2) INSTEAD OF COMPUTING PROB. OF <u>SYMBOL ERROR</u> P_M, CONVERT TO PROB. OF A <u>BIT ERROR</u> P_b.

<u>EXAMPLE</u>: FOR $M=4$

$$
\begin{aligned}
P_M &= P(\text{ERROR} \mid \underline{s}_1) = P(\text{SYMBOL ERROR} \mid \underline{s}_1) \\
&= P(y_2 > y_1 \text{ OR } y_3 > y_1 \text{ OR } y_4 > y_1) \\
&= P(y_2 > y_1 \text{ OR } y_3 > y_1 \text{ OR } y_4 > y_1 \mid y_2 \text{ MAX})\, P(y_2 \text{ MAX}) \\
&+ P(\quad '' \quad \mid y_3 \text{ MAX})\, P(y_3 \text{ MAX}) \\
&+ P(\quad \mid y_4 \text{ MAX})\, P(y_4 \text{ MAX})
\end{aligned}
$$

y_2, y_3, y_4 ARE IID CONDITIONAL ON $\underline{s}_1$

$$
\begin{aligned}
\Rightarrow P_M &= 3\, P(y_2 > y_1 \text{ OR } y_3 > y_1 \text{ OR } y_4 > y_1,\ y_2 \text{ MAX}) \\
&= 3\, P(\underline{s}_2 \text{ CHOSEN})
\end{aligned}
$$

FOR $M=4 \Rightarrow k=2$ BITS

$\underline{s}_1 \Rightarrow 00$

$\underline{s}_2 \Rightarrow$ 1 BIT ERROR $\qquad \underline{s}_2 \Rightarrow 01$

$\underline{s}_3 \Rightarrow$ 1 BIT ERROR $\qquad \underline{s}_3 \Rightarrow 10$

$\underline{s}_4 \Rightarrow$ 2 BIT ERRORS $\qquad \underline{s}_4 \Rightarrow 11$

EQUIPROBABLE OF DECODING AS

1 BIT

2 BITS IN ERROR

NUMBER OF BIT ERRORS $= \binom{2}{1}, \binom{2}{2}$

AVERAGE NUMBER OF BIT ERRORS =

$$
1 \cdot \binom{2}{1} \cdot P(\text{CHOOSE } \underline{s}_2 \text{ OR } \underline{s}_3) + 2 \cdot \binom{2}{2} P(\text{CHOOSE } \underline{s}_4)
$$

$$
= \left[1 \cdot \binom{2}{1} + 2 \cdot \binom{2}{2} \right] P(\text{CHOOSE } \underline{s}_2)
$$

P_b = (AVERAGE NUMBER OF BIT ERRORS)/k

IN GENERAL, $P_b = \left(\sum_{n=1}^{k} n \binom{k}{n} \frac{P_M}{M-1} \right) / k$

BUT $\sum_{n=0}^{k} n \binom{k}{n} \frac{1}{2^k}$ = EXPECTED VALUE OF BINOMIAL RANDOM VARIABLE WITH $p = \frac{1}{2}$

$= k/2$

$$\Rightarrow \quad \sum_{n=0}^{k} n \binom{k}{n} \frac{1}{2^k} = \frac{k}{2}$$

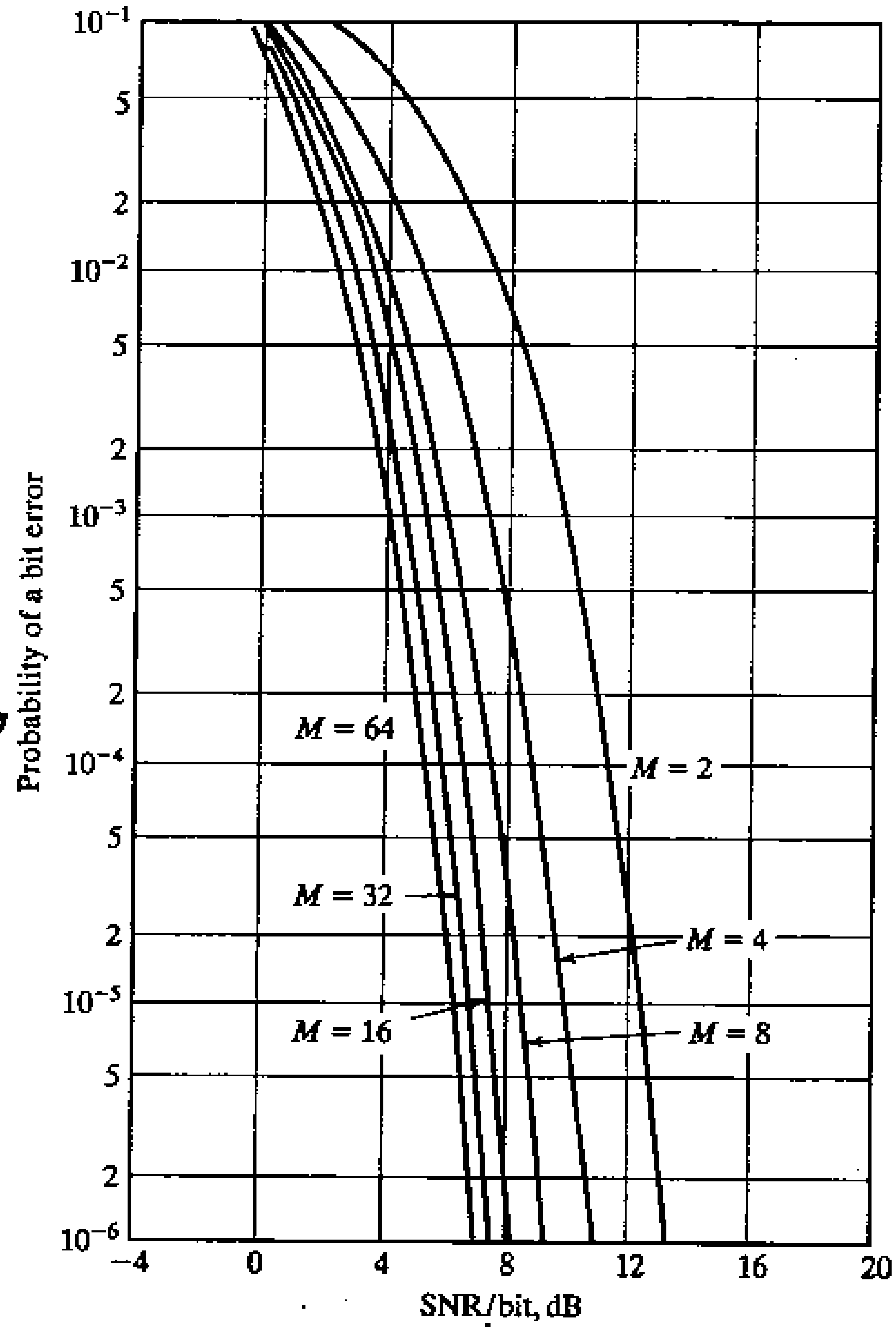
$$\sum_{n=1}^{k} n \binom{k}{n} = k\, 2^{k-1}$$

$$P_b = \left(k\, 2^{k-1} \frac{P_M}{2^k - 1} \right) / k = \frac{2^{k-1}}{2^k - 1} P_M \approx P_M / 2$$

k LARGE

PLOTTING P_b VS SNR/BIT SHOWS P_b <u>DECREASES</u> AS M INCREASES - SEEMS TOO GOOD TO BE TRUE!

WE PLOTTED P_b VS $\mathcal{E}_b/N_0$. RECALL $\mathcal{E}_s = k\mathcal{E}_b$. AS $M = 2^k$ INCREASES, k INCREASES $\Rightarrow$ $\mathcal{E}_s$ INCREASES $\Rightarrow$ CAN INCREASE AMPLITUDE OF EACH ORTHOGONAL SIGNAL.

