T$ (T IS LARGER NOW $\Rightarrow$ Δf IS SMALLER)

$K = W/\Delta f$ = NUMBER OF <u>SUB</u>CARRIERS WE CAN ALLOCATE



CHANNEL IS NEARLY IDEAL OVER THIS SMALL BANDWIDTH

Figure 11.9 Subdivision o
channel bandwidth W into n
subchannels of equal width Δ

USING SUBCARRIER FREQUENCIES IS CALLED FREQ. DIVISION MULTIPLEXING (FDM)

$$x_k(t) = \cos 2\pi f_k t \qquad k = 0, 1, \ldots, K-1$$

FURTHERMORE, LETTING $\Delta f = 1/T$ WE HAVE <u>ORTHOGONAL</u> SUBCARRIERS OR

$$\int_0^T \overbrace{A_k \cos(2\pi f_k t + \phi_k)}^{\text{QAM SYMBOL}} A_j \cos(2\pi f_j t + \phi_j)\, dt = 0$$

CALLED <u>ORTHOGONAL</u> FDM = OFDM

USED FOR WIRELESS LOCAL AREA NETWORKS (LAN) - WIRELESS ROUTERS

OFDM

ASSUME WE USE QAM FOR EACH SUBCARRIER
LET $1/T_s$ = SYMBOL RATE FOR SINGLE CARRIER
WE NOW DECREASE THIS TO $1/T = \frac{1}{KT_s}$ SO THAT $T \gg T_m$.

$$u_k(t) = \sqrt{2/T}\, A_{kc} \cos 2\pi f_k t + \sqrt{2/T}\, A_{ks} \sin 2\pi f_k t \qquad 0 \le t \le T$$
$$k = 0, 1, \ldots, K-1$$

WHERE $f_k = f_c + k/T$

WE TRANSMIT $u(t) = \sum_{k=0}^{K-1} u_k(t)$.

LET $\sqrt{A_{kc}^2 + A_{ks}^2} = A_k$

$$\theta_k = \text{ARCTAN}\, \frac{-A_{ks}}{A_{kc}}$$

$$u_k(t) = \sqrt{2/T}\, A_k \cos(2\pi f_k t + \theta_k)$$

$$= Re\left(\sqrt{2/T}\, \underbrace{A_k e^{j\theta_k}}_{x_k \leftarrow \text{COMPLEX NUMBER}} e^{j2\pi f_k t}\right)$$

NOTE THAT X_k IS THE SIGNAL CONSTELLATION POINT $(Re(X_k), Im(X_k))$

THE RECEIVED SIGNAL ON k^{th} SUBCHANNEL IS MODIFIED BY CHANNEL RESPONSE

$C(f_k) = |C(f_k)|\, e^{j\phi(f_k)} = |C_k|\, e^{j\phi_k} = C_k$

SO THAT $r(t) = \sum_{k=0}^{K-1} r_k(t)$, WHERE

$$r_k(t) = \sqrt{2/T}\,|C_k|\,A_{kc}\cos(2\pi f_k t + \phi_k) + \sqrt{2/T}\,|C_k|\,A_{ks}\sin(2\pi f_k t + \phi_k) + n_k(t)$$

$$= \mathrm{Re}\left(\sqrt{2/T}\, C_k X_k\, e^{j2\pi f_k t}\right) + n_k(t)$$

USUALLY $C_k = C(f_k)$ IS ESTIMATED PRIOR TO TRANSMISSION.

TO DEMODULATE: USE

ESTIMATE

$$\psi_{1_k}(t) = \sqrt{2/T}\cos(2\pi f_k t + \hat{\phi}_k)$$

$$\psi_{2_k}(t) = \sqrt{2/T}\sin(2\pi f_k t + \hat{\phi}_k)$$

THIS YIELDS $y_k = (|C_k|A_{kc},\ |C_k|A_{ks}) + (n_{kc},\ n_{ks})$

OR $\dfrac{y_k}{|\hat{C}_k|} = (A_{kc}, A_{ks}) + (n'_{kc}, n'_{ks})$

USE MINIMUM DISTANCE RECEIVER TO DECODE.

FINALLY, NOTE THAT TO DEMODULATE

$$y_{kc} = \int_0^T r(t)\,\psi_{1_k}(t)\,dt$$

$$y_{ks} = \int_0^T r(t)\,\psi_{2_k}(t)\,dt$$

$$y_{kc} = \int_0^T r(t)\sqrt{2/T}\cos(2\pi f_k t + \hat{\phi}_k)\,dt$$

$$y_{ks} = \int_0^T r(t)\sqrt{2/T}\sin(2\pi f_k t + \hat{\phi}_k)\,dt$$

$\tilde{y}_k = y_{kc} - j\,y_{ks}$ $\qquad \sim \Rightarrow$ COMPLEX

$$= \int_0^T r(t)\Big[\sqrt{2/T}\cos(2\pi f_k t + \hat{\phi}_k) - \sqrt{2/T}\,j\sin(2\pi f_k t + \hat{\phi}_k)\Big]\,dt$$

$$= \int_0^T r(t)\sqrt{2/T}\,e^{-j(2\pi f_k t + \hat{\phi}_k)}\,dt$$

$$= \sqrt{2/T}\,e^{-j\hat{\phi}_k}\underbrace{\int_0^T r(t)\,e^{-j2\pi f_k t}\,dt}$$

$k = 0, 1, \ldots, K-1$

= FOURIER TRANSFORM AT $f = f_k$

IF $f_k = k\Delta f = k\frac{W}{K}$ AND $\Delta t = T/K$

$$\int_0^T r(t)\,e^{-j2\pi f_k t}\,dt \approx \sum_{n=0}^{K-1} r(n\Delta t)\,e^{-j2\pi k\Delta f n\Delta t}\,\Delta t$$

$$= \Delta t \sum_{n=0}^{K-1} r(n\Delta t)\,e^{-j2\pi k\frac{W}{K} n\frac{T}{K}}$$

BUT $\frac{WT}{K} = \frac{W}{K}\frac{1}{\Delta f} = 1$

$$\Rightarrow \int_0^T r(t)\, e^{-j2\pi f_k t}\, dt$$

$$\simeq \frac{T}{K} \sum_{n=0}^{K-1} r(n\Delta t)\, e^{-j2\pi \frac{kn}{K}}$$

$$k = 0, 1, \ldots, K-1$$

THIS IS IN THE FORM OF A DFT

$\Rightarrow$ USE FFT TO REDUCE COMPUTATION.

SEE BOOK FOR FURTHER DETAILS.

