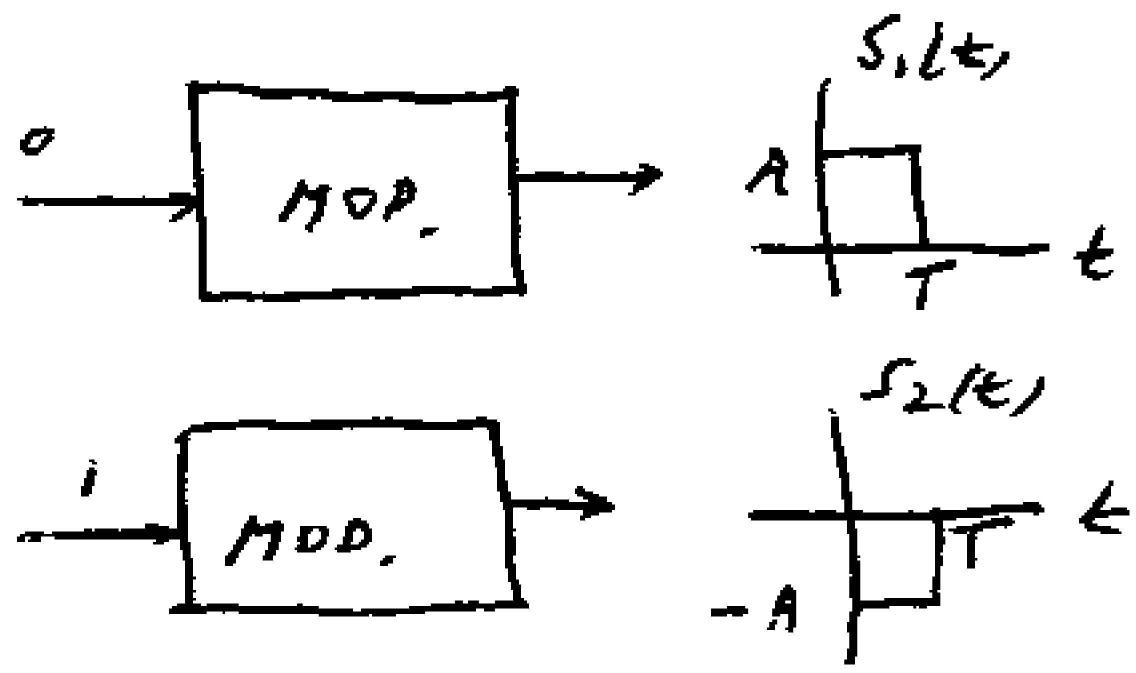


→ TO CHANNEL

$M = 2$ MESSAGES

↑ BINARY

COULD ALSO DEFINE MESSAGE AS <u>BLOCK</u> OF 2 BITS

$0\,0 \rightarrow S_1(t)$
$0\,1 \rightarrow S_2(t)$ $M = 4$
$1\,0 \rightarrow S_3(t)$
$1\,1 \rightarrow S_4(t)$

FOR k BITS NEED $M = 2^k$ WAVEFORMS. ADVANTAGEOUS TO REPRESENT USING <u>BASIS</u> WAVEFORMS OR SIGNAL.

<u>EXAMPLE</u> : BINARY CASE ABOVE

ENERGY = 1

$$S_1(t) = A\sqrt{T}\,\psi(t)$$
$$S_2(t) = -A\sqrt{T}\,\psi(t)$$

MORE GENERALLY FOR M WAVEFORMS

$$S_m(t) = \sum_{n=1}^{N} S_{mn}\,\psi_n(t) \qquad m = 1, 2, \ldots, M$$

COEFFICIENTS ORTHONORMAL

THIS IS A VECTOR SPACE REPRESENTATION.

BY ORTHOGONAL WE MEAN

$$\int_{-\infty}^{\infty} \psi_m(t)\,\psi_n(t)\,dt = 0 \qquad m \neq n$$

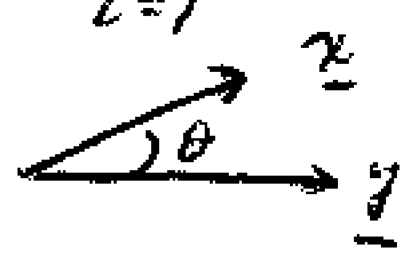
BY ORTHO<u>NORMAL</u>

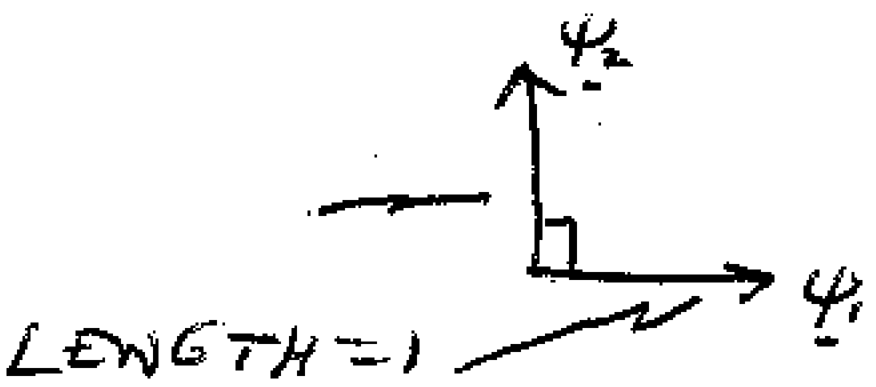
$$\int_{-\infty}^{\infty} \psi_m(t)\,\psi_n(t)\,dt = \begin{matrix} 0 & m \neq n \\ 1 & m = n \end{matrix}$$

JUST LIKE EUCLIDEAN VECTORS IN R^3

$$\underline{x} \cdot \underline{y} = x_1 y_1 + x_2 y_2 + x_3 y_3 = \sum_{i=1}^{3} x_i y_i$$

$$= \|\underline{x}\|\,\|\underline{y}\| \cos\theta$$





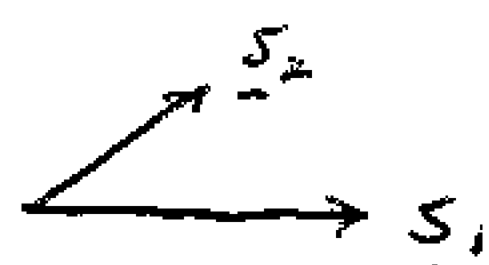
$$\text{LENGTH}^2 = \underline{\psi}_1 \cdot \underline{\psi}_1$$

$$= \|\underline{\psi}_1\|\,\|\underline{\psi}_1\| \cos 0$$

$$= \|\underline{\psi}_1\|^2 = 1$$

IF WE ARE GIVEN $\underline{s}_1, \underline{s}_2$

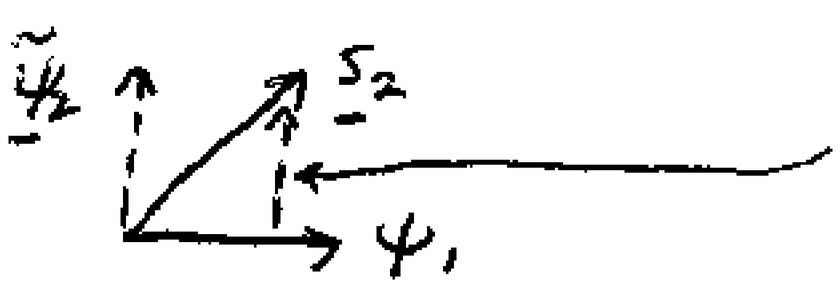
HOW CAN WE FIND $\underline{\psi}_1, \underline{\psi}_2$?



GRAM-SCHMIDT ORTHOGONALIZATION

$$\underline{\psi}_1 = \frac{\underline{s}_1}{\|\underline{s}_1\|} \qquad \text{NORMALIZE LENGTH}$$

$\Rightarrow$



$$\tilde{\underline{\psi}}_2 = \underline{s}_2 - \underline{s}_2 \cdot \underline{\psi}_1$$

$$\underline{\psi}_2 = \frac{\underline{s}_2 - \underline{s}_2 \cdot \underline{\psi}_1}{\|\underline{s}_2 - \underline{s}_2 \cdot \underline{\psi}_1\|}$$

TAKE EACH $\underline{s}_k$ AND PROJECT IT ONTO SUBSPACE SPANNED BY $\{\underline{s}_1, \underline{s}_2, \ldots, \underline{s}_{k-1}\}$, FIND ERROR VECTOR (WILL BE $\perp$ TO SUBSPACE) AND NORMALIZE TO UNITY LENGTH.

$$\Rightarrow \quad \underline{\psi}_1 = \frac{\underline{s}_1}{\|\underline{s}_1\|}$$

FOR $k = 2, \ldots, M$

$$\underline{\psi}_k = \frac{\underline{s}_k - \sum_{i=1}^{k-1} \underline{s}_k \cdot \underline{\psi}_i}{\left\| \underline{s}_k - \sum_{i=1}^{k-1} \underline{s}_k \cdot \underline{\psi}_i \right\|}$$

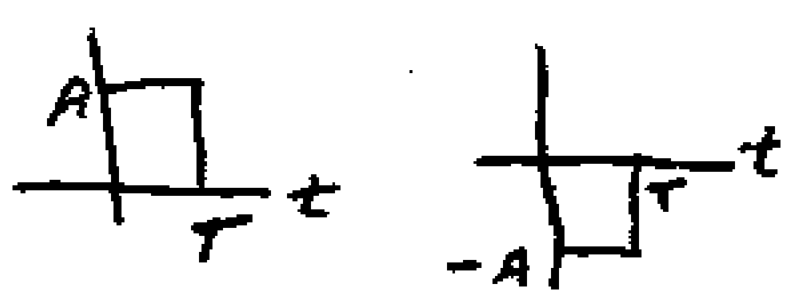
SAME PROCEDURE FOR WAVEFORMS WITH

$$\underline{s}_m \longrightarrow s_m(t)$$

$$\underline{s}_m \cdot \underline{s}_n \longrightarrow \int_{-\infty}^{\infty} s_m(t)\, s_n(t)\, dt$$