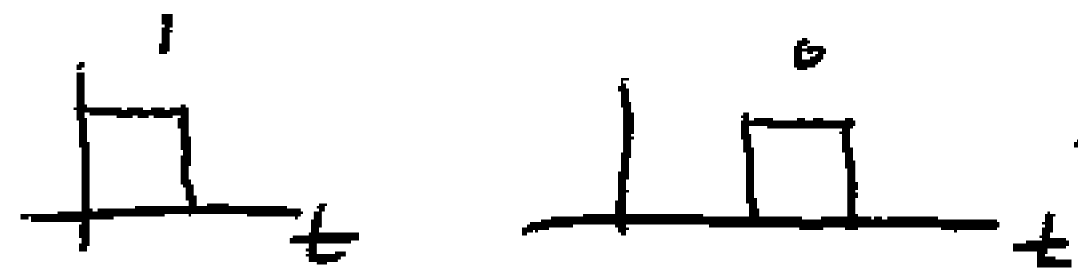


P_b

E_b/N_0

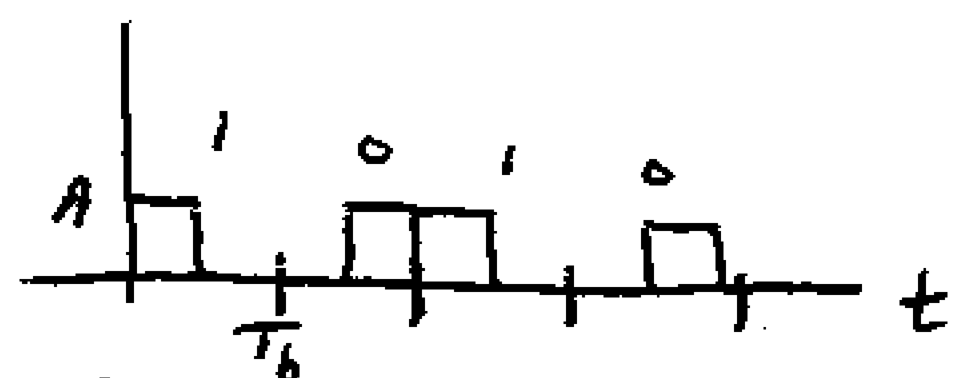
Figure 8.46 Probability of a bit error for optimum detection of orthogonal signals.

EXAMPLE : PPM



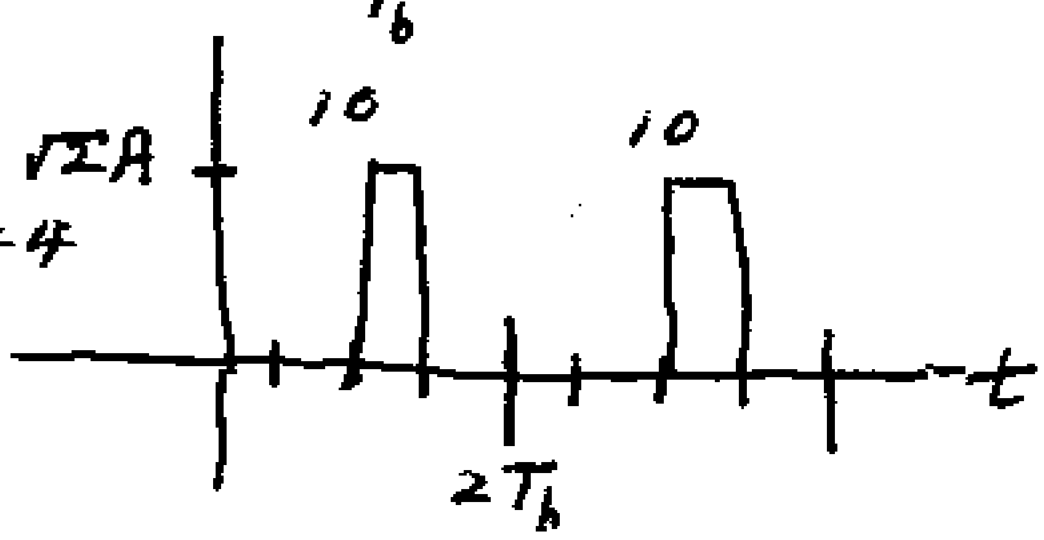
M = 2

M = 2



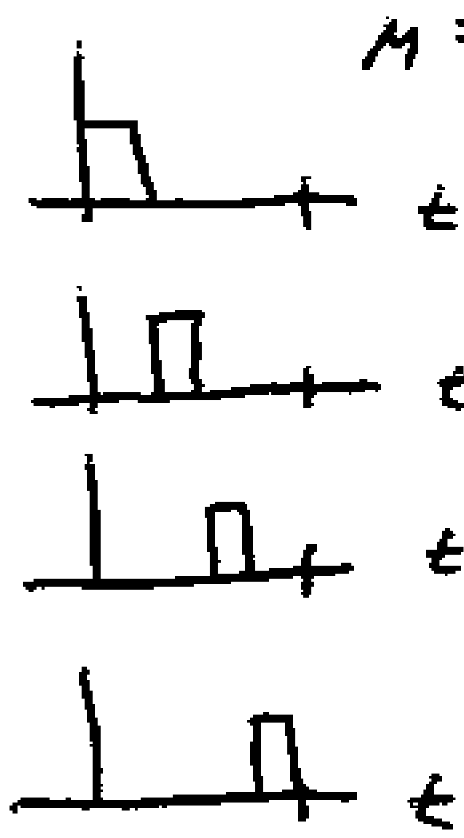
1010
$\longrightarrow t$

M = 4



SAME AVERAGE POWER

M = 4



AMPLITUDE GOES UP BUT MUST "SEARCH" OVER

MORE TIME SLOTS. ALSO, DECODING DELAY INCREASES (WHY?)

FINALLY, FOR k BITS OCCURING AT RATE R_b BPS, HAVE kT_b SEC TO ENCODE. NUMBER OF TIME SLOTS $= 2^k$. THUS, EACH TIME SLOT IS

$$\frac{kT_b}{2^k} \text{ SEC} \rightarrow 0 \quad \begin{array}{l} \text{AS } k \rightarrow \infty \\ \text{AS } M \rightarrow \infty \end{array}$$

CAN WE TRANSMIT THESE PULSES?

RECAP: APPEARS AS IF BY INCREASING M FOR ORTHOGONAL SIGNALS, WE CAN HAVE $P_M \rightarrow 0$. THIS IS GENERALLY TRUE BUT WAS NOT THOUGHT POSSIBLE UNTIL CLAUDE SHANNON! (BELL LABS - 1948)

UNION BOUND ON P_M (FOR ANY M-ARY SCHEME - NOT JUST ORTHOGONAL)

DERIVES FROM SIMPLE IDEA THAT

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\leq P(A) + P(B) \qquad = \text{ HOLDS IFF } A \cap B = \phi$$

OR $P\left(\bigcup_{1}^{M} A_i\right) \leq \sum_{i=1}^{M} P(A_i)$

NOW CONSIDER M-ARY SIGNALING $P(s_m) = 1/M$

$$P_m = P(\text{ERROR} \mid s_m(t) \text{ SENT})$$

$$= P(s_1(t) \text{ DECODED} \cup s_2(t) \text{ DECODED}$$

$$\cup\, s_{m-1}(t) \text{ DECODED} \cup s_{m+1}(t) \text{ DECODED} \ldots \mid s_m(t))$$

UNION BOUND

$$\le \sum_{\substack{i=1 \\ i \ne m}}^{M} P(s_i(t) \text{ DECODED} \mid s_m(t))$$

BUT $P(s_i(t) \text{ DECODED} / s_m(t)) =$

$$P(\underbrace{D(\underline{y}, \underline{s}_i) < D(\underline{y}, \underline{s}_m)}_{A} \cap \overbrace{D(\underline{y}, \underline{s}_i) < D(\underline{y}, \underline{s}_k)}^{B} \mid s_m(t))$$

$k = 1, \ldots, M$
$k \ne i$

$P(A \cap B) \le P(A)$ WHY?

$$\Rightarrow P(s_i(t) \text{ DECODED} \mid s_m(t)) \le \underbrace{P(D(\underline{y}, \underline{s}_i) < D(\underline{y}, \underline{s}_m) \mid s_m(t))}$$