SPREAD SPECTRUM

USED TO MITIGATE EFFECTS OF INTERFERENCE

EXAMPLE: IN MILITARY APPLICATIONS - INTENTIONAL JAMMER, IN COMMERCIAL APPLICATIONS - CROSSTALK DUE TO MULTIPLE CELL PHONE USERS IN SAME CELL

IN EITHER CASE IDEA IS TO SPREAD SIGNAL IN BANDWIDTH. WE DO THIS USING PSEUDORANDOM CODE.

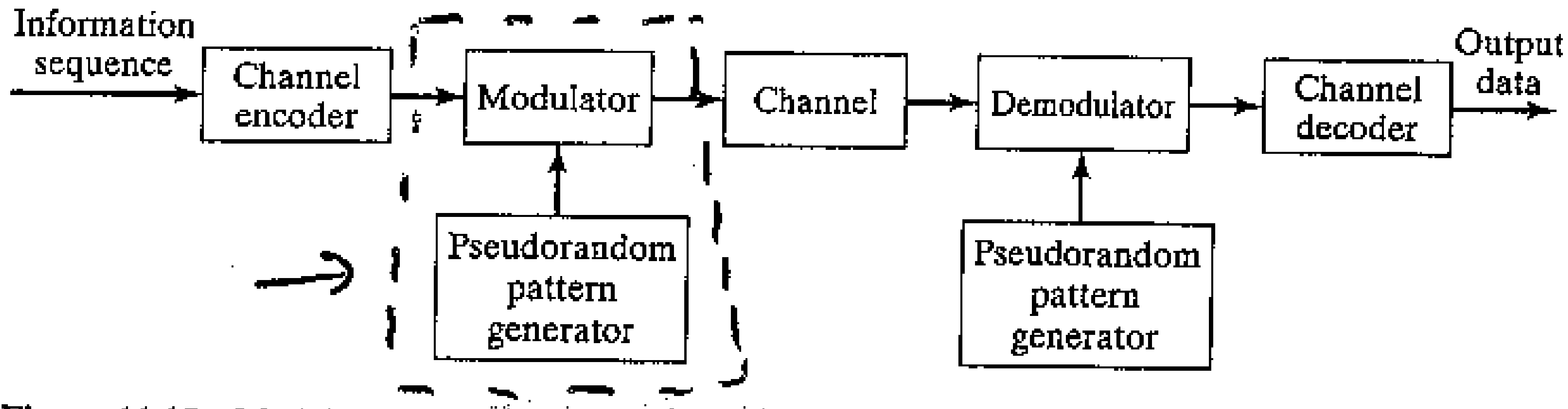
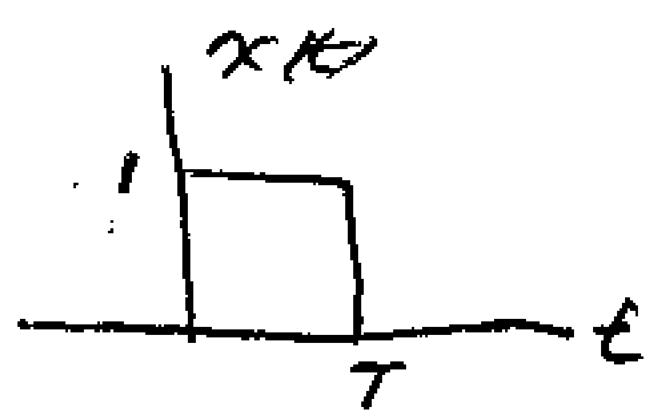
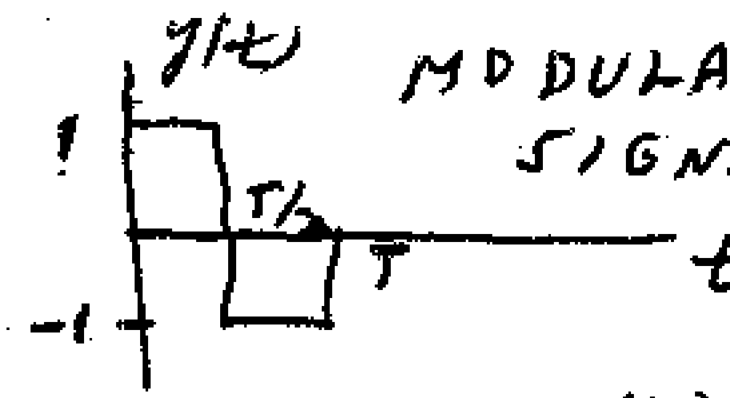


Figure 11.15 Model of a spread-spectrum digital communication system.

BY MODULATING WAVEFORM WE INCREASE ITS BANDWIDTH

EXAMPLE :



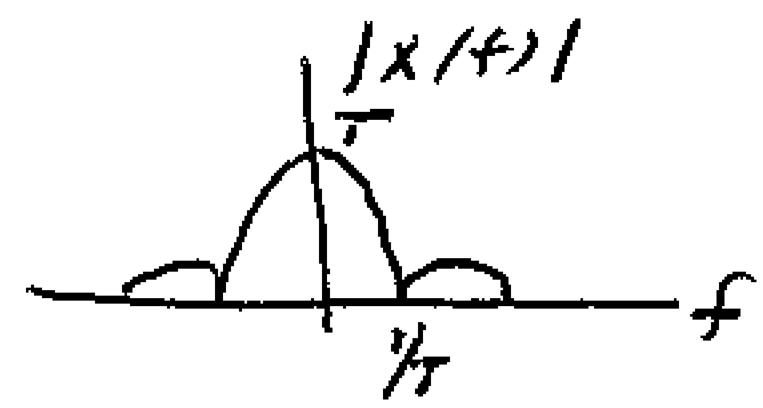


MODULATING SIGNAL

$y(t) = s(t) - s(t - T/2)$

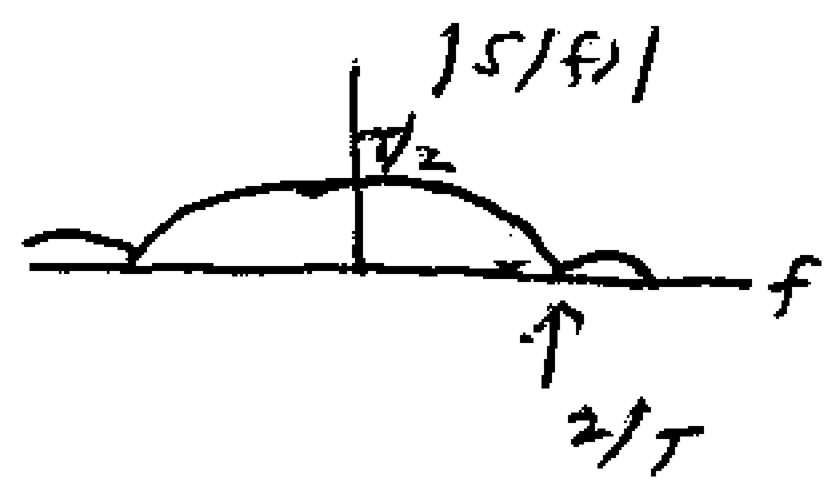
$x(t)\, y(t) = y(t)$

$|X(f)| = \left| T \dfrac{\sin \pi f T}{\pi f T} \right|$



$Y(f) = S(f) - S(f) e^{-j 2\pi f T/2}$

$|Y(f)| = \underbrace{|S(f)|}_{\left| T/2 \frac{\sin \pi f T/2}{\pi f T/2} \right|} \left| 1 - e^{-j 2\pi f T/2} \right|$