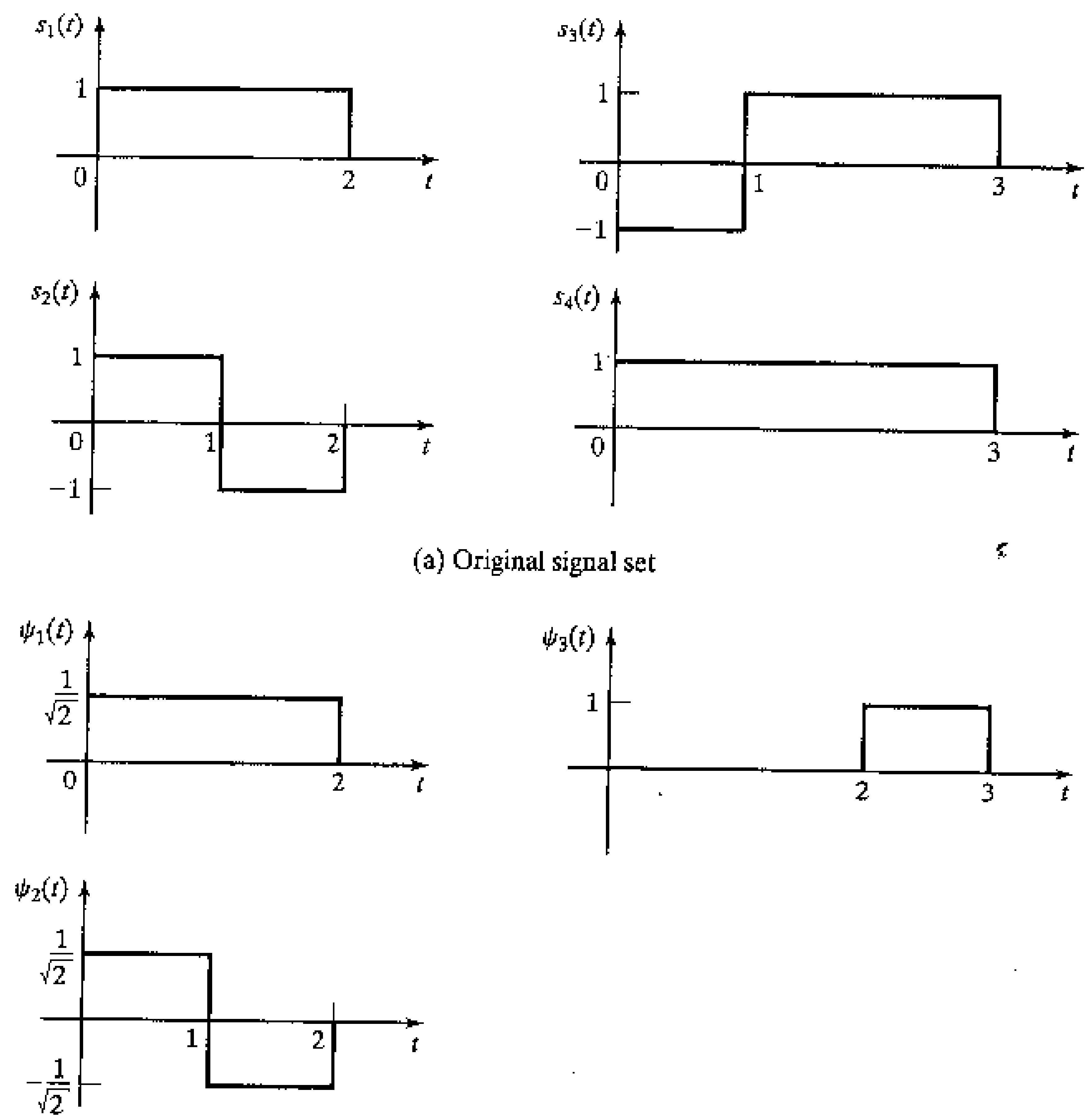
$$\underline{s}_m \cdot \underline{s}_m \longrightarrow \int_{-\infty}^{\infty} s_m^2(t)\, dt = \text{ENERGY} = \mathcal{E}_m$$

FOR M WAVEFORMS CAN HAVE $N \leq M$ ORTHONORMAL BASIS SIGNALS



$N = 1$
$M = 2$

EXAMPLE :



(b) Orthonormal waveforms

Figure 8.1 Application of the Gram–Schmidt orthogonalization procedure to signals $\{s_i(t)\}$.

1) NORMALIZE $s_1(t)$ $\Rightarrow$ $\psi_1(t) = s_1(t)/\sqrt{E_1} = s_1(t)/\sqrt{2}$

2) PROJECT $s_2(t)$ ONTO $\psi_1(t)$ $\Rightarrow$ $d_2(t) = s_2(t) - c_{21}\psi_1(t)$

$$c_{21} = \int_{-\infty}^{\infty} s_2(t)\psi_1(t)\,dt$$

$$= 0 \quad \Rightarrow \quad d_2(t) = s_2(t) \qquad s_1 \perp s_2$$

$$\psi_2(t) = s_2(t)/\sqrt{E_2} = s_2(t)/\sqrt{2}$$

3) PROJECT $s_3(t)$ ONTO $\psi_1(t)$, $\psi_2(t)$

$$d_3 = s_3(t) - c_{31}\psi_1(t) - c_{32}\psi_2(t)$$

$$c_{31} = \int_{-\infty}^{\infty} s_3(t)\,\psi_1(t)\,dt$$

$$= \int_0^2 \frac{1}{\sqrt{2}}\, s_3(t)\,dt = 0$$

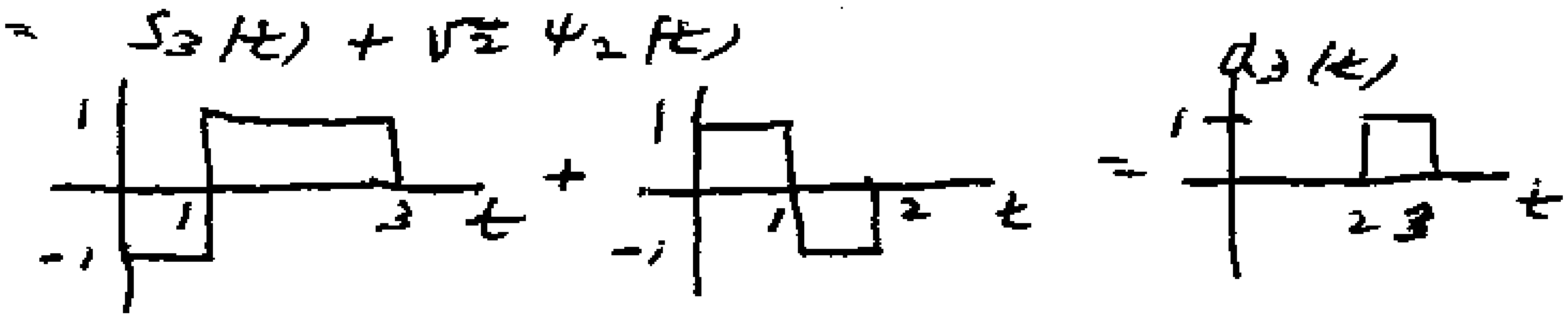
$$c_{32} = \int_{-\infty}^{\infty} s_3(t)\,\psi_2(t)\,dt$$

$$= \int_0^2 s_3(t)\,\psi_2(t)\,dt$$

$$= \int_0^1 + \int_1^2 = \int_0^1 (-1)(1/\sqrt{2})\,dt$$
$$+ \int_1^2 (1)(-1/\sqrt{2})\,dt$$

$$= -2/\sqrt{2} = -\sqrt{2}$$

$$d_3(t) = s_3(t) + \sqrt{2}\,\psi_2(t)$$



ETC.

NOW WE HAVE

$$s_m(t) = \sum_{n=1}^{N} s_{mn}\,\psi_n(t) \qquad m = 1, 2, \ldots, M$$

KNOWN

HOW DO WE FIND THESE?

$$\int_{-\infty}^{\infty} s_m(t)\,\psi_i(t)\,dt = \int_{-\infty}^{\infty} \sum_{n=1}^{N} s_{mn}\,\psi_n(t)\,\psi_i(t)\,dt$$

$$= \sum_{n=1}^{N} s_{mn} \underbrace{\int_{-\infty}^{\infty} \psi_n(t)\,\psi_i(t)\,dt}_{\delta_{ni} = \begin{cases} 1 & n = i \\ 0 & n \neq i \end{cases}}$$

$$= \sum_{n=1}^{N} s_{mn}\,\delta_{ni}$$

$$= s_{mi}$$

$$s_{mn} = \int_{-\infty}^{\infty} s_m(t)\,\psi_n(t)\,dt$$

NOW LET "COORDINATES" OF $s_m(t)$, WHICH ARE

$s_{m1}, s_{m2}, \ldots, s_{mN}$ BE REPRESENTED AS VECTOR (EUCLIDEAN) IN R^N, i.e.,

$$\underline{s}_m = (s_{m1}, s_{m2}, \ldots, s_{mN}) = \begin{bmatrix} s_{m1} \\ s_{m2} \\ \vdots \\ s_{mN} \end{bmatrix} \quad (N \times 1)$$

THEN $$\mathcal{E}_m = \int_{-\infty}^{\infty} s_m^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} \sum_n s_{mn} \psi_n(t) \sum_k s_{mk} \psi_k(t)\,dt$$

$$= \sum_n \sum_k s_{mn} s_{mk} \underbrace{\int_{-\infty}^{\infty} \psi_n(t) \psi_k(t)\,dt}_{\delta_{nk}}$$

$$= \sum_{n=1}^{N} s_{mn}^2 = \underline{s}_m \cdot \underline{s}_m$$

EUCLIDEAN DOT PRODUCT

SIMILARLY, $\int_{-\infty}^{\infty} s_m(t) s_n(t)\,dt = \underline{s}_m \cdot \underline{s}_n$

GIVEN BASIS SIGNALS WE HAVE REDUCED WAVEFORM SPACE TO SIMPLER EUCLIDEAN SPACE (CAN NOW USE LINEAR ALGEBRA).