BINARY EQUIPROBABLE PROBLEM

ERROR PROBABILITY $= Q\left(\sqrt{\dfrac{2E_b}{N_0}}\right)$ (8.5.1)

$$= Q\left(\sqrt{\frac{2}{N_0}}\, \frac{d}{2}\right)$$

$$= Q\left(d / \sqrt{2N_0}\right)$$

$\leftarrow d \rightarrow$

$-\sqrt{E_b}$ 0 $\sqrt{E_b}$

$d = 2\sqrt{E_b}$

LET $d \to d_{mi}$

WHY?

$$P_m \le \sum_{\substack{i=1 \\ i \ne m}}^{M} Q\left(\frac{d_{mi}}{\sqrt{2N_0}}\right) \le \sum_{\substack{i=1 \\ i \ne m}}^{M} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

WHERE d_{min} IS THE MINIMUM DISTANCE BETWEEN $\underline{s}_m$ AND ALL OTHER $\underline{s}_i$'s.

$$P_m \le (M-1)\, Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$ $s_m(t)$ WAS TRANSMITTED

$$P_M = \sum_{k=1}^{M} P_k \frac{1}{M}$$

↑ SAME BOUND FOR ALL k

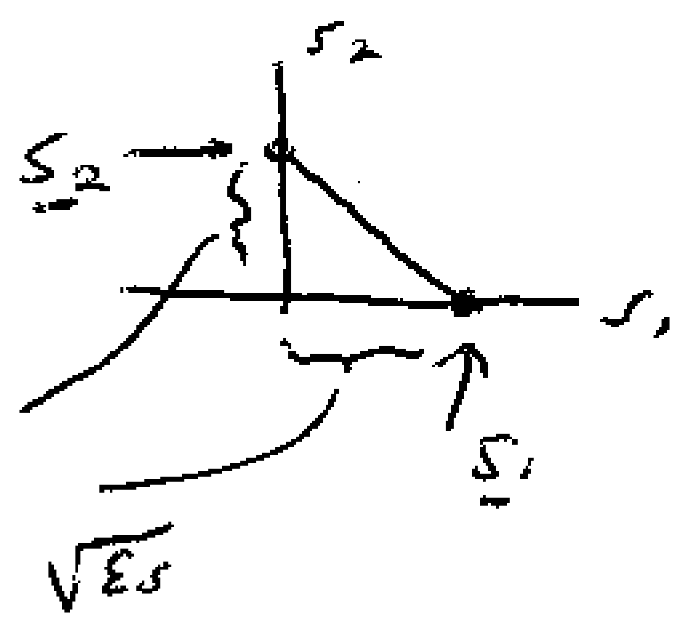
$$\le \sum_{k=1}^{M} (M-1)\, Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)\frac{1}{M} = (M-1)\, Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

BUT IT CAN BE SHOWN THAT

$$Q(x) \le \frac{1}{2} e^{-\frac{1}{2}x^2}$$

$$\therefore\ P_M \le \frac{M-1}{2} e^{-d_{min}^2/4N_0}$$

<u>EXAMPLE</u>: M-ARY ORTHOGONAL SIGNALING



$M = 2$

$$d^2 = 2\mathcal{E}_s$$

↑ SAME DISTANCE FOR ANY PAIR OF SIGNALS

$$\Rightarrow d_{min} = \sqrt{2\mathcal{E}_s} = \sqrt{2k\mathcal{E}_b}$$

$$P_M \le \frac{M-1}{2} e^{-d_{min}^2/4N_0} \le M e^{-d_{min}^2/4N_0}$$

$$= M e^{-k\mathcal{E}_b/2N_0} = 2^k e^{-k\mathcal{E}_b/2N_0}$$

$$= e^{\ln 2^k} e^{-k\,\mathcal{E}_b/2N_0}$$

$$= e^{-k(\mathcal{E}_b/2N_0 - \ln 2)}$$

$$\therefore\ P_M \leq e^{-\frac{1}{2}k(\mathcal{E}_b/N_0 - 2\ln 2)}$$

IF $\frac{\mathcal{E}_b}{N_0} > 2\ln 2 = 1.39 = 1.42\,dB$

$P_M \to 0$ AS $k \to \infty$ OR $M \to \infty$

$\Rightarrow$ PERFECT (ERRORLESS) COMMUNICATION!

ACTUALLY, LOWER BOUND FOR $P_M \to 0$ IS
$\mathcal{E}_b/N_0 > \ln 2 = -1.6\ dB$

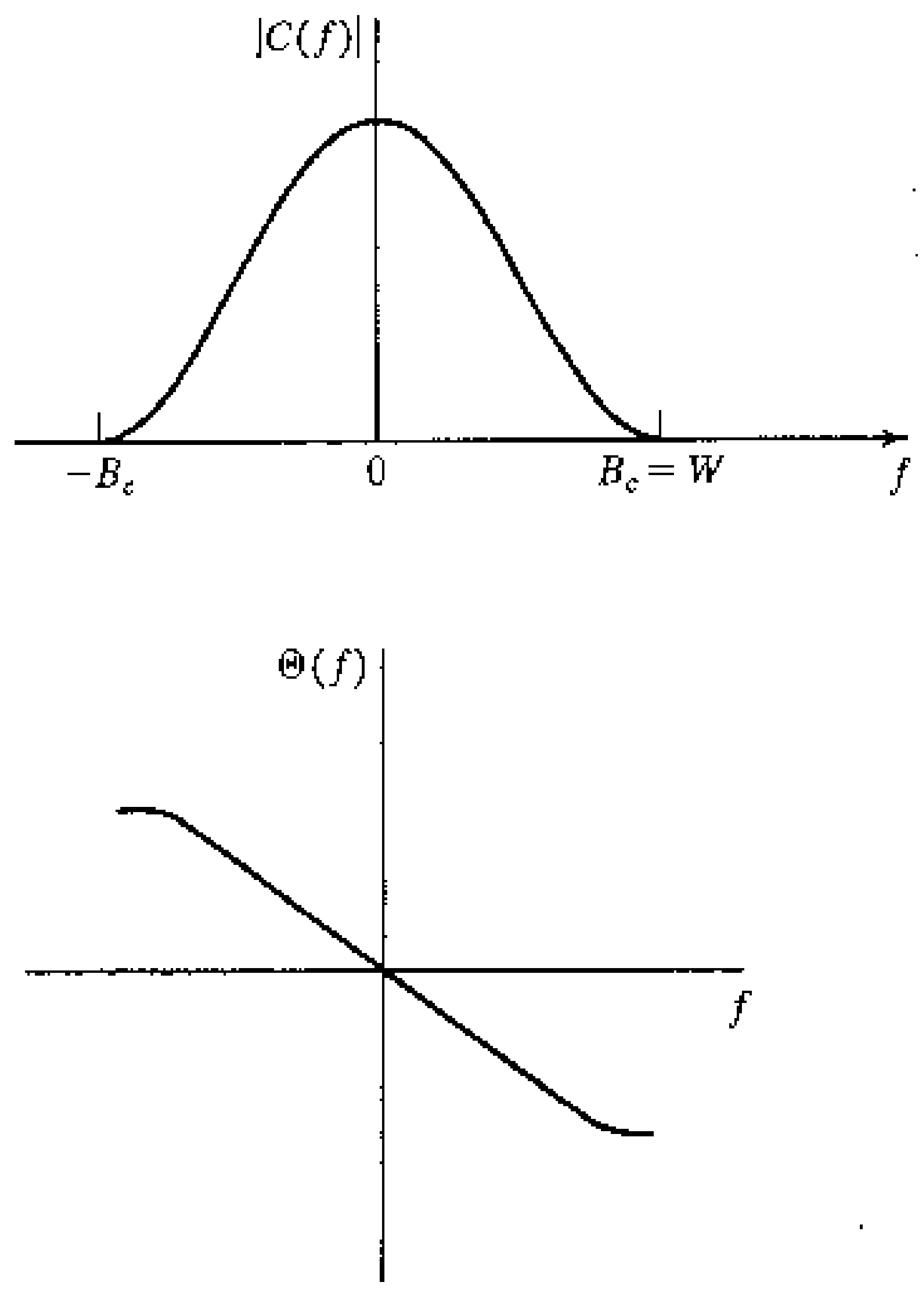
SO CALLED <u>SHANNON LIMIT</u> FOR AWGN CHANNEL.