WANT MODULATING SIGNAL TO BE WIDEBAND, MUCH LIKE WHITE NOISE (NOT SINUSOIDAL)

PSEUDORANDOM NOISE (PN) USED, ALSO ADVANTAGEOUS SINCE TRANSMITTED SIGNAL

APPEARS TO AN ADVERSARY AS RECEIVER NOISE (COVERT TRANSMISSION).

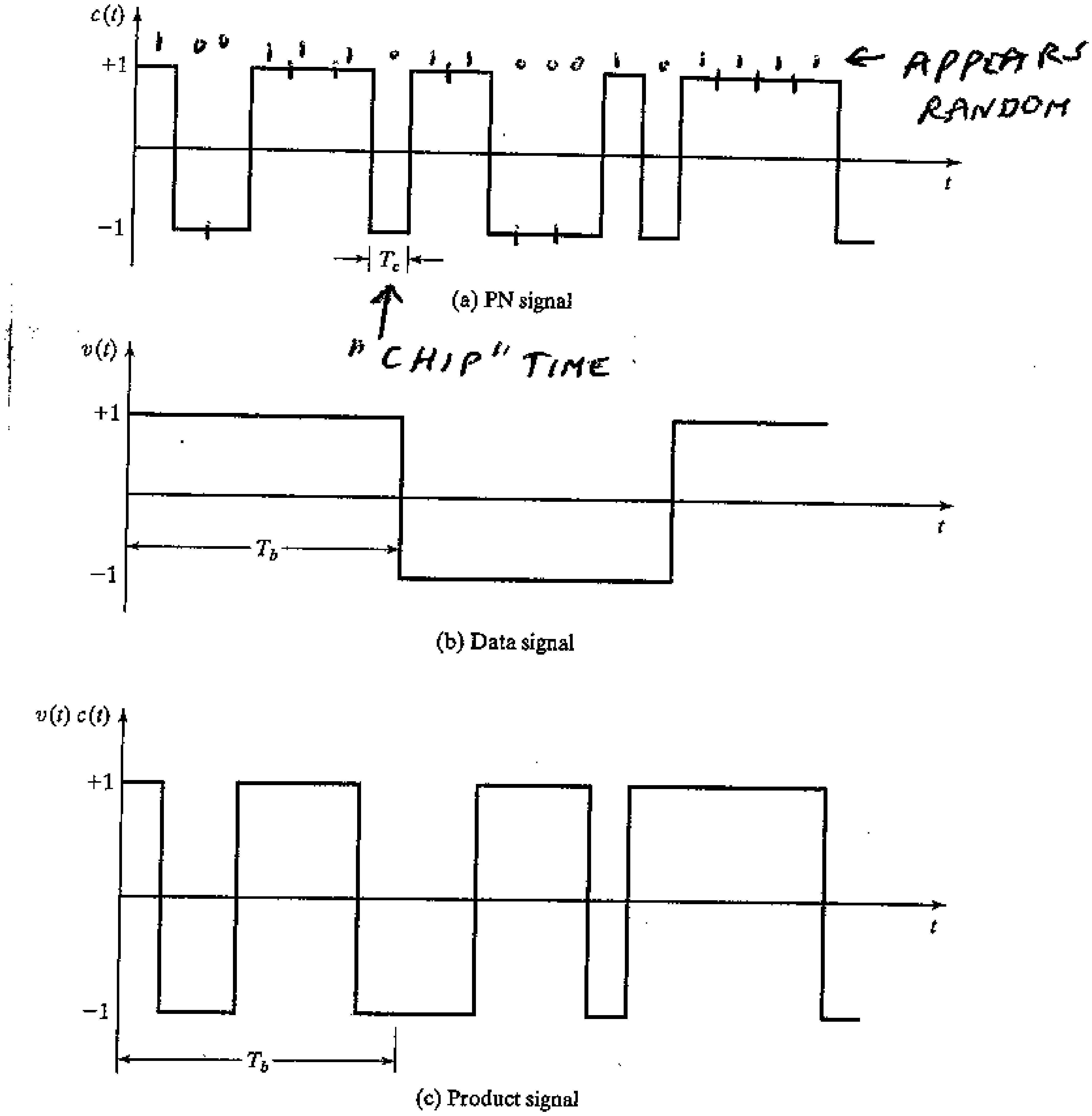


Figure 11.16 Generation of a DS spread-spectrum signal.

INFORMATION RATE $= R_b = 1/T_b$ BUT $T_c << T_b \Rightarrow$ BANDWIDTH OF CHANNEL NEEDS TO BE MUCH LARGER OR

$$\frac{1}{T_c} \simeq B_c >> R_b = \frac{1}{T_b}$$

THE EXPANSION FACTOR IS $\frac{B_c}{R_b} = \frac{T_b}{T_c}$.

CALLED A DIRECT SEQUENCE (DS) SPREAD SPECTRUM SIGNAL.

NOTE: $v(t) = \sum_{n=-\infty}^{\infty} \overset{\pm 1}{a_n}\, g_{T_b}(t - nT_b)$