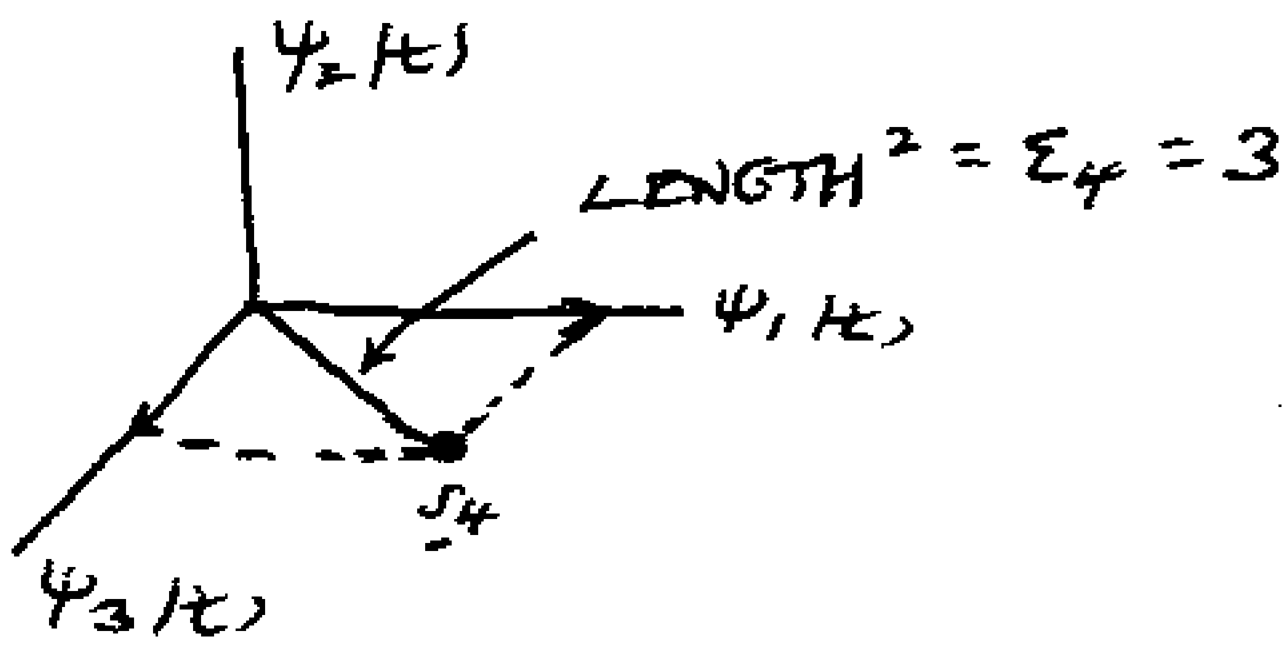
<u>EXAMPLE</u>: SEE FIG. 8.1 (PREVIOUS EXAMPLE)

$$s_4(t) = \sum_{n=1}^{3} s_{4n} \psi_n(t)$$

$s_4(t) = 1 \qquad 0 \le t \le 3$

$$\Rightarrow \quad s_{4n} = \int_{-\infty}^{\infty} s_4(t)\,\psi_n(t)\,dt = \int_0^3 1 \cdot \psi_n(t)\,dt$$

$$= \begin{cases} \sqrt{2} & n = 1 \\ 0 & n = 2 \\ 1 & n = 3 \end{cases} \quad \Rightarrow \quad \underline{s_4} = \begin{bmatrix} \sqrt{2} \\ 0 \\ 1 \end{bmatrix}$$



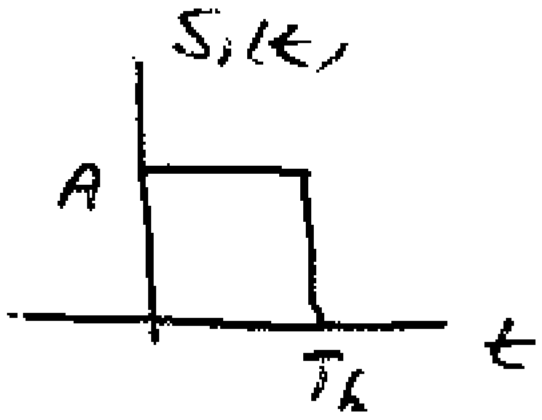
<u>NOTE</u>: SET OF BASIS SIGNALS NOT UNIQUE - DEPENDS ON WHICH IS CHOSEN FIRST, SECOND, ETC.

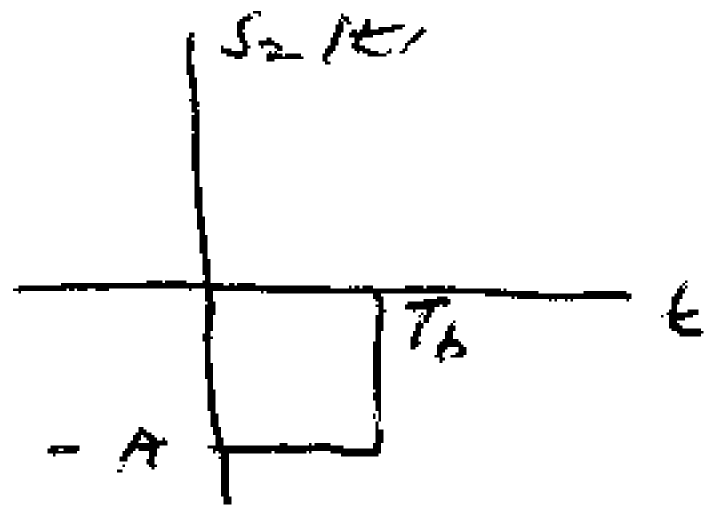
<u>BINARY PULSE MODULATION</u>

1) AMPLITUDE - PAM
2) POSITION - PPM

WE WILL ASSUME THAT WE NEED TO TRANSMIT $R_b = 1/T_b$ BPS $\Rightarrow$ LENGTH OF PULSE INTERVAL $= T_b$ SEC.

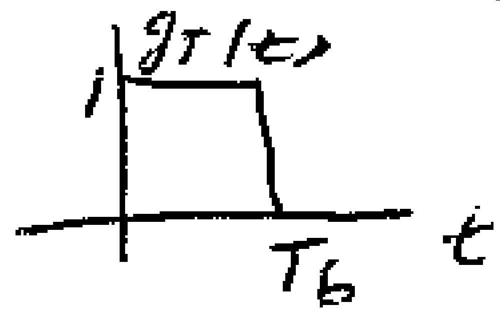
PAM





$s_m(t) = A_m\, g_T(t)$

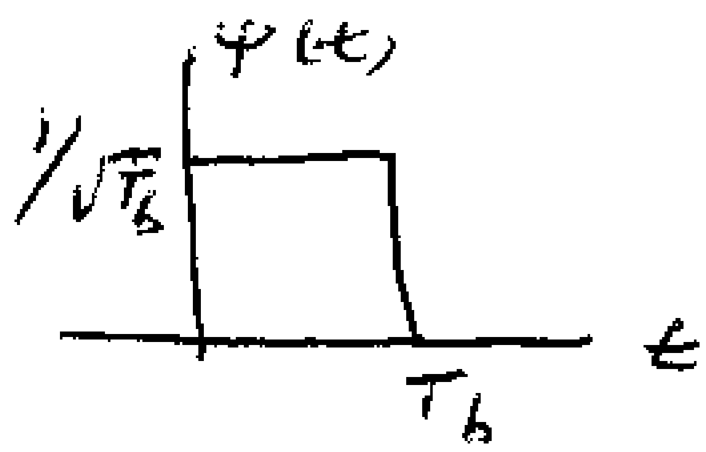
$m = 1, 2$



$A_1 = A$
$A_2 = -A$

CALLED ANTIPODAL SIGNALS

$s_2(t) = -s_1(t)$



$$s_m(t) = A_m \sqrt{T_b}\, \psi(t)$$

$$\mathcal{E}_m = \int_0^{T_b} A_m^2 T_b\, \psi^2(t)\, dt$$

$$= A_m^2 T_b = A^2 T_b$$

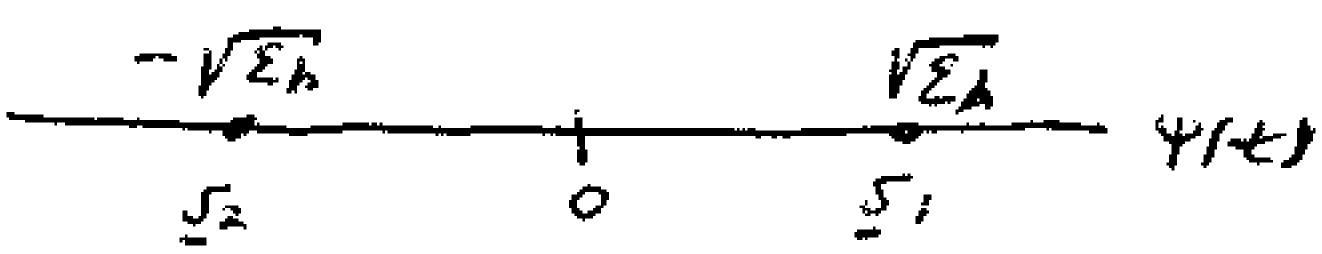
$\Rightarrow$ LET $\mathcal{E}_m = \mathcal{E}_b$ (BOTH SIGNALS HAVE SAME ENERGY)

$$s_m(t) = A_m \frac{\sqrt{\mathcal{E}_b}}{|A|}\, \psi(t) \qquad (T_b = \mathcal{E}_b / A^2)$$

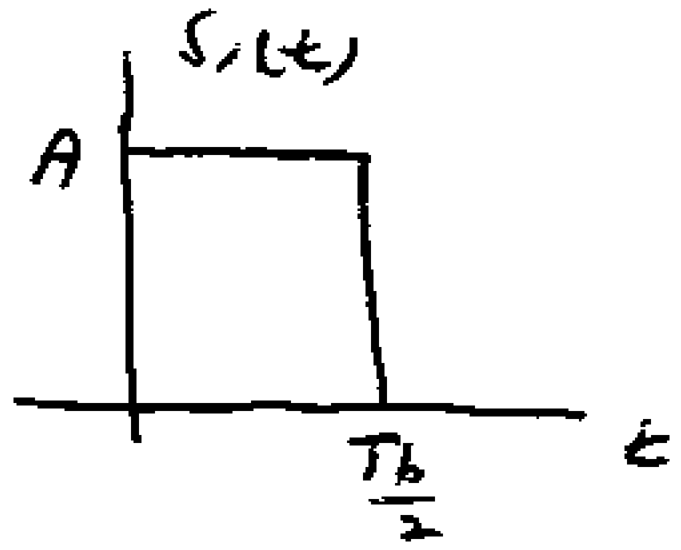
$$= \sqrt{\mathcal{E}_b}\, \psi(t) \quad m = 1$$

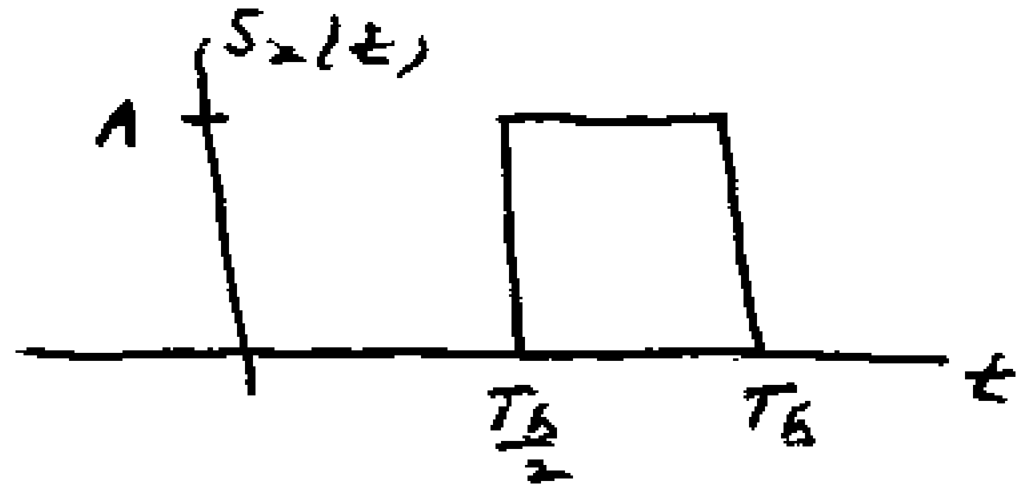
$$-\sqrt{\mathcal{E}_b}\, \psi(t) \quad m = 2$$