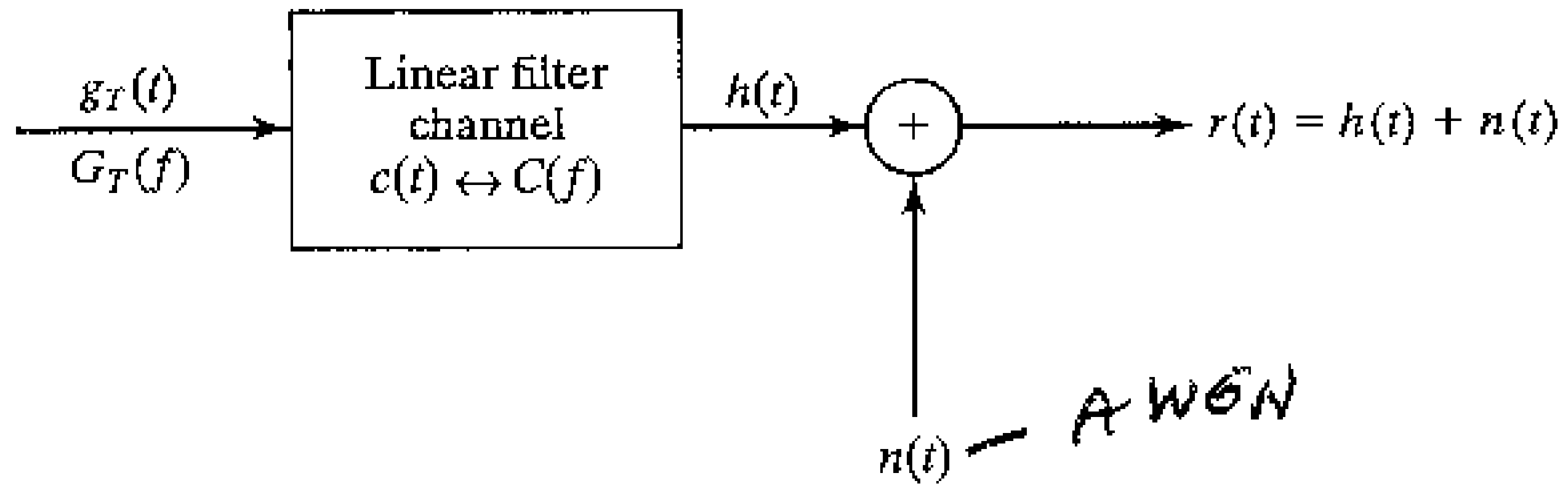
(NEED BETTER BOUNDING TECHNIQUES TO SHOW THIS)

<u>CHAPTER 9 - TRANSMISSION VIA BANDLIMITED AWGN CHANNELS</u> (9.1 - 9.4)

ALL BASEBAND CHANNELS ARE BANDLIMITED
(WHY WE REFER TO THEM AS BASEBAND!)

ASSUME BANDWIDTH = B_c Hz





— AWGN

Figure 9.2 Linear filter model for a bandlimited channel.

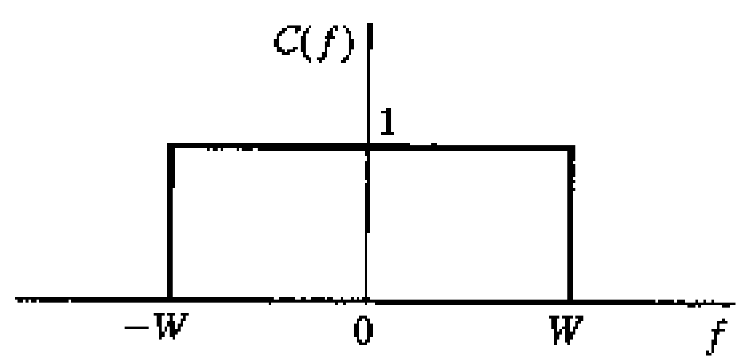
OUTPUT SIGNAL OF CHANNEL

NON DISTORTED

$$h(t) = g_T(t) * c(t)$$

OR $$H(f) = C(f)\,G_T(f)$$

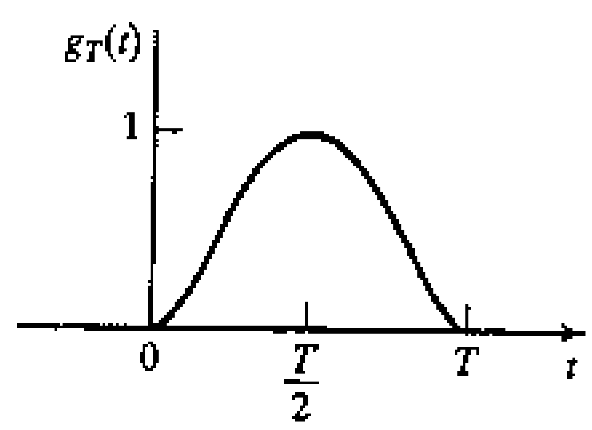
<u>EXAMPLE</u>:



$\theta(f) = 0$

(a)

$$g_T(t) = \frac{1}{2}\left[1 + \cos\frac{2\pi}{T}\left(t - \frac{T}{2}\right)\right], \quad 0 \le t \le T$$



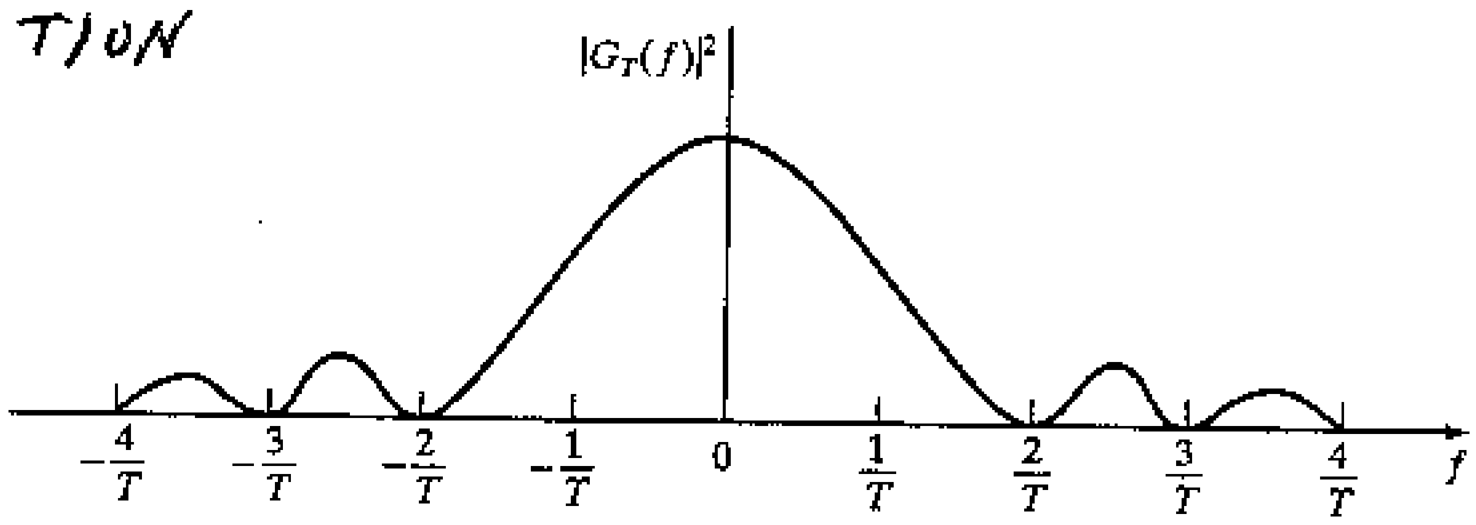
(b)

FOR NO DISTORTION

NEED $W \approx \frac{5}{T}$

FOR BINARY PAM

$W \approx \frac{5}{T_b} = 5 R_b$



(c)

Figure 9.3 The signal pulse in (b) is transmitted through the ideal bandlimited channel shown in (a). The spectrum of $g_T(t)$ is shown in (c).