$c(t) = \sum_{n=-\infty}^{\infty} c_n\, g_{T_c}(t - nT_c)$

$C_n = \pm 1$ – CALLED SPREADING SEQUENCE OR CODE

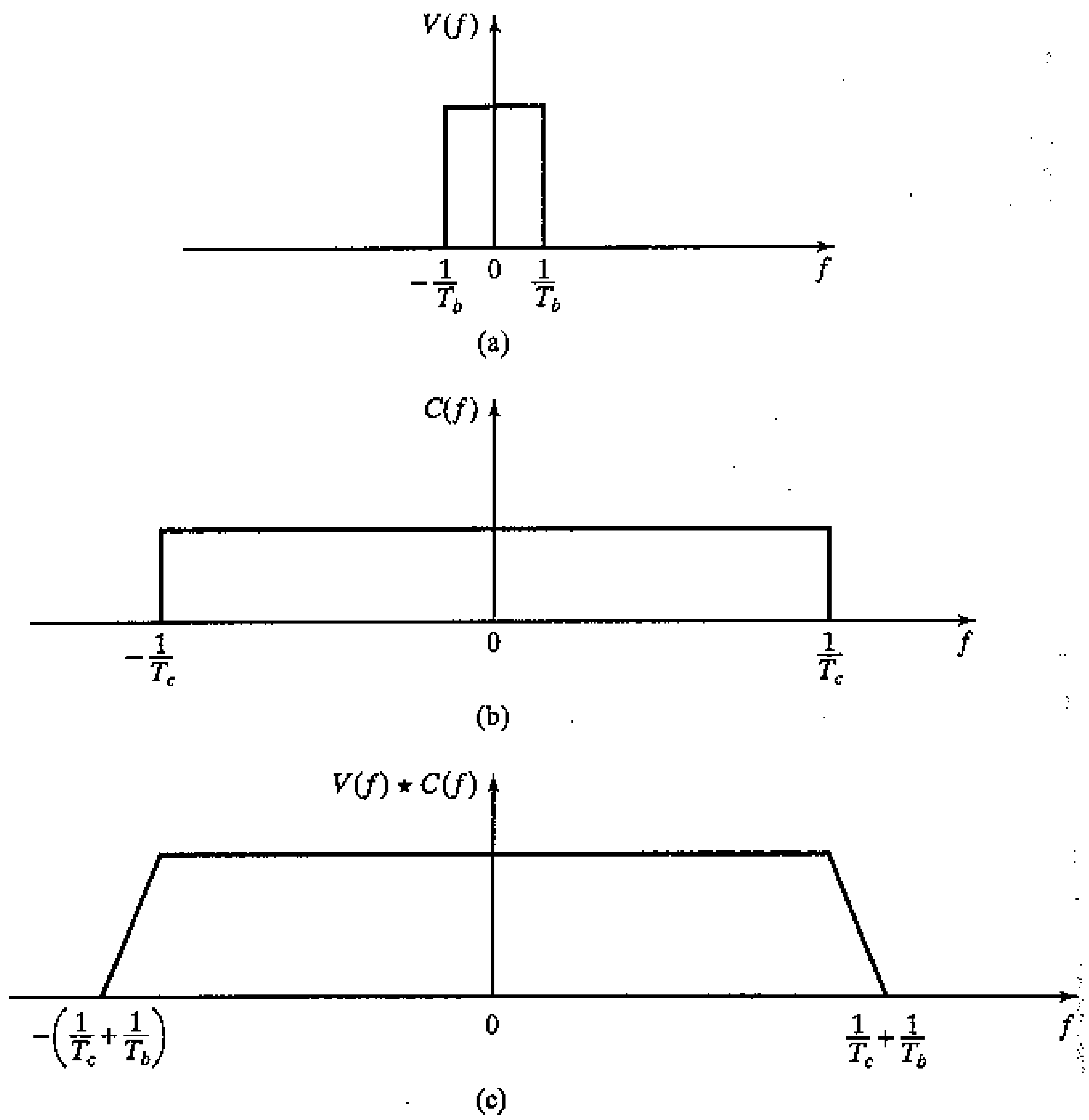


Figure 11.17 Convolution of the spectra of the (a) data signal with the (b) PN code signal.

TRANSMITTED SIGNAL IS

$$u(t) = A_c \underbrace{v(t)\,c(t)}\cos 2\pi f_c t$$

$$\mathfrak{F}\{v(t)\,c(t)\} = V(f) * C(f)$$

↑ CAUSES SPREADING

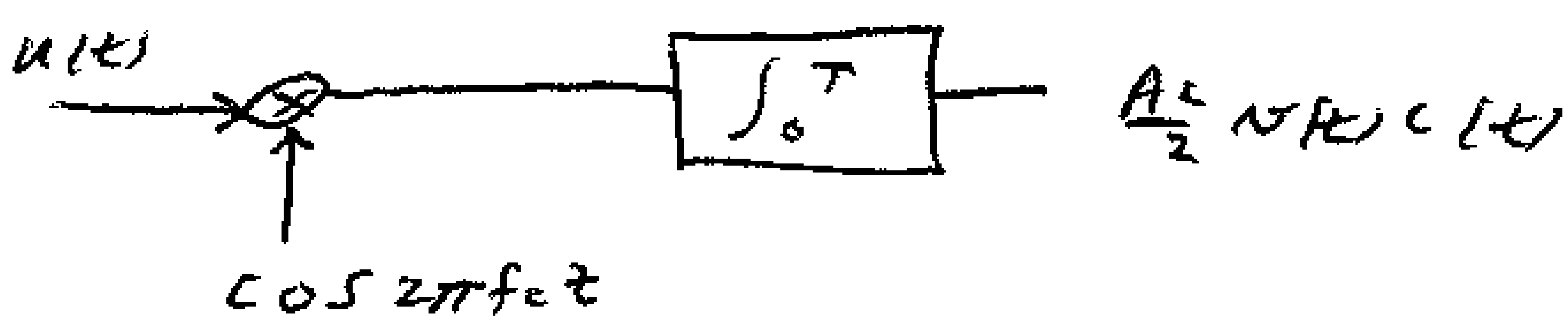
ALSO, SINCE $v(t) = \pm 1$, $c(t) = \pm 1$,
$v(t)\,c(t) = \pm 1$ AND

$$u(t) = A_c \cos(2\pi f_c t + \theta(t))$$

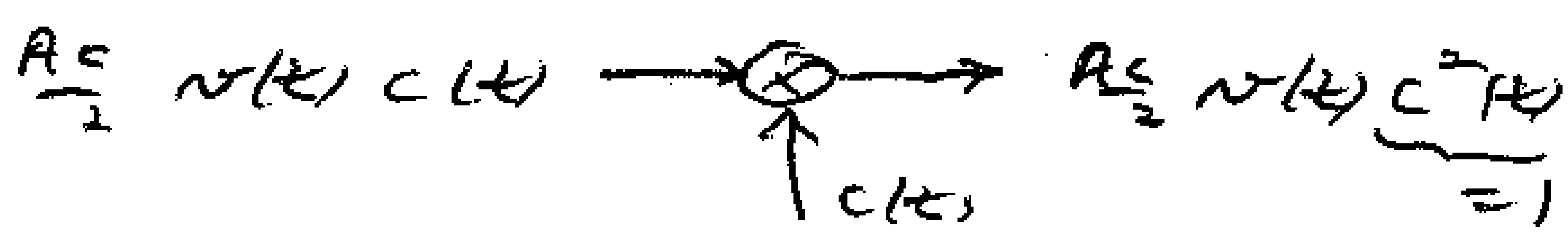
$$\theta(t) = \begin{cases} 0 & \text{IF } v(t)c(t) = 1 \\ \pi & \text{IF } v(t)\,c(t) = -1 \end{cases}$$

$\Rightarrow$ TRANSMITTED SIGNAL IS BPSK AT THE HIGHER BIT RATE $1/T_c$.