OR $\quad s_1 = \sqrt{\mathcal{E}_b}, \quad s_2 = -\sqrt{\mathcal{E}_b} \qquad (s_m(t) = s_m\, \psi(t))$



PPM



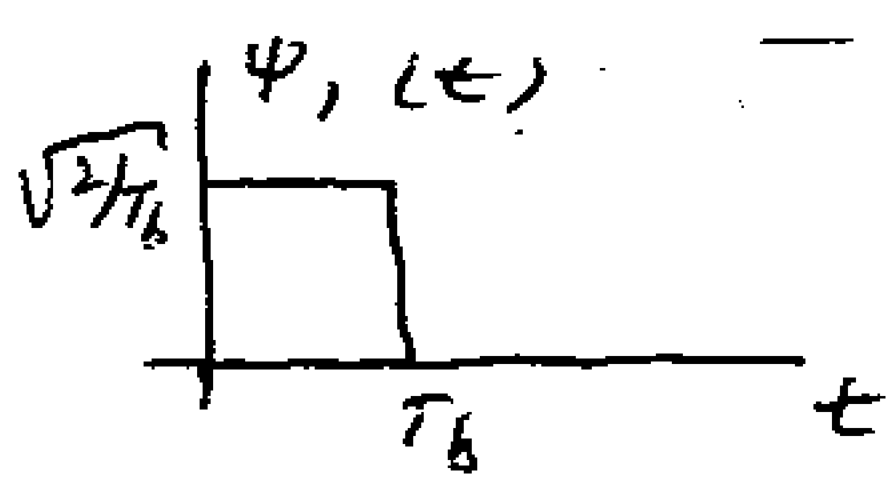


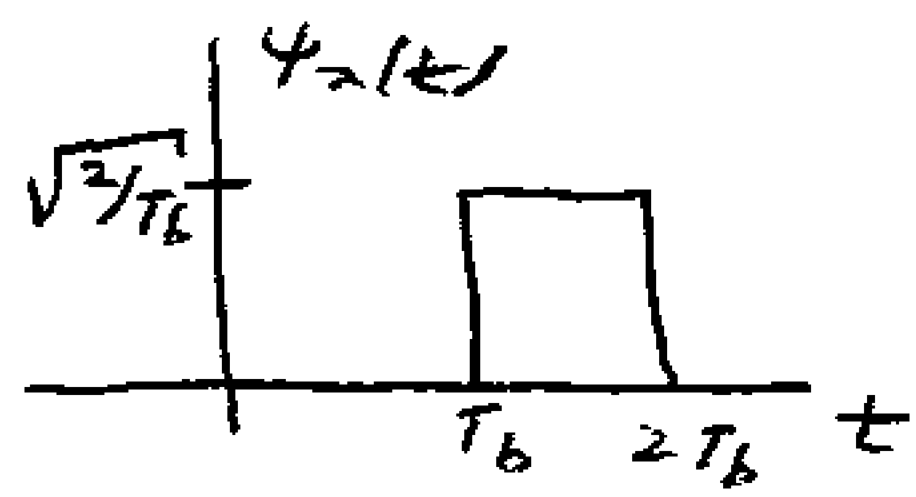
$$\mathcal{E}_b = A^2 T_b/2 \Rightarrow A = \sqrt{2\mathcal{E}_b/T_b}$$

NOTE: SIGNALS ARE NOW ORTHOGONAL

$$\int_0^{T_b} s_1(t)\, s_2(t)\, dt = 0$$

JUST NEED TO NORMALIZE TO FIND $\psi_1(t)$, $\psi_2(t)$



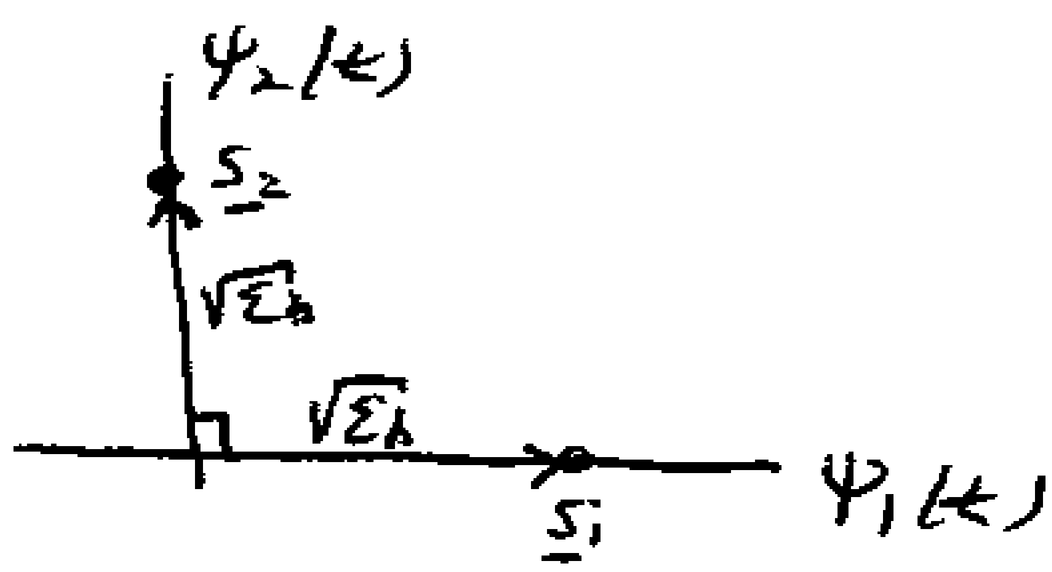


$$s_1(t) = s_1\, \psi_1(t) = s_1 \sqrt{2/T_b} \qquad 0 \le t \le T_b$$

$$A = s_1 \sqrt{2/T_b}$$

$$\sqrt{2\mathcal{E}_b/T_b} = s_1 \sqrt{2/T_b} \Rightarrow s_1 = \sqrt{\mathcal{E}_b}$$

$$s_1(t) = \sqrt{\mathcal{E}_b}\, \psi_1(t) \qquad s_2(t) = \sqrt{\mathcal{E}_b}\, \psi_2(t)$$



$$\underline{s}_1 = \begin{bmatrix} \sqrt{\mathcal{E}_b} \\ 0 \end{bmatrix}$$

$$\underline{s}_2 = \begin{bmatrix} 0 \\ \sqrt{\mathcal{E}_b} \end{bmatrix}$$

EXAMPLE : ANOTHER SET OF ORTHOGONAL SIGNALS

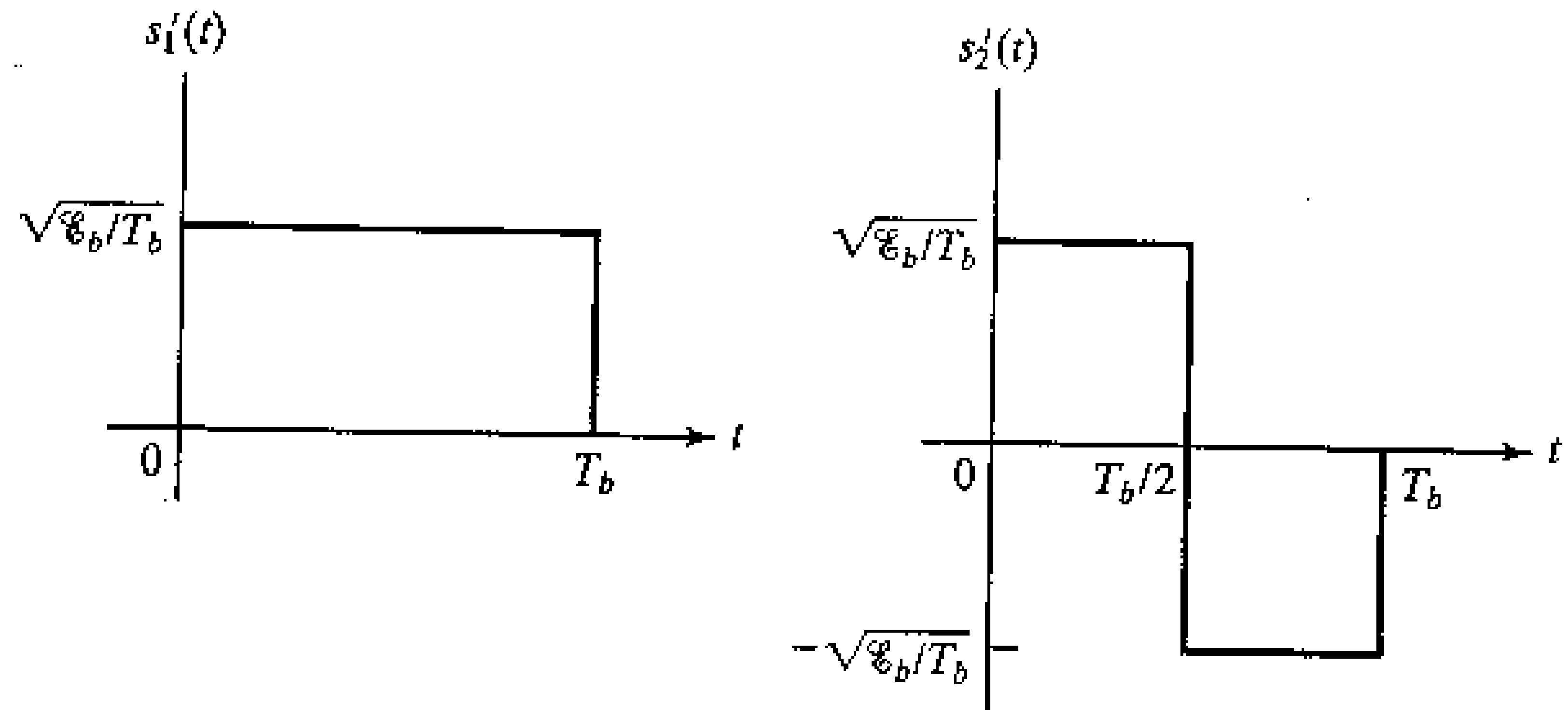
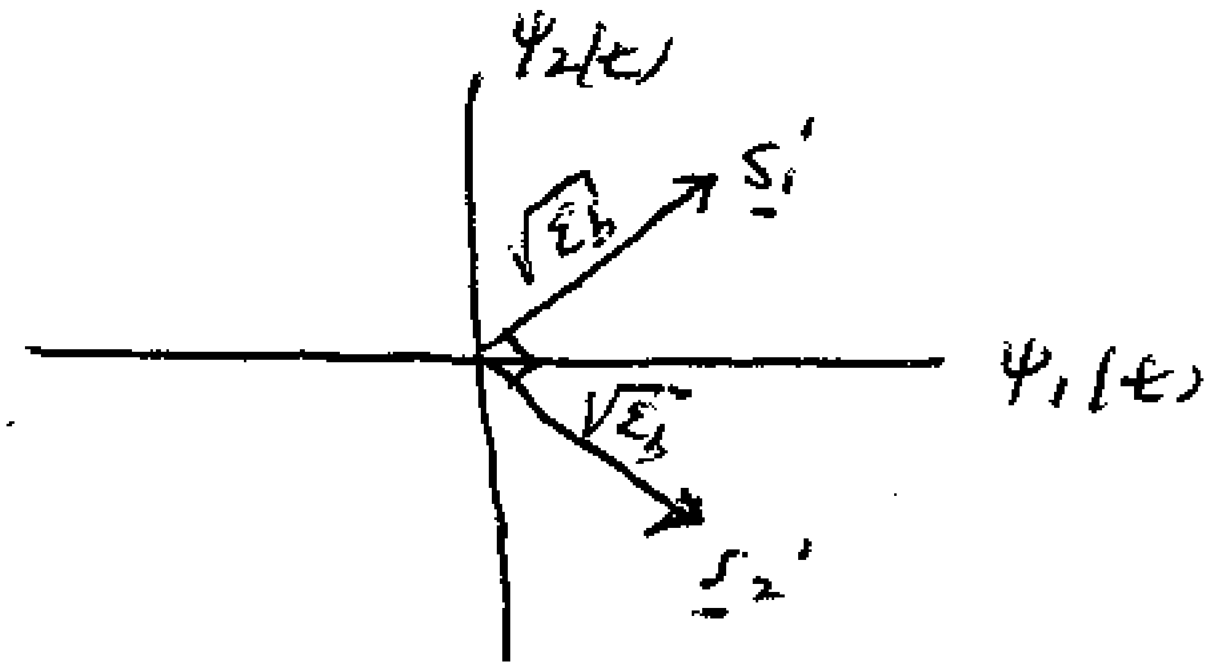