RECALL LPC-10, $R_b = 2400$ BPS $\Rightarrow$
NEED $W = 12{,}000$ Hz. THIS IS TOO LARGE FOR TELEPHONE CHANNEL. ONLY HAVE ABOUT 4000 Hz.

DIGRESSION - MATCHED FILTER
IN FREQ. DOMAIN
(SEE PAGE #81)

RECALL THAT $h(t) = s(T-t) \qquad 0 \le t \le T$

$$H(f) = \int_0^T h(t)\, e^{-j2\pi f t}\, dt = \int_0^T s(\underbrace{T-t}_{u})\, e^{-j2\pi f t}\, dt \qquad \Rightarrow t = T-u$$

$$= \int_0^T s(u)\, e^{-j2\pi f (T-u)}\, du$$

$$= \int_0^T s(u)\, e^{j2\pi f u}\, du \; e^{-j2\pi f T}$$

$$= S^*(f)\, e^{-j2\pi f T} \qquad T = \text{SAMPLE TIME}$$

NOW FIND RESPONSE TO $s(t)$

$$y_s(t) = h(t) * s(t)$$

$$Y_s(f) = H(f)\, S(f)$$

$$= S^*(f)\, e^{-j2\pi f T}\, S(f)$$

$$= |S(f)|^2\, e^{-j2\pi f T}$$

NOT DEPENDENT ON PHASE OF $S(f)$

$$y_s(t) = \int_{-\infty}^{\infty} Y_s(f)\, e^{j2\pi f t}\, df$$

$$= \int_{-\infty}^{\infty} |S(f)|^2\, e^{-j2\pi f (T-t)}\, df$$

AT THE SAMPLE TIME

$$y_s(T) = \int_{-\infty}^{\infty} |S(f)|^2 \, df = \mathcal{E}_s$$

NOW THE OUTPUT SIGNAL OF CHANNEL IS $g_T(t) * c(t) = s(t)$, OR $S(f) = G_T(f)\,C(f)$. THUS, OUR MATCHED FILTER SHOULD BE

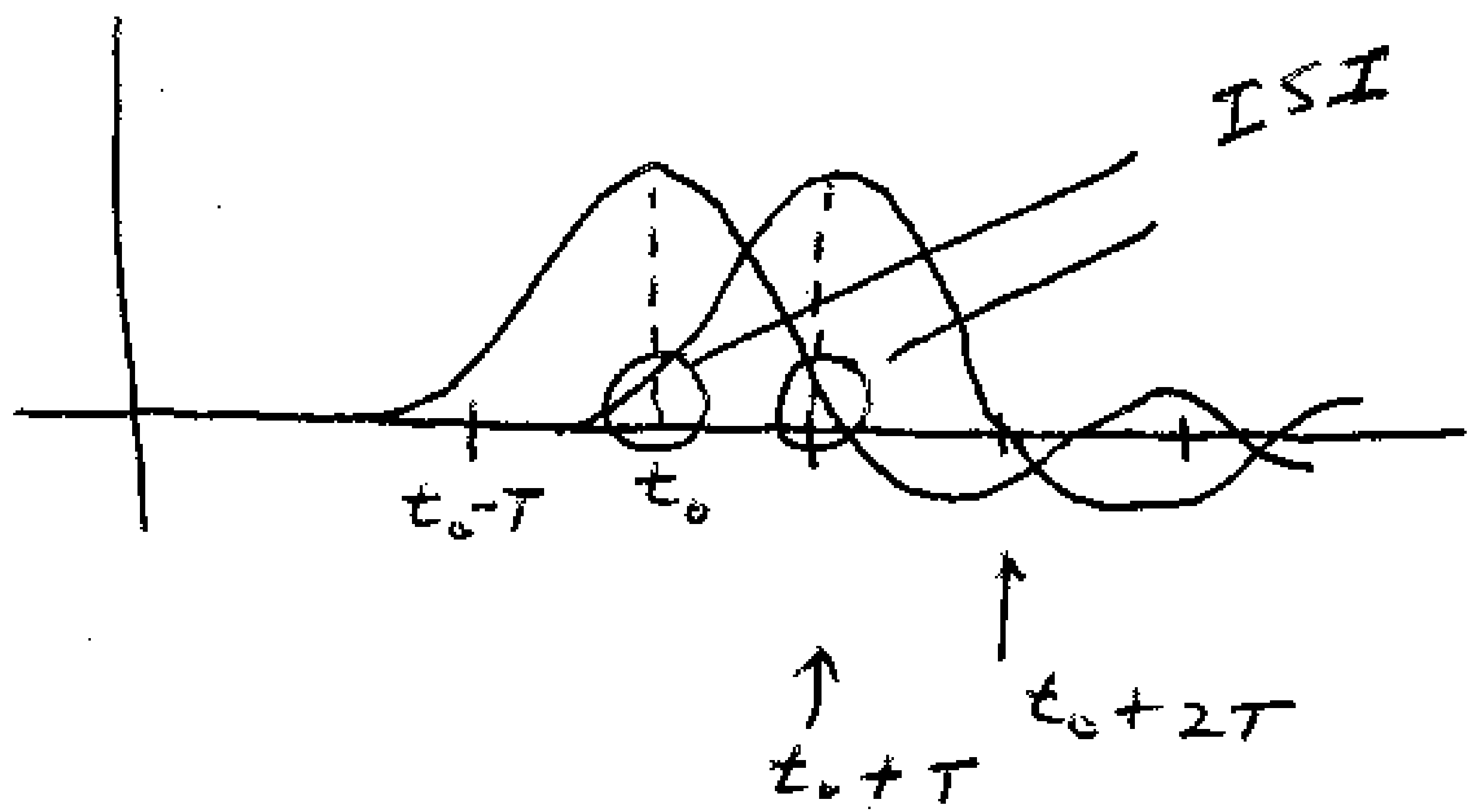
$$S^*(f)\, e^{-j2\pi f T} = G_T^*(f)\, C^*(f)\, e^{-j2\pi f T}$$

↑ NEED TO KNOW THIS

FOR NOW WE WILL ASSUME $C(f)$ IS KNOWN.

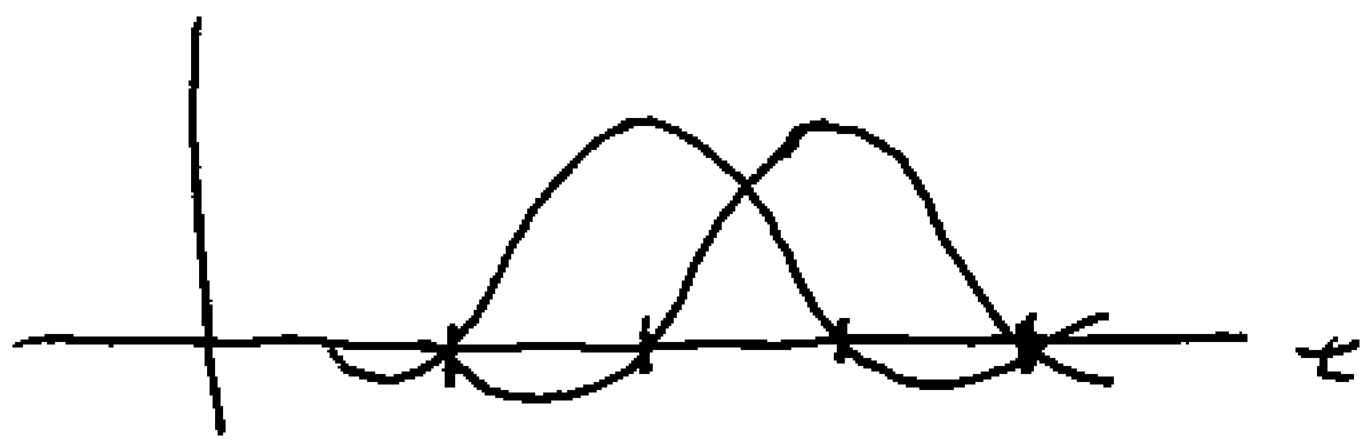
EFFECT OF WAVEFORM DISTORTION IS INTERSYMBOL INTERFERENCE (ISI), SINCE WE NOW CONSIDER A <u>SEQUENCE</u> OF RECEIVED PULSES (NO LONGER ONE-SHOT TRANSMISSION)

<u>EXAMPLE</u>: TWO SUCCESSIVE PULSE OUTPUTS OF MATCHED FILTER



t_0 = CHANNEL DELAY

WOULD BE NICE IF AT SAMPLE TIMES, t_0, $t_0 + T$, $t_0 + 2T$, ... ALL INTERFERING PULSES WENT THROUGH ZERO.