TO DEMODULATE



TO DESPREAD



MITIGATION OF NARROWBAND INTERFERENCE

$$r(t) = A_c\, v(t)\, c(t) \cos 2\pi f_c t + i(t)$$

INTERFERENCE

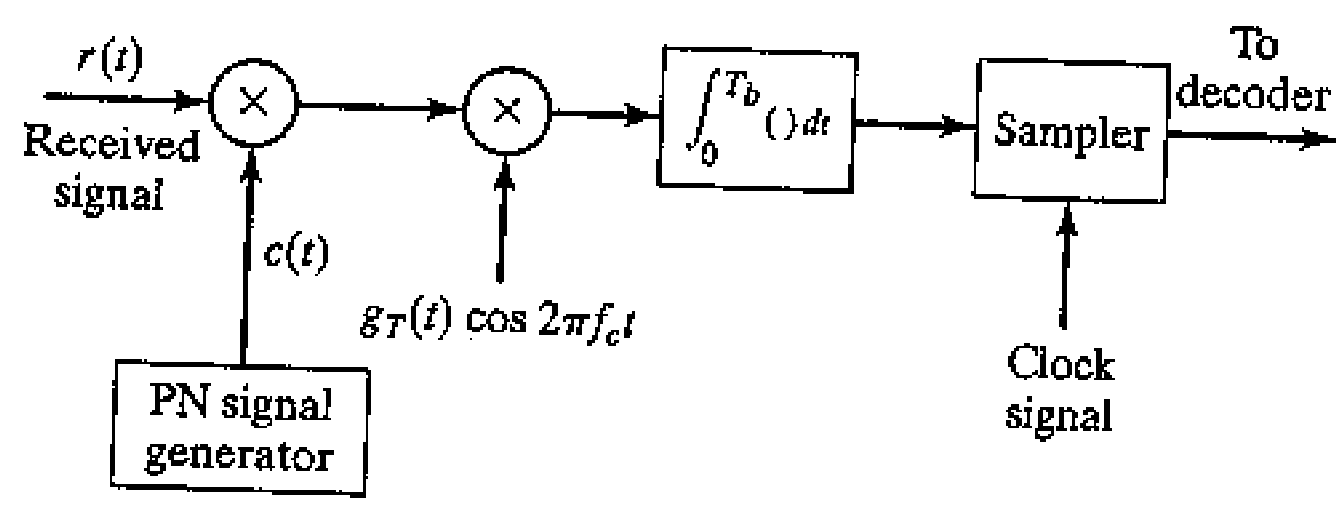


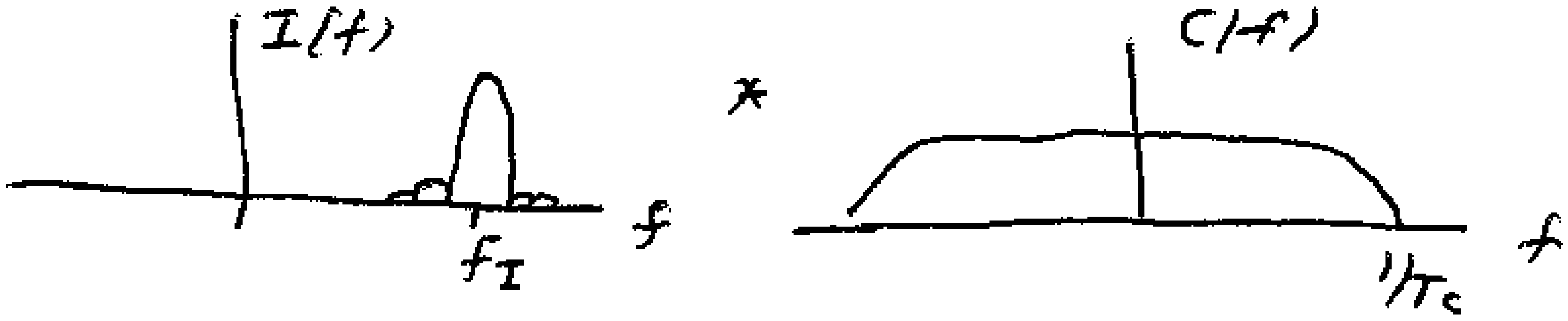
Figure 11.18 Demodulation of DS spread-spectrum signal.

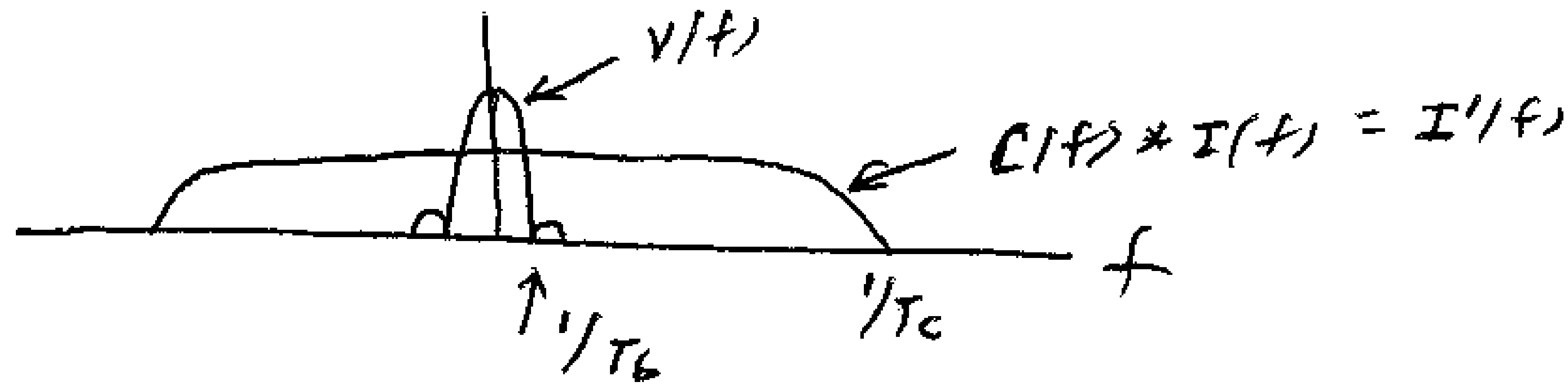
DESPREADING

$$r(t)\,c(t) = A_c\, v(t) \underbrace{c^2(t)}_{1} \cos 2\pi f_c t + c(t)\,i(t)$$

INTERFERENCE IS NOW $c(t)\,i(t)$. DATA SIGNAL $v(t)$ IS LOCATED IN BAND OF WIDTH $1/T_b = R$ Hz. BUT HOW ABOUT $i'(t) = c(t)\,i(t)$?