Figure 8.13 Two orthogonal signal waveforms.

$$s_1'(t) = \sqrt{\frac{\mathcal{E}_b}{2}}\,\psi_1(t) + \sqrt{\frac{\mathcal{E}_b}{2}}\,\psi_2(t)$$
$$s_2'(t) = \sqrt{\frac{\mathcal{E}_b}{2}}\,\psi_1(t) - \sqrt{\frac{\mathcal{E}_b}{2}}\,\psi_2(t)$$

$$\Rightarrow \quad \underline{s}_1' = \begin{bmatrix} \sqrt{\mathcal{E}_b/2} \\ \sqrt{\mathcal{E}_b/2} \end{bmatrix} \qquad \underline{s}_2' = \begin{bmatrix} \sqrt{\mathcal{E}_b/2} \\ -\sqrt{\mathcal{E}_b/2} \end{bmatrix}$$

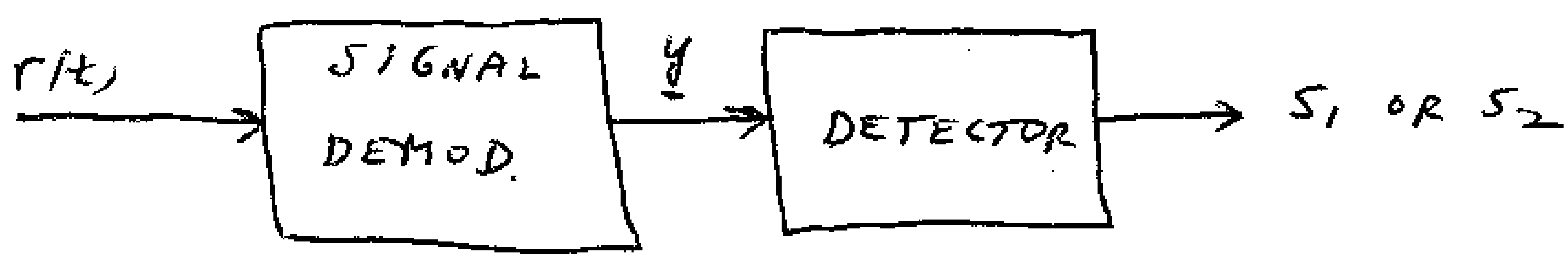


JUST ROTATED FROM PPM

OPTIMUM RECEIVER

AWGN

RECEIVE $r(t) = s_m(t) + n(t)$



IF NO NOISE, $\underline{y} = \underline{s}_1$ OR $\underline{s}_2$

DEMODULATOR

CONVERTS WAVEFORMS TO VECTORS FOR INPUT TO DETECTOR. CONSIDER ANTIPODAL SIGNALS FOR NOW.

$$r(t) = s_m \psi(t) + n(t) \qquad 0 \le t \le T_b$$

$$m = 1, 2$$

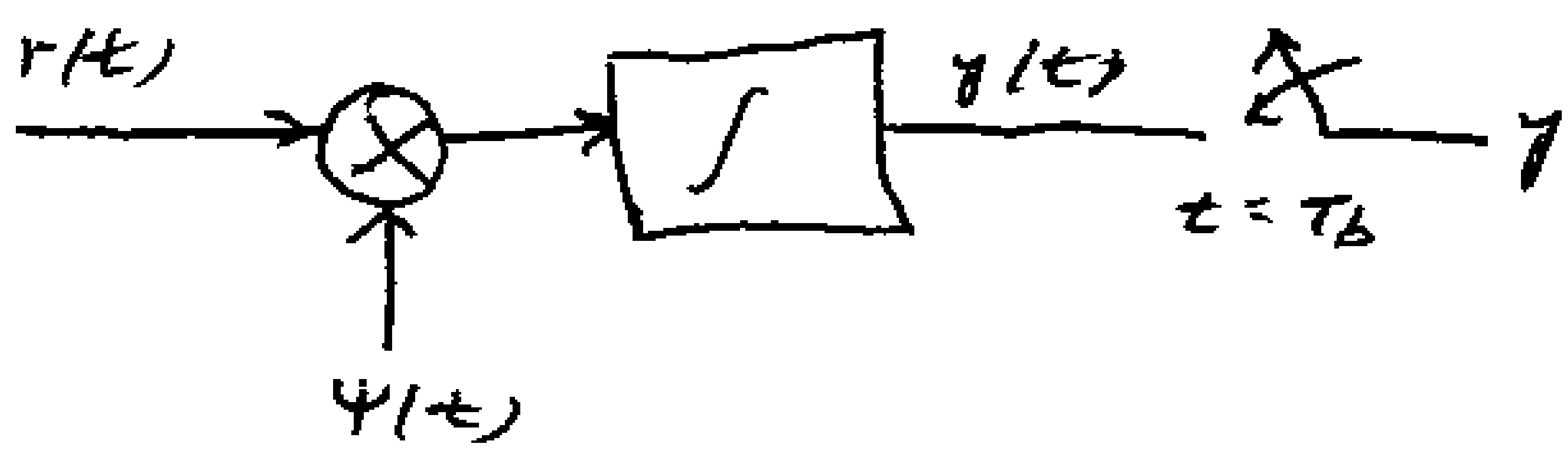
TO RECOVER s_m

$$s_1 = \sqrt{E_b}$$
$$s_2 = -\sqrt{E_b}$$

$$y = \int_0^{T_b} r(t)\,\psi(t)\,dt =$$

$$\int_0^{T_b} [s_m \psi^2(t) + n(t)\psi(t)]\,dt$$

$$= s_m + \underbrace{\int_0^{T_b} n(t)\psi(t)\,dt}_{\text{EFFECT OF NOISE}} = s_m + n$$



CROSS-CORRELATOR

$r(t)$ AND $\psi(t)$

NOISE PRODUCES THE PERTURBATION

$$n = \int_0^{T_b} \psi(\tau)\, n(\tau)\, d\tau$$

RECALL THAT $n(t)$ is AWGN $\Rightarrow$

$$E(n(t)) = 0 \qquad -\infty < t < \infty$$

$$R_n(\tau) = N_0/2\ \delta(\tau), \quad S_n(f) = \frac{N_0}{2} \quad -\infty < f < \infty$$

ALSO, SINCE n IS A LINEAR TRANSFORMATION OF GAUSSIAN, IT TOO IS GAUSSIAN.

$$E[n] = E\left[\int_0^{T_b} \psi(\tau) n(\tau) d\tau\right]$$

$$= \int_0^{T_b} \psi(\tau) \underbrace{E[n(\tau)]}_{=0} d\tau = 0$$

$$VAR(n) = \sigma_n^2 = E(n^2) = E\left[\int_0^{T_b} \psi(t) n(t) dt \int_0^{T_b} \psi(\tau) n(\tau) d\tau\right)$$

$$= \iint \psi(t)\psi(\tau) \underbrace{E[n(t) n(\tau)]}_{\frac{N_0}{2}\delta(t-\tau)} dt\, d\tau$$

$$= \int_0^{T_b} \psi^2(t) \frac{N_0}{2} dt = N_0/2$$

$$\Rightarrow \quad n \sim N(0, N_0/2)$$

$$\Rightarrow \quad y \sim N(S_m, N_0/2)$$

NOTE THAT WE GENERALLY MODEL THE OCCURENCE OF A 0 OR 1 AS A RANDOM EVENT. THUS, S_m IS ACTUALLY A RANDOM VARIABLE WITH TYPICALLY

$$P(S_m = S_1 = \sqrt{E_b}) = P(S_m = S_2 = -\sqrt{E_b}) = \frac{1}{2}$$

HENCE, WE WRITE THE CONDITIONAL PDF

AS $y \mid s_m \sim N(s_m, N_0/2)$ OR

$$f(y \mid s_m) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(y - s_m)^2}$$

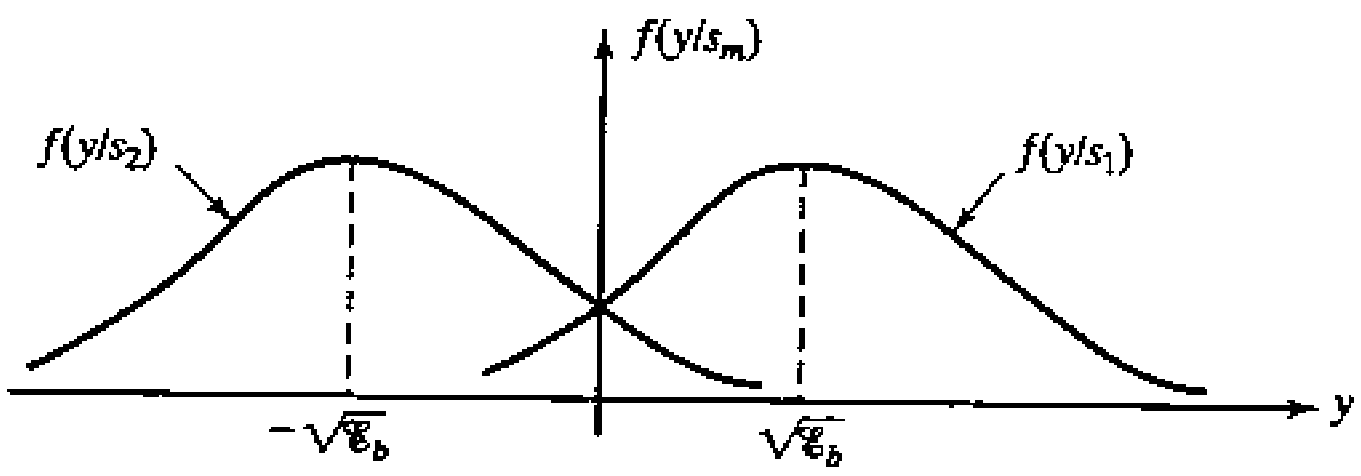


Figure 8.18 The conditional probability density functions of the correlator output for binary antipodal signaling.