NO ISI!

PAM TRANSMISSION

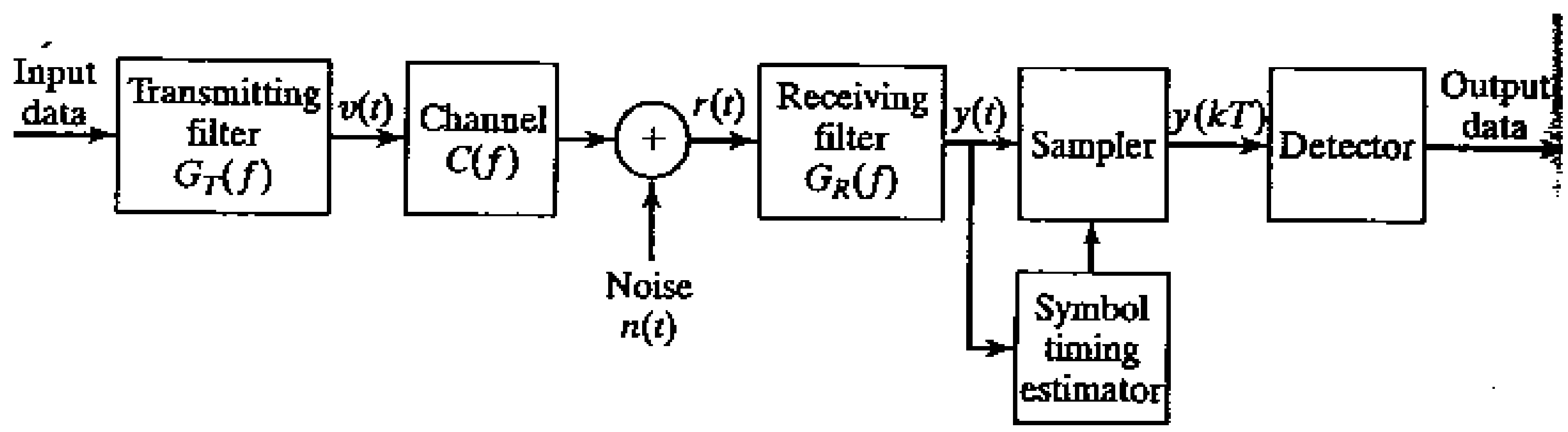


Figure 9.4 Block diagram of a digital PAM system.

NEEDED TO DECIDE WHEN TO SAMPLE - WANT TO SAMPLE AT PEAK (t_0 MAY NOT BE KNOWN)

$G_R(f)$ ADDED TO SHAPE OUTPUT PULSE TO HAVE ZEROS AT OTHER SAMPLING TIMES.

CONSIDER M-ARY PAM - CODE k BITS INTO ONE OF $M = 2^k$ LEVELS. LET a DENOTE THE LEVEL AND a_n THE LEVEL AT TIME nT. (RECALL $T = kT_b$). SEQUENCE OF PAM LEVELS ARE ..., a_{-1}, a_0, a_1, ...

$$v(t) = \sum_{n=-\infty}^{\infty} a_n \, g_T(t-nT)$$

LET $h(t) = g_T(t) * c(t)$

OUTPUT OF CHANNEL IS

$$r(t) = \sum_{n=-\infty}^{\infty} a_n \underbrace{g_T(t-nT) * c(t)}_{h(t-nT)} + n(t)$$

$\leftarrow$ SHOW THIS

AT THE RECEIVING FILTER OUTPUT:

$$y(t) = \sum_{n=-\infty}^{\infty} a_n \, h(t-nT) * g_R(t) + n(t) * g_R(t)$$

LET $x(t) = h(t) * g_R(t) = g_T(t) * c(t) * g_R(t)$

$$y(t) = \sum_{n=-\infty}^{\infty} a_n \, x(t-nT) + \underbrace{n(t) * g_R(t)}_{w(t)}$$

AT THE SAMPLE TIMES, WHICH WE ASSUME ARE $\ldots, -T, 0, T, \ldots$ (ADJUSTED FOR CHANNEL DELAY)

$$y(mT) = \sum_{n=-\infty}^{\infty} a_n \, x((m-n)T) + w(mT)$$

NEXT ASSUME THAT $x(t)$ HAS ITS PEAK AT TIME $t=0$ $\Rightarrow$ FOR A ONE-SHOT TRANSMISSION AT $n=0$ (IGNORES CHANNEL DELAY)

$$y(mT) = a_0 \, x(mT) + w(mT)$$

$y(0) = a_0 x(0) + w(0)$

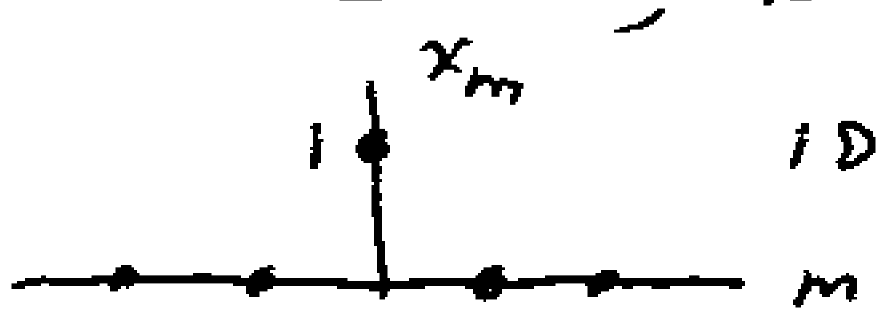
THUS, WE SAMPLE AT $t = 0$ TO DETERMINE a_0 OR IN GENERAL $y(mT)$ USED TO DECODE a_m.

LET $y_m = y(mT)$, ETC

$$y_m = \sum_{n=-\infty}^{\infty} a_n x_{m-n} + w_m$$

$$= a_m x_0 + \underbrace{\sum_{\substack{n=-\infty \\ n \neq m}}^{\infty} a_n x_{m-n}}_{ISI} + w_m$$

TO HAVE $ISI = 0$, REQUIRE $x_{m-n} = 0$ FOR $m \neq n$



IDEAL SEQUENCE

SIGNAL DESIGN

FIRST CONSIDER AN IDEAL CHANNEL

$$C(f) = \begin{cases} c_0 e^{-j2\pi f t_0} & |f| \leq W \\ 0 & |f| > W \end{cases}$$

t_0 IS TRANSMISSION DELAY, c_0 IS CHANNEL GAIN ($c_0 > 0$). IF INPUT SIGNAL IS $v(t)$, OUTPUT SIGNAL IS $c_0 v(t - t_0)$.