LET $i(t) = A_I \cos 2\pi f_I t$ SO THAT AVERAGE POWER IS $A_I^2/2$. ALSO $c(t)\,i(t)$ HAS SAME AVERAGE POWER (WHY?) BUT BANDWIDTH OF $c(t)\,i(t)$ IS FOUND FROM $C(f) * I(f) \approx C(f) \Rightarrow$ BANDWIDTH $\approx 1/T_c$ Hz





AT THE DEMODULATOR OUTPUT (AFTER MATCHED FILTERING OF N(t) WAVEFORM) THE INTERFERENCE POWER IS REDUCED DUE TO ALL THE FREQ. COMPONENTS ABOVE $1/T_b$ BEING FILTERED OUT.

TOTAL INTERFERENCE POWER AT MF OUTPUT =

$$2\int_0^{1/T_c} |H(f)|^2 \, P_{I'}(f)\, df$$

↑ MATCHED FILTER ≈ 1 OVER $|f| \le 1/T_b$

$$P_{I'}(f) = \frac{A_I^2/2}{2/T_c} \qquad |f| \le 1/T_c$$

(TOTAL POWER UNCHANGED)

$$\approx 2\int_0^{1/T_b} P_{I'}(f)\, df = \frac{A_I^2/2}{2/T_c} \cdot \frac{2}{T_b}$$

$$\frac{\text{INTERFERENCE POWER OUT}}{\text{INTERFERENCE POWER IN}} = \frac{T_c}{T_b} \ll 1$$

OR T_b/T_c = <u>PROCESSING GAIN</u>

= IMPROVEMENT IN SIGNAL-TO-INTERENCE RATIO (SIR)

$$= \frac{W}{R_b}$$

W = BANDWIDTH OF SPREAD SPECTRUM SIGNAL

OTHER APPLICATIONS ARE:

1) LOW PROBABILITY OF INTERCEPT (LPI) - CODE ONLY KNOWN TO INTENDED RECEIVER AND SPECTRAL SPREADING MAKES SIGNAL APPEAR AS NOISE

2) CODE DIVISION MULTIPLE ACCESS (CDMA) - USED IN CELL PHONE NETWORKS SINCE IF WE CORRELATE (DESPREAD) USING WRONG CODE GET SMALL OUTPUT ⇒ NO "CROSSTALK"

SEE BOOK

PN SEQUENCES

SEQUENCE OF 0'S AND 1'S THAT LOOK RANDOM (WHITE NOISE) BUT ARE A DETERMINISTIC SEQUENCE.

LET b_n = n^{th} BIT OF L LENGTH SEQUENCE

ONE IMPORTANT EXAMPLE IS AN m BIT MAXIMAL LENGTH SHIFT-REGISTER SEQUENCE, WHERE $L = 2^m - 1$ BITS. IT IS PERIODIC WITH PERIOD L. CHOOSE m SO THAT IT DOESN'T REPEAT FOR LENGTH DESIRED.

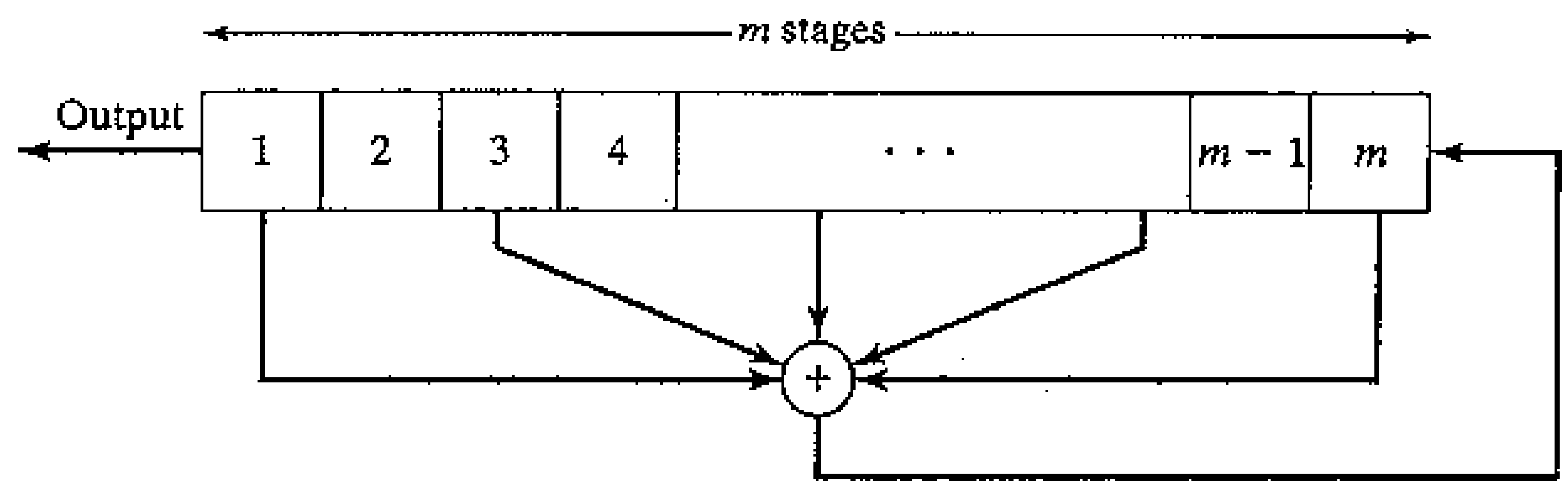
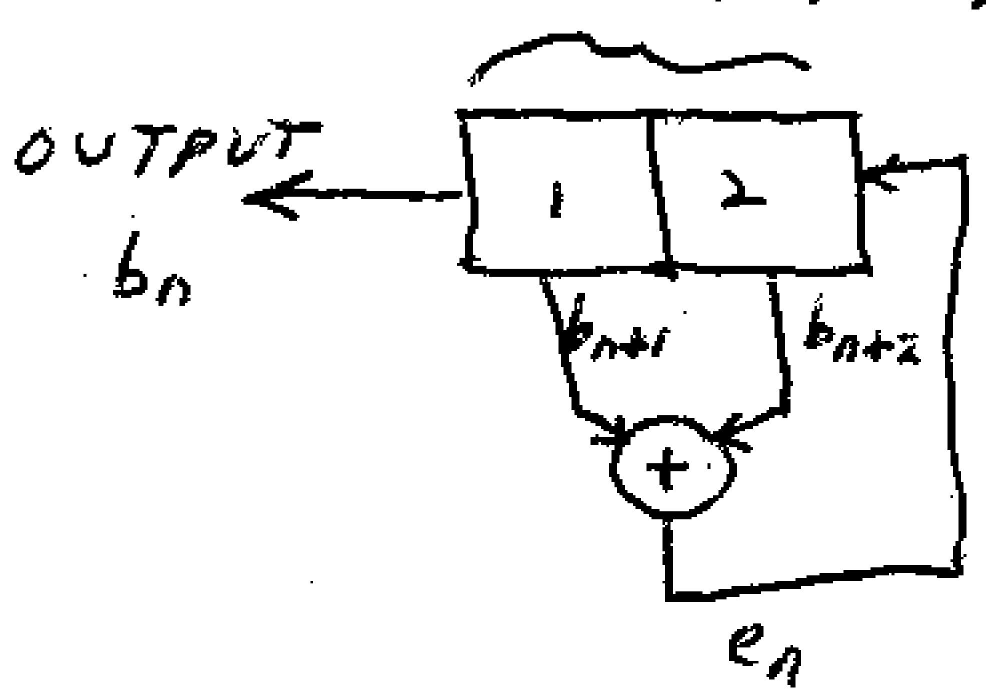