OPTIMAL DETECTOR $\xrightarrow{y} \boxed{\text{DET.}} \rightarrow s_1$ OR s_2 ?

ORTHOGONAL SIGNALS

NOW NEED TWO DEMODULATORS SINCE

$$s_m(t) = s_{m1}\psi_1(t) + s_{m2}\psi_2(t)$$

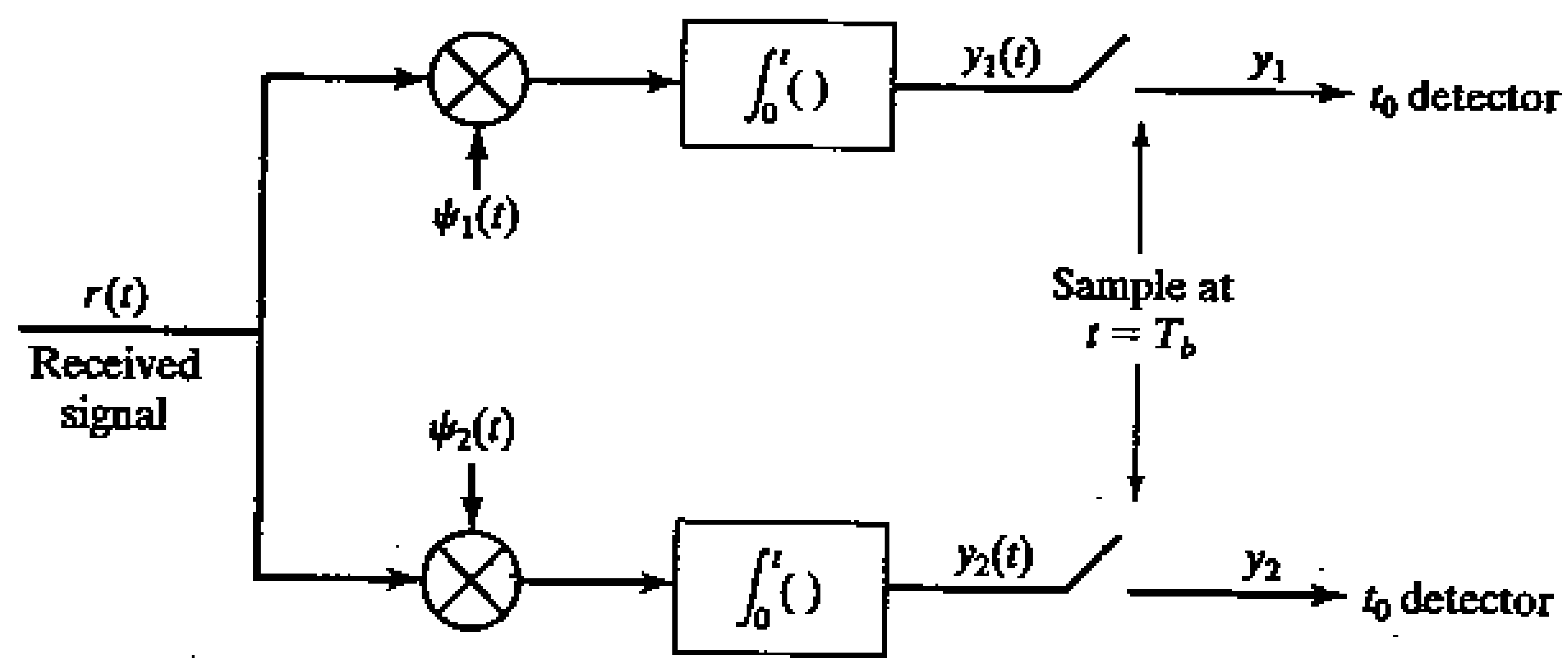


Figure 8.20 Correlation-type demodulator for binary orthogonal signals.

$$y_1 = y_1(T_b) = \int_0^{T_b} r(\tau)\psi_1(\tau)\, d\tau$$

WHERE $r(\tau) = s_{m1}\psi_1(\tau) + s_{m2}\psi_2(\tau) + n(\tau)$

$$y_1 = \int_0^{T_b} (s_{m1}\psi_1(\tau) + \underbrace{s_{m2}\psi_2(\tau) + n(\tau))\psi_1(\tau)}_{=0} d\tau$$

$$= s_{m1} + \underbrace{\int_0^{T_b} n(\tau)\psi_1(\tau)\,d\tau}_{n_1}$$

ONLY PASSES $\psi_1(t)$ COMPONENT OF SIGNAL

SIMILARLY

$$y_2 = s_{m2} + \underbrace{\int_0^{T_b} n(\tau)\psi_2(\tau)\,d\tau}_{n_2}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} s_{m1} \\ s_{m2} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

AS BEFORE (ANTIPODAL), $n_1 \sim N(0, N_0/2)$
$n_2 \sim N(0, N_0/2)$ AND NOW

$$\mathrm{COV}(n_1, n_2) = E(n_1 n_2) = \iint \overbrace{E(n(t)n(\tau))}^{N_0/2\ \delta(t-\tau)} \cdot \psi_1(t)\psi_2(\tau)\,dt\,d\tau$$

$$= \int_0^{T_b} \frac{N_0}{2} \psi_1(t)\psi_2(t)\,dt = 0$$

$\Rightarrow$ n_1, n_2 ARE UNCORRELATED AND SINCE JOINTLY GAUSSIAN ARE <u>INDEPENDENT</u>

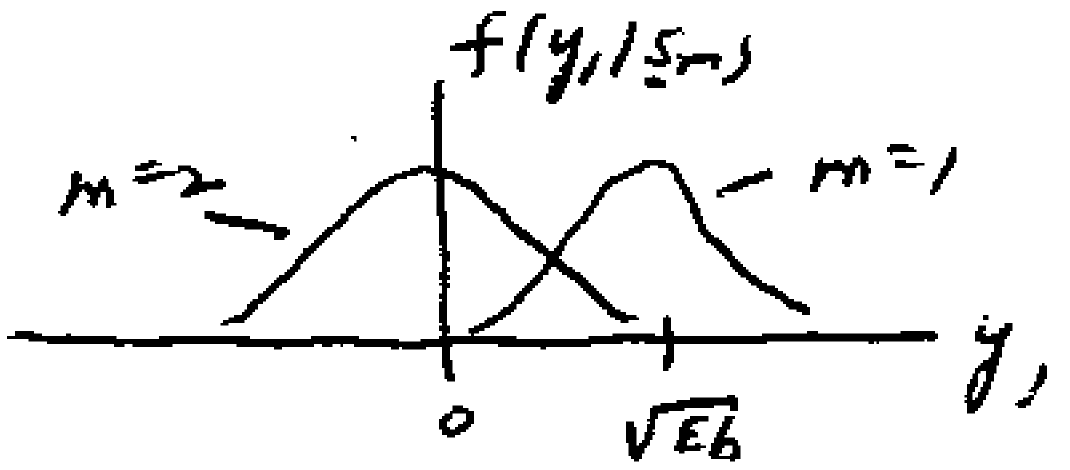
$$f(y_1, y_2 / \underline{s}_m) = \left(\frac{1}{\sqrt{\pi N_0}}\right)^2 e^{-\frac{1}{N_0}[(y_1 - s_{m1})^2 + (y_2 - s_{m2})^2]}$$

$$= f(y_1 / \underline{s}_m) f(y_2 / \underline{s}_m)$$

FINALLY, FOR ORTHOGONAL SIGNALS

$$\underline{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}, \quad \underline{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

$$f(y_1 | \underline{s}_m) = \begin{cases} \dfrac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(y_1 - \sqrt{E_b})^2} & m=1 \\ \dfrac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0} y_1^2} & m=2 \end{cases}$$

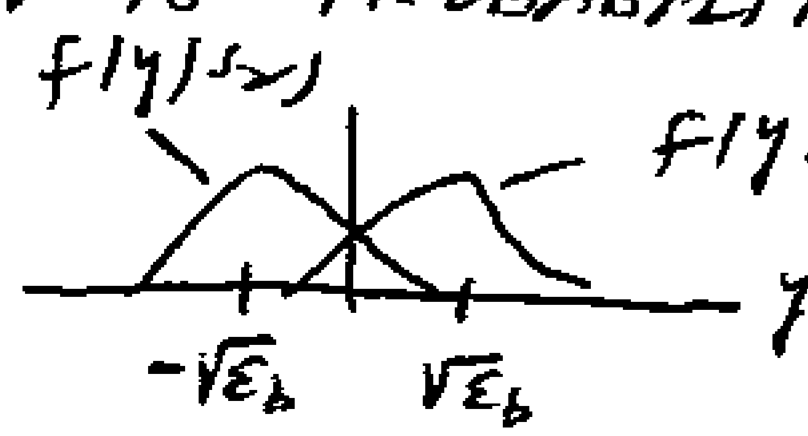


SAME FOR $f(y_2|\underline{s}_m)$

OPTIMAL DETECTOR

WISH TO DECIDE IF $s_1(t)$ OR $s_2(t)$ WAS TRANSMITTED BASED ON y FOR ANTIPODAL SIGNALS OR (y_1, y_2) FOR ORTHOGONAL SIGNALS. CONSIDER ANTIPODAL FIRST.

CRITERION IS PROBABILITY OF ERROR, AND SINCE



DECIDE $s_1(t)$ IF $y > \alpha$ AND DECIDE $s_2(t)$ IF $y \leq \alpha$

WHAT SHOULD α (THRESHOLD) BE?