FOR $g_T(t)$ BANDLIMITED TO W Hz AND THE RECEIVING FILTER A MATCHED FILTER

$$x(t) = g_T(t) * c(t) * g_R(t)$$
$$= c_0\, g_T(t - t_0) * g_R(t)$$

LET $c_0 = 1$, $t_0 = 0$ FOR SIMPLICITY

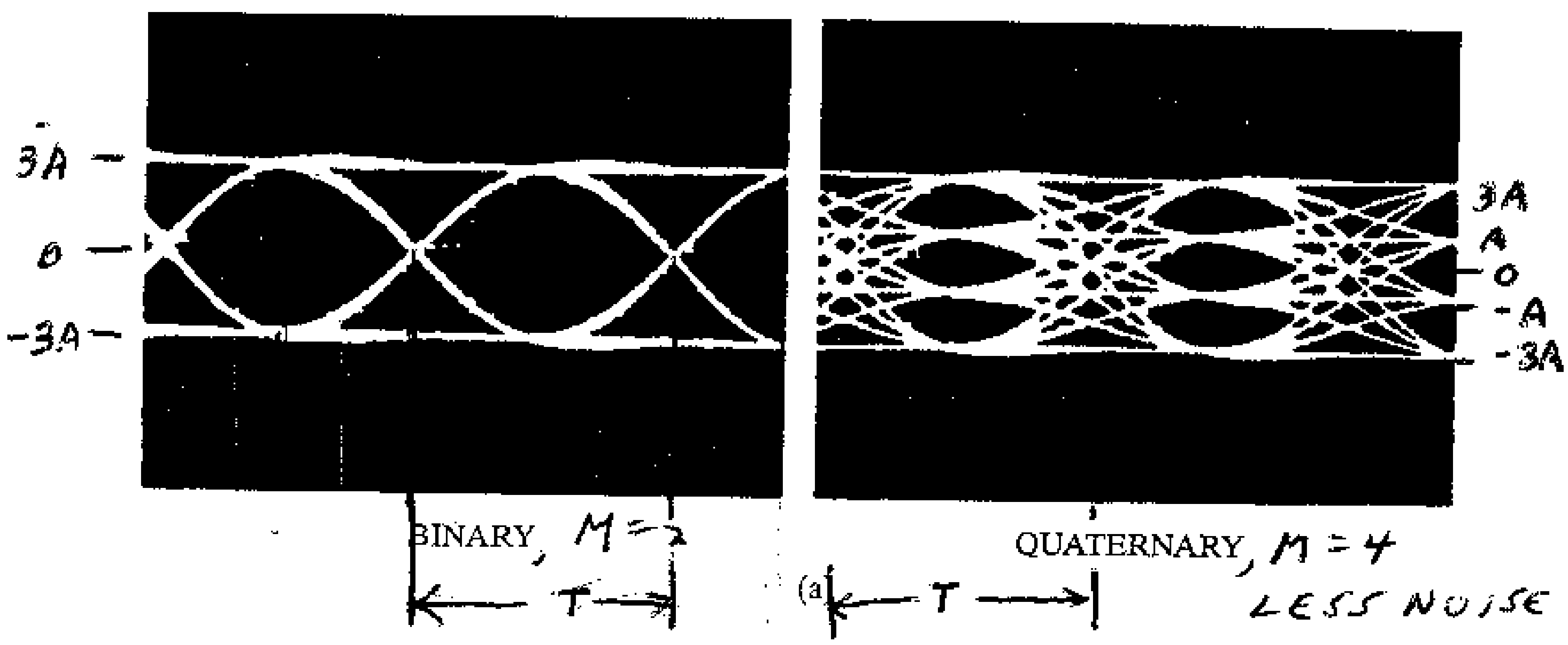
$$\Rightarrow \quad x(t) = g_T(t) * g_R(t)$$

$$x_m = g_T(t) * g_R(t) \Big|_{t = mT}$$

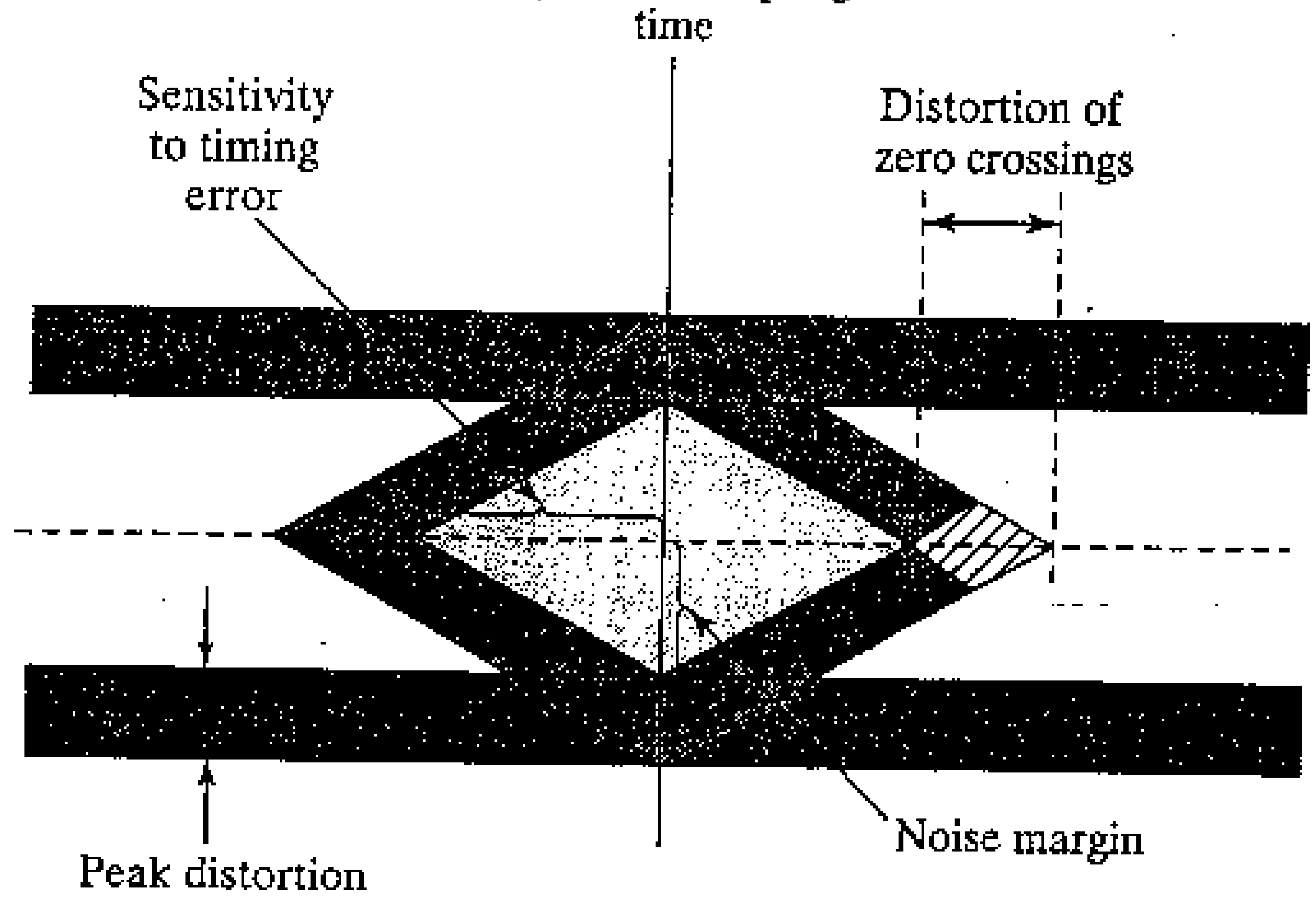
$\uparrow$ MF

OVERALL x_m MAY NOT BE ZERO FOR $m \neq 0$

EFFECT OF ISI CAN BE DISPLAYED USING "EYE" DIAGRAM.