Figure 11.20 General m-stage shift register with linear feedback.

EXAMPLE : $m = 2 \Rightarrow L = 2^2 - 1 = 3$

SHIFT REGISTER (CONTAINS 00 OR 01 OR 10 OR 11)



$\oplus$ = MODULO 2 ADDER

$e_n = b_{n+1} \oplus b_{n+2}$

BUT $b_n = e_{n-3} \Rightarrow b_n = b_{n-1} \oplus b_{n-2}$

ASSUME INITIALLY WE HAVE $b_0 = 1$, $b_1 = 1$

$\Rightarrow b_2 = 1 \oplus 1 = 0$

$b_3 = b_2 \oplus b_1 = 0 \oplus 1 = 1$

$b_4 = b_3 \oplus b_2 = 1 \oplus 0 = 1$

$b_5 = b_4 \oplus b_3 = 1 \oplus 1 = 0$

$b_6 = b_5 \oplus b_4 = 0 \oplus 1 = 1$

$b_7 = b_6 \oplus b_5 = 1 \oplus 0 = 1$

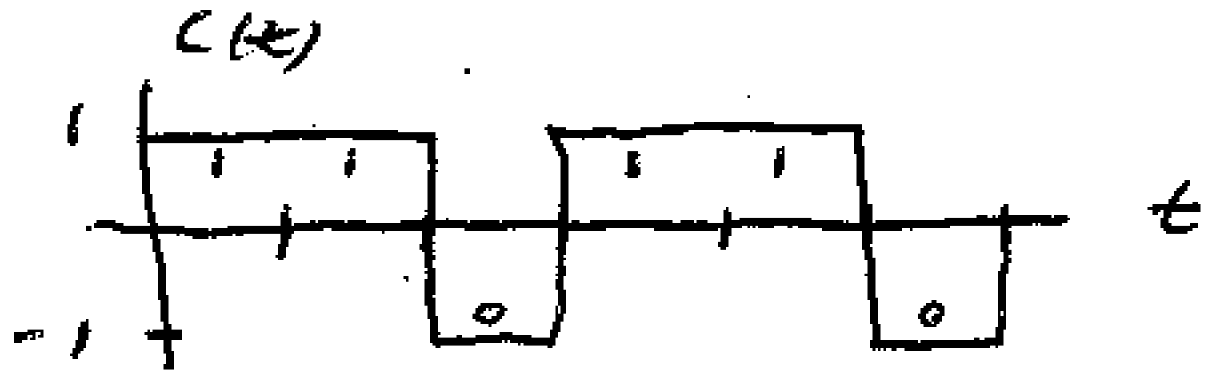
1 1 0 1 1 0 1 1 0 ...

$\uparrow$ $\uparrow$ $\longrightarrow$ TIME

$n=0$ $n=1$ ...

PERIOD $= L = 2^m - 1 = 4 - 1 = 3$

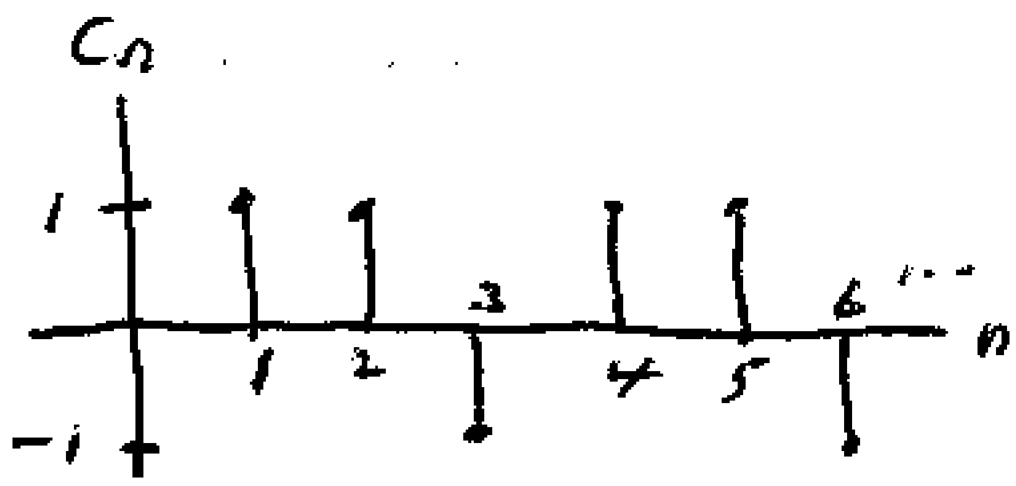
CONVERT TO ± 1 WITH $1 \to +1,\ 0 \to -1$ (BIPOLAR SEQUENCE)