PROB OF ERROR

$$P_2(\alpha) = P(y \leq \alpha | s_1) P(s_1) + P(y > \alpha | s_2) P(s_2)$$

$$= \int_{-\infty}^{\alpha} f(y|s_1)\,dy \; p(s_1) + \int_{\alpha}^{\infty} f(y|s_2)\,dy \; p(s_2)$$

NOTE: $p(s_1)$, $p(s_2)$ ARE CALLED A PRIORI PROBABILITIES

$$\frac{\partial P_e(\alpha)}{\partial \alpha} = f(\alpha|s_1)\,p(s_1) + \frac{\partial}{\partial \alpha}\left[1 - \int_{-\infty}^{\alpha} f(y|s_2)\,dy\right] \cdot p(s_2)$$

$$= f(\alpha|s_1)\,p(s_1) - f(\alpha|s_2)\,p(s_2) = 0$$

$\Rightarrow$ CHOOSE THRESHOLD AS SOLUTION OF

$$\frac{f(\alpha|s_1)}{f(\alpha|s_2)} = \frac{p(s_2)}{p(s_1)}$$

USUALLY $p(s_1) = p(s_2) = \frac{1}{2}$ WHY ?

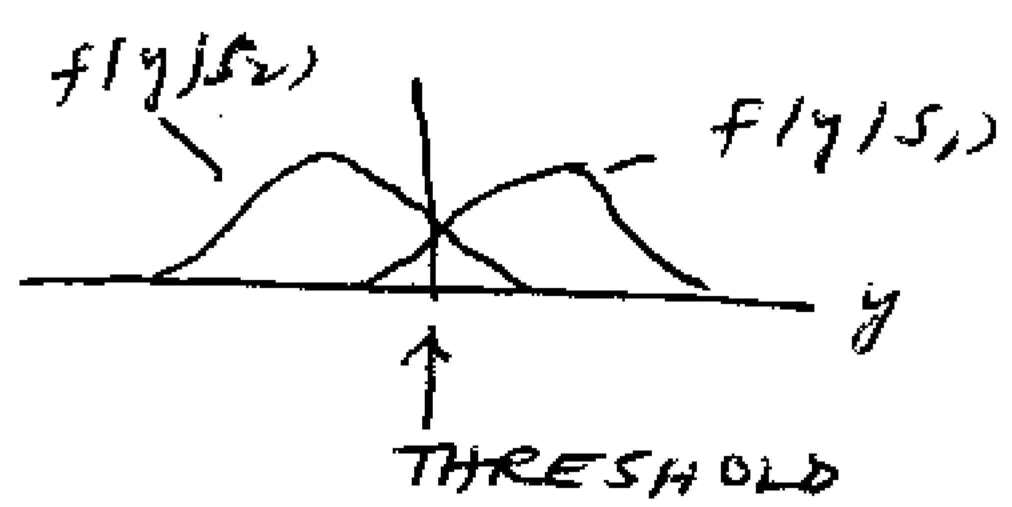
$\Rightarrow$ $f(\alpha|s_1) = f(\alpha|s_2)$

$$\frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(\alpha - \sqrt{E_b})^2} = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(\alpha + \sqrt{E_b})^2}$$

$\Rightarrow$ $(\alpha - \sqrt{E_b})^2 = (\alpha + \sqrt{E_b})^2$

$$\alpha^2 - 2\alpha\sqrt{E_b} + E_b = \alpha^2 + 2\alpha\sqrt{E_b} + E_b$$

$\therefore$ $\alpha^* = 0$



DECIDE $s_1(t)$ IF $y > 0$

$s_2(t)$ IF $y \leq 0$ = SIGN ARBITRARY (WHY?)

TO FIND PROB. OF ERROR:

$$P_2(0) = P_2 = \int_{-\infty}^{0} f(y|s_1)\,dy\,\tfrac{1}{2} + \int_{0}^{\infty} f(y|s_2)\,dy\,\tfrac{1}{2}$$

$N(\sqrt{E_b}, N_0/2)$ $\quad$ $N(-\sqrt{E_b}, \frac{N_0}{2})$

$$= \underbrace{\int_0^{\infty} f(-y|s_1)\,dy}\,\tfrac{1}{2} + \underbrace{\int_0^{\infty} f(y|s_2)\,dy}\,\tfrac{1}{2}$$

= WHY?

$$= \int_0^{\infty} f(y|s_2)\,dy = \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(y+\sqrt{E_b})^2}\,dy$$

LET $y' = y + \sqrt{E_b}$

$$P_2 = \int_{\sqrt{E_b}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}y'^2}\,dy'$$

LET $z = y'/\sqrt{N_0/2}$ $\qquad dy' = \sqrt{N_0/2}\,dz$

$$P_2 = \int_{\sqrt{E_b/N_0/2}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{2}z^2}\sqrt{N_0/2}\,dz$$

$$= \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}}_{\text{PDF FOR } N(0,1)}\,dz$$

$\therefore \quad P_E = Q\left(\sqrt{2\mathcal{E}_b/N_0}\right)$ $\qquad Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$

"Q" FUNCTION

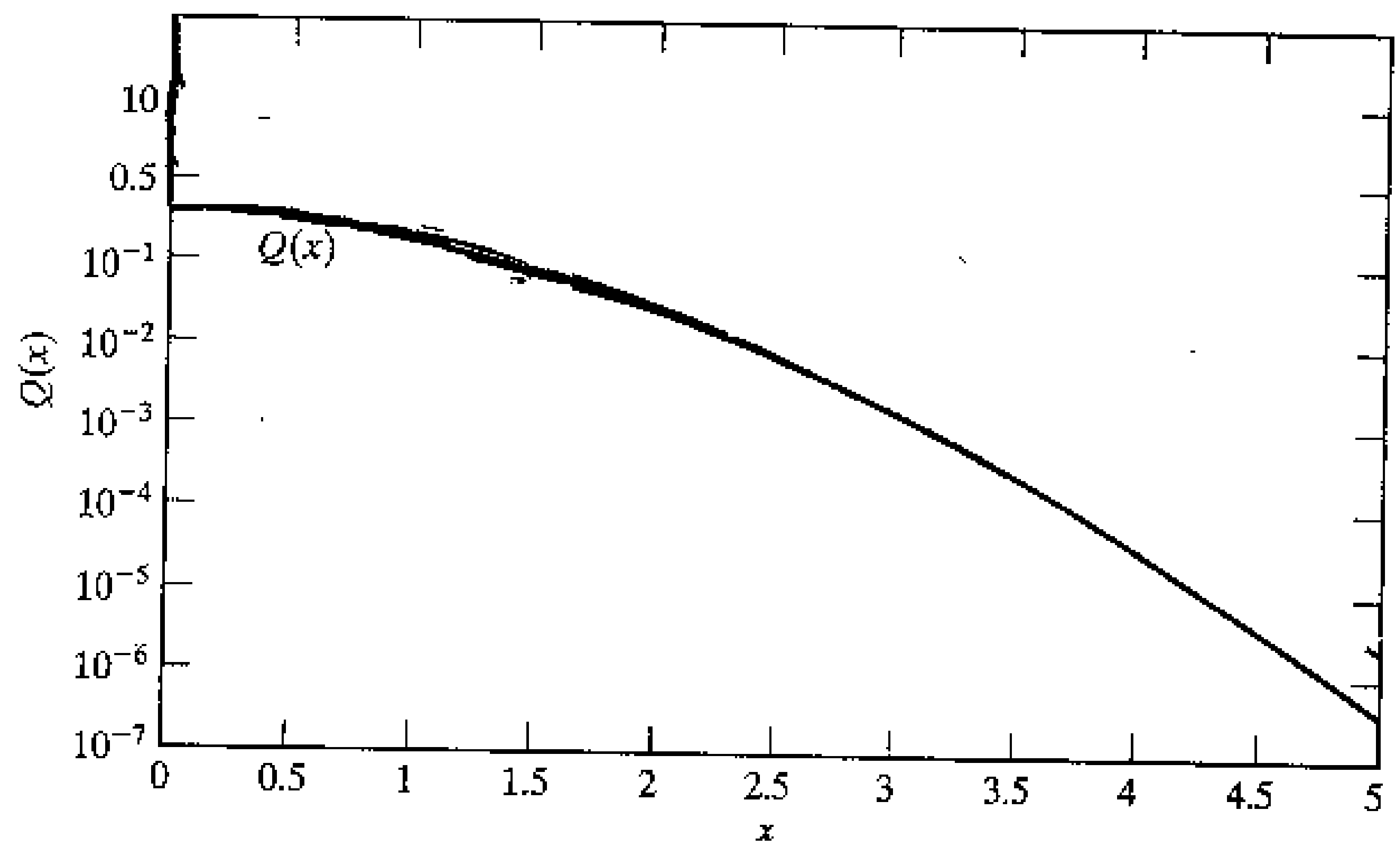


Figure 5.10 Bounds on the Q-function.

NEED $\sqrt{\frac{\mathcal{E}_b}{N_0/2}} = 5.2$ FOR $P_E = 10^{-7}$

$$\frac{\mathcal{E}_b}{N_0/2} = \frac{\text{SIGNAL ENERGY}}{\text{NOISE PSD LEVEL}}$$

CALLED AN ENERGY-TO-NOISE RATIO (ENR)

NOTE: REQUIRE $\text{ENR} = 10\log_{10} \frac{\mathcal{E}_b}{N_0/2}$

$= 10\log_{10}(5.2)^2 \approx 14 \text{ dB}$

FOR GOOD PERFORMANCE.

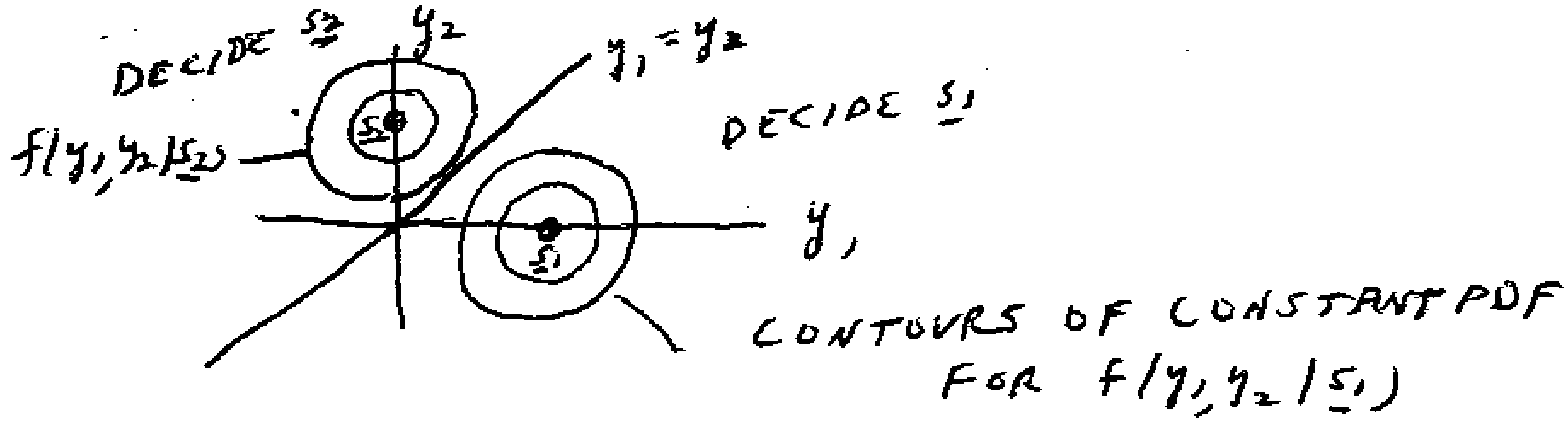
NOW CONSIDER ORTHOGONAL SIGNALS

RECALL THAT WE HAVE AS INPUT TO DETECTOR

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad \text{FOR } \underline{s}_1$$

$$\begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad \text{FOR } \underline{s}_2$$

$$\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \frac{N_0}{2} \underline{\underline{I}} \right) \qquad \underline{\underline{I}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



WILL SHOW LATER
THAT WE SHOULD DECIDE $\underline{s}_1$ IF $y_1 > y_2$
(LARGER OUTPUT OF DEMODULATOR INDICATES SIGNAL)

$$P_2 = P(y_2 > y_1 | \underline{s}_1) \tfrac{1}{2} + P(y_2 \le y_1 | \underline{s}_2) \tfrac{1}{2}$$

$$= P(y_2 > y_1 | \underline{s}_1) \qquad \text{BY SYMMETRY}$$

$$= P(y_2 - y_1 > 0 | \underline{s}_1)$$

BUT $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \Big| \underline{s}_1 \sim N\left(\begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}, \frac{N_0}{2} \underline{\underline{I}} \right)$

$\Rightarrow \quad y_2 - y_1 | \underline{s}_1 \sim N(-\sqrt{E_b}, N_0) \qquad$ WHY?

$$P_2 = \phi\left(\frac{0 - (-\sqrt{E_b})}{\sqrt{N_0}} \right) = Q\left(\sqrt{\frac{E_b}{N_0}} \right)$$

3 dB POORER THAN ANTIPODAL

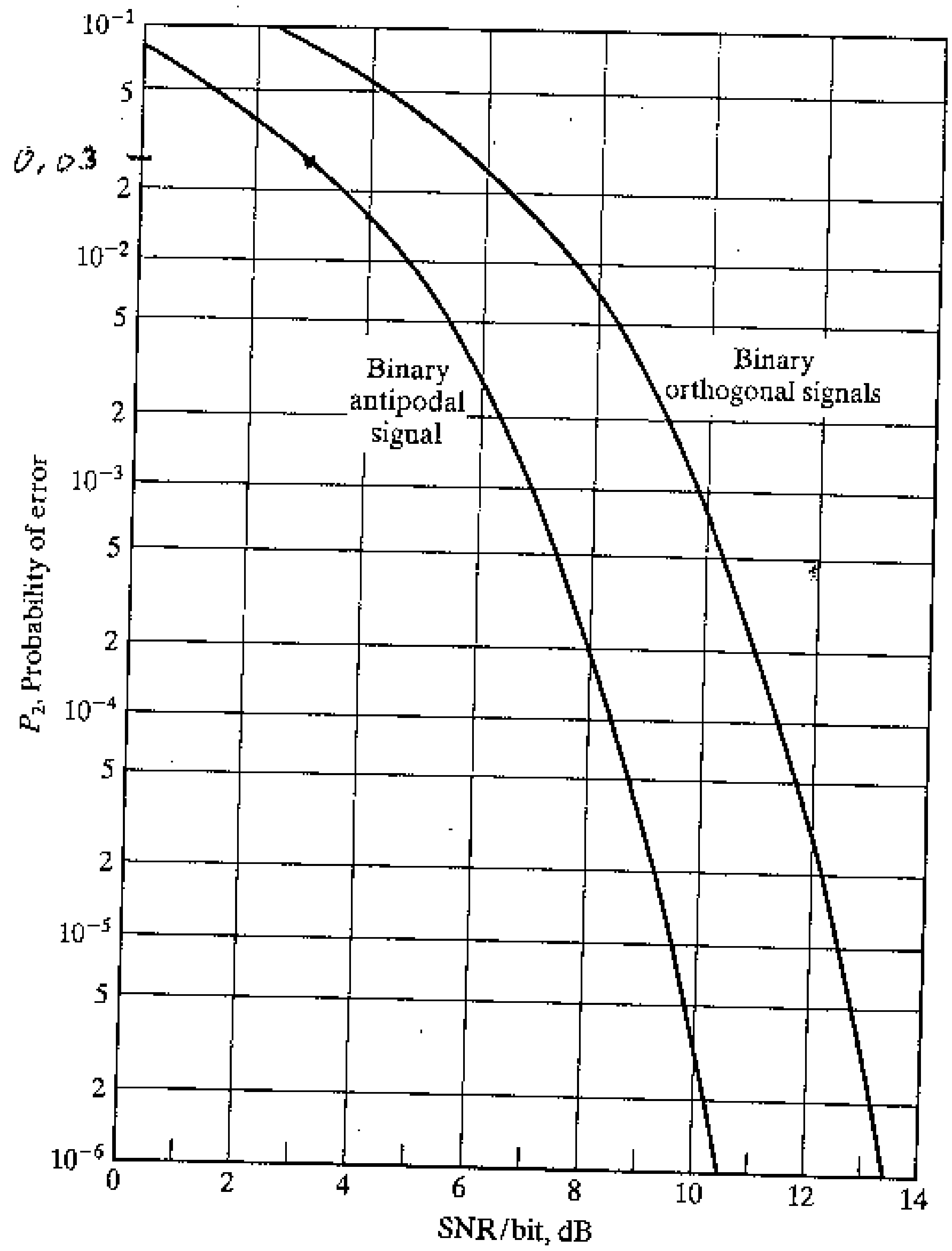


Figure 8.26 Probability of error for binary signals.

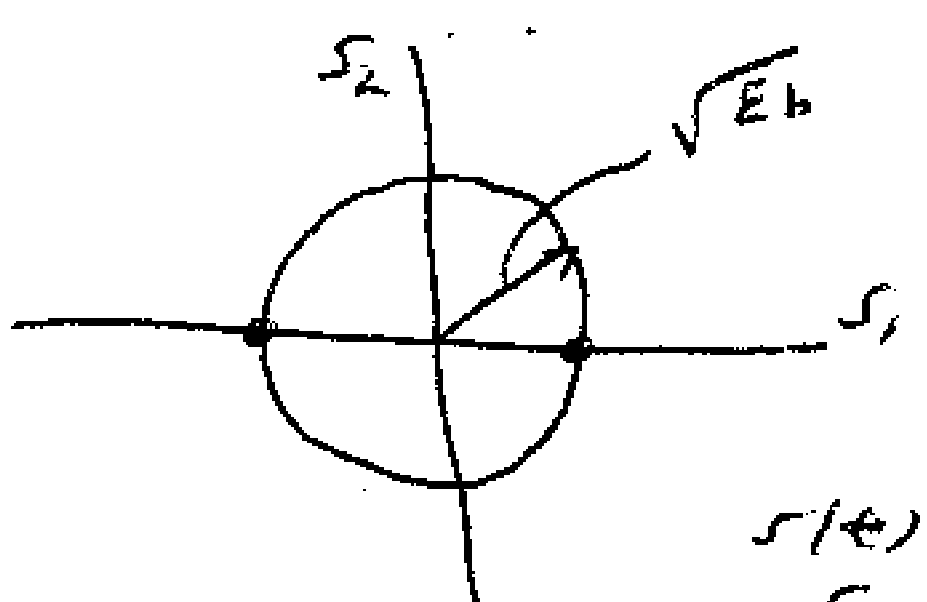
SNR IS DEFINED AS

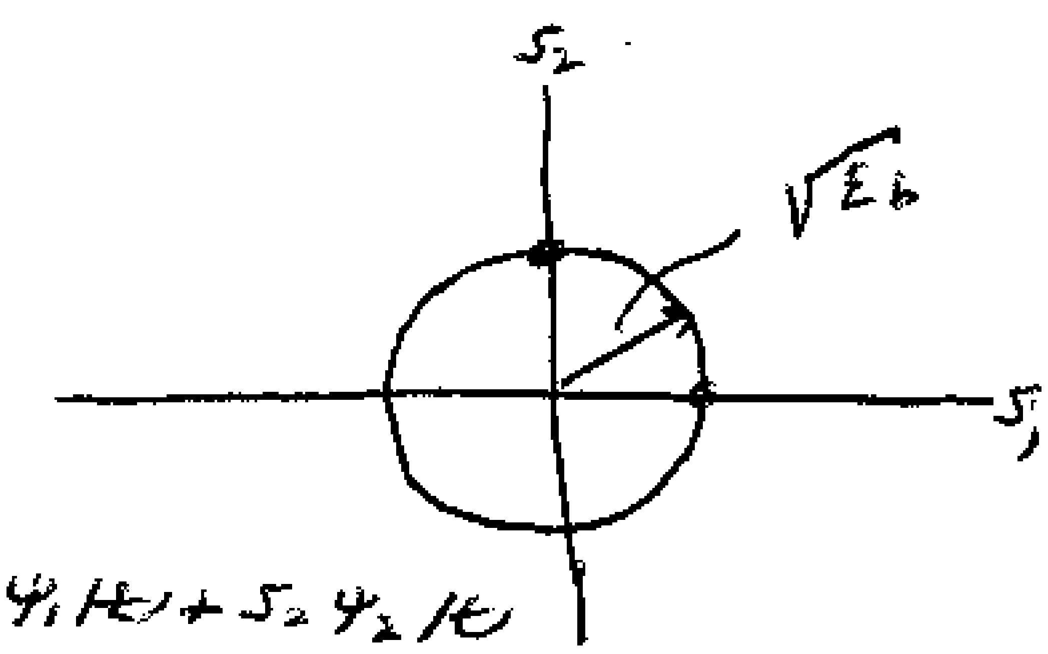
$10 \log_{10} \mathcal{E}_b/N_0$

NOT

$10 \log_{10} \frac{\mathcal{E}_b}{N_0/2}$

(HISTORICAL REASONS ?)





$s(t) = s_1 \varphi_1(t) + s_2 \varphi_2(t)$

$\mathcal{E}_b = s_1^2 + s_2^2$

ANTIPODAL

ORTHOGONAL