SUPPOSED TO BE NOISE-LIKE

DEFINE THE AUTOCORRELATION FUNCTION AS

$$R_c(m) = \sum_{n=1}^{L} c_n c_{n+m} \qquad m = 0, 1, \ldots, L-1$$



$$R_c(m) = \sum_{n=1}^{3} c_n c_{n+m} \qquad m = 0, 1, 2$$

$$R_c(0) = \sum_{n=1}^{3} c_n^2 = 3$$

$$R_c(1) = \sum_{n=1}^{3} c_n c_{n+1} = c_1 c_2 + c_2 c_3 + c_3 c_4$$

$$= 1 \cdot 1 + 1 \cdot (-1) + (-1) \cdot 1$$

$$= -1$$

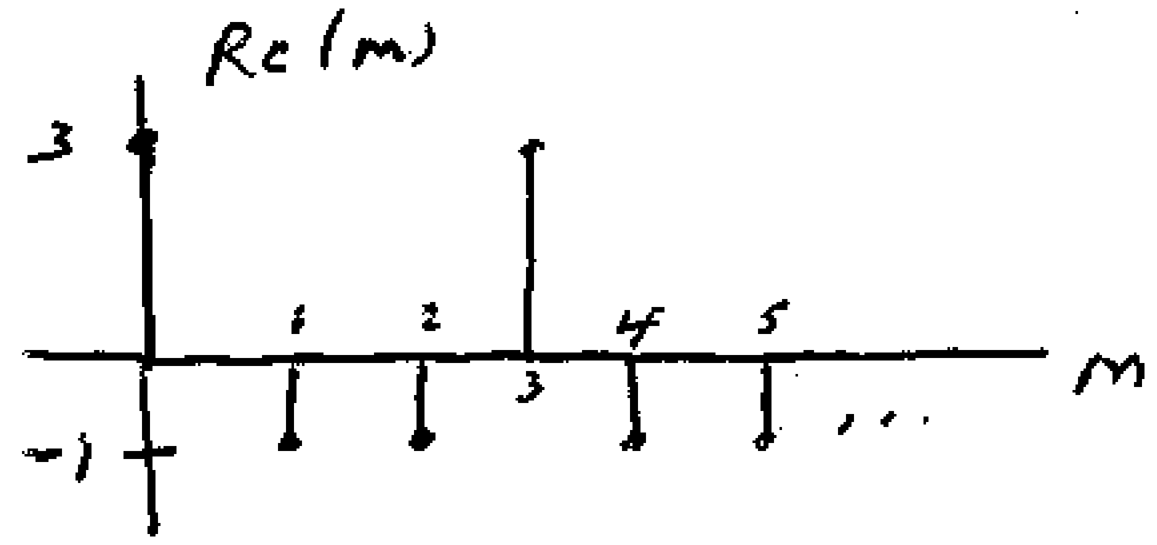
$$R_c(2) = \sum_{n=1}^{3} c_n c_{n+2} = c_1 c_3 + c_2 c_4 + c_3 c_5$$

$$= 1 \cdot (-1) + 1 \cdot 1 + (-1) \cdot 1$$

$$= -1$$

$R_c(3) = R_c(0)$, $R_c(4) = R_c(1)$, ...

PERIODIC WITH SAME PERIOD OF $L = 3$



IN GENERAL,

$$R_c(m) = \begin{cases} L & m = 0 \\ -1 & m = 1, 2, \ldots, L-1 \end{cases}$$

OR NORMALIZED AS $r_c(m) = R_c(m)/R_c(0)$

$$r_c(m) = \begin{cases} 1 & m = 0 \\ -1/L & m = 1, 2, \ldots, L-1 \end{cases}$$

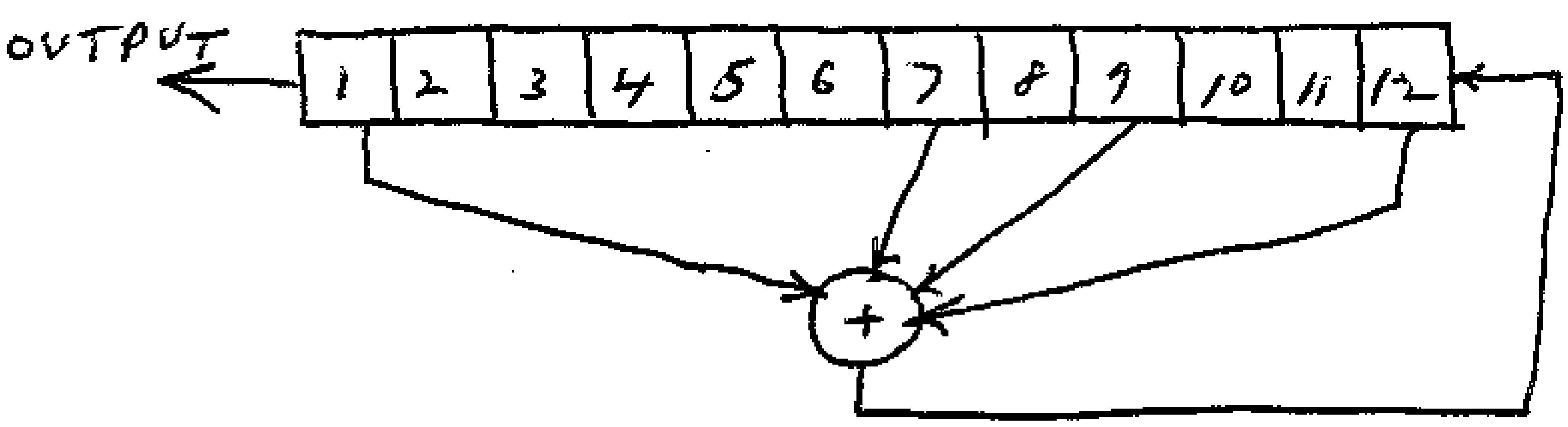
NEARLY THE AUTOCORRELATION OF WHITE NOISE! FOR EXAMPLE, IF $m = 12$ BIT REGISTER, $L = 2^m - 1 = 4095$

$1/L = 0.00244$.

THE SHIFT REGISTER CONNECTIONS ARE:

TABLE 11.2 SHIFT-REGISTER CONNECTIONS FOR GENERATING MAXIMUM-LENGTH SEQUENCES

m	Stages connected to modulo-2-adder	m	Stages connected to modulo-2-adder	m	Stages connected to modulo-2 adder
2	1, 2	13	1, 10, 11, 13	24	1, 18, 23, 24
3	1, 3	14	1, 5, 9, 14	25	1, 23
4	1, 4	15	1, 15	26	1, 21, 25, 26
5	1, 4	16	1, 5, 14, 16	27	1, 23, 26, 27
6	1, 6	17	1, 15	28	1, 26
7	1, 7	18	1, 12	29	1, 28
8	1, 5, 6, 7	19	1, 15, 18, 19	30	1, 8, 29, 30
9	1, 6	20	1, 18	31	1, 29
10	1, 8	21	1, 20	32	1, 11, 31, 32
11	1, 10	22	1, 22	33	1, 21
12	1, 7, 9, 12	23	1, 19	34	1, 8, 33, 34



$$b_n = b_{n-1} \oplus b_{n-7} \oplus b_{n-9} \oplus b_{n-12}$$

START WITH REGISTER CONTAINING ANY 12 BIT SEQUENCE EXCEPT <u>ALL</u> <u>ZEROS</u> (WHY?)

<u>RADIO CHANNELS</u>

NOW CONSIDER LINE OF SIGHT (LOS) MICROWAVE CHANNELS (AT GHz FREQ.S.)