(a)

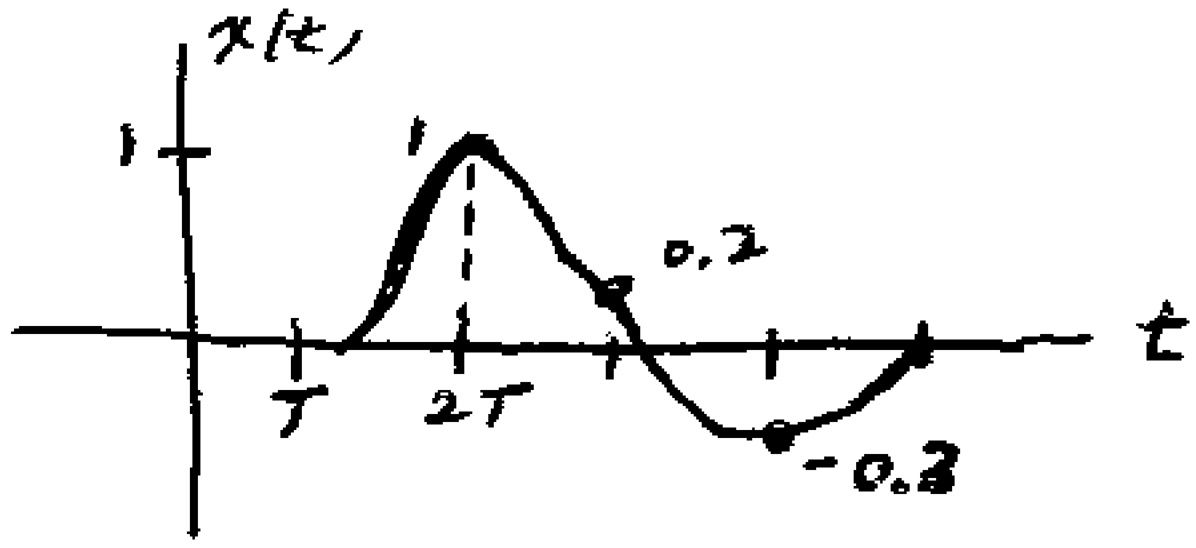


(b)

Figure 9.5 Eye patterns. (a) Examples of eye patterns for binary and quaternary PAM and (b) the effect of ISI on eye opening.

EXAMPLE: BINARY PAM, $a_m = \pm 1$

$$y_m = a_m + 0.2\, a_{m-1} - 0.3\, a_{m-2} + w_m$$

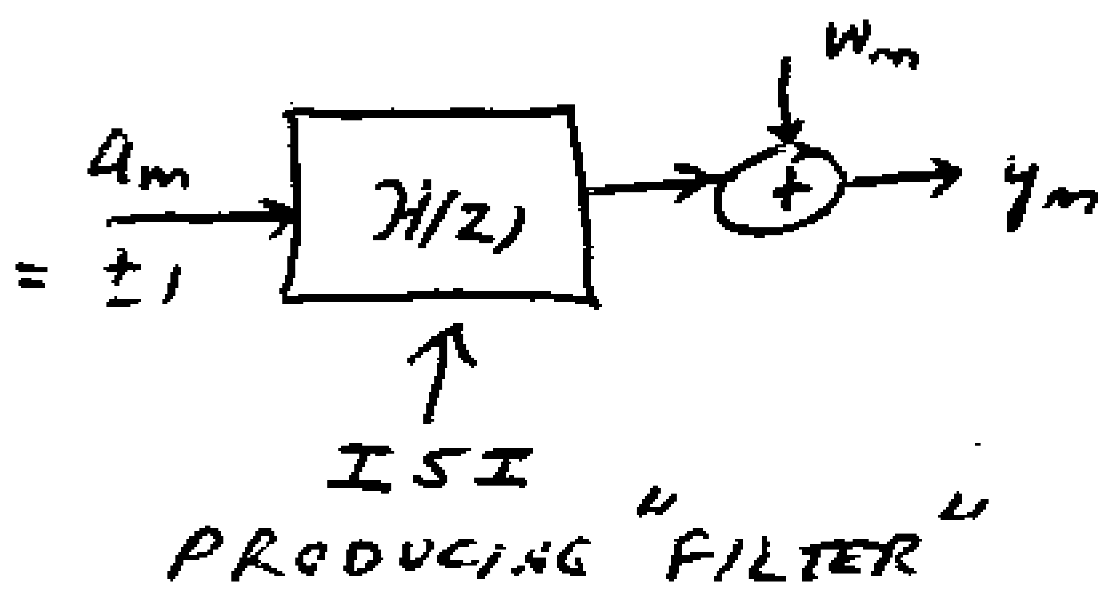


$t_0 = 2T$

WORST CASE OCCURS IF $a_m = 1$, $a_{m-1} = -1$, $a_{m-2} = 1 \Rightarrow y_m = 1 - 0.2 - 0.3 = 0.5$

$\Rightarrow$ 50% AMPLITUDE REDUCTION.

NOTE:



$H(z) = 1 + 0.2\, z^{-1} - 0.3\, z^{-2}$

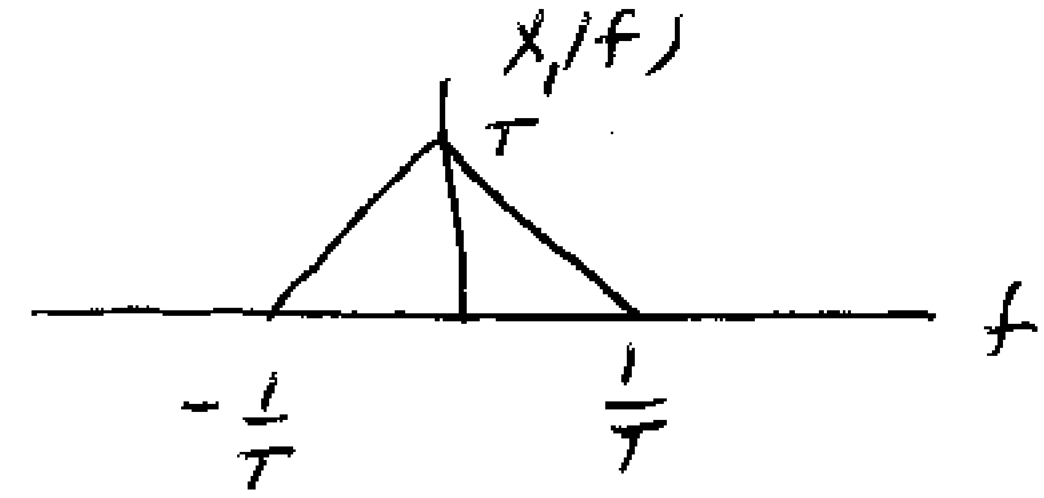
NYQUIST CRITERION FOR ZERO ISI

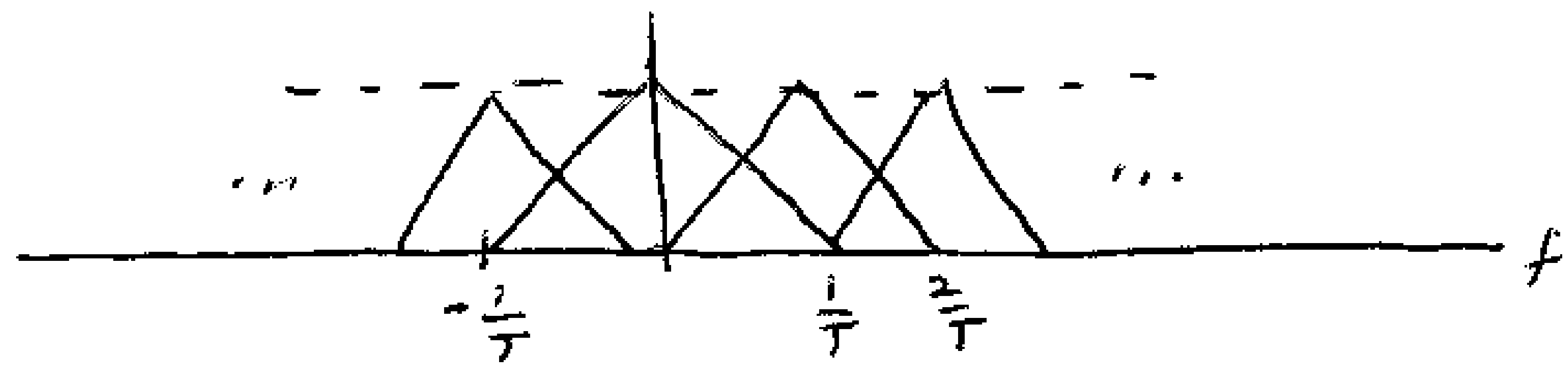
WANT (FOR $t_0 = 0$), $x(nT) = 1$, $n = 0$ AND $x(nT) = 0$ $n \neq 0$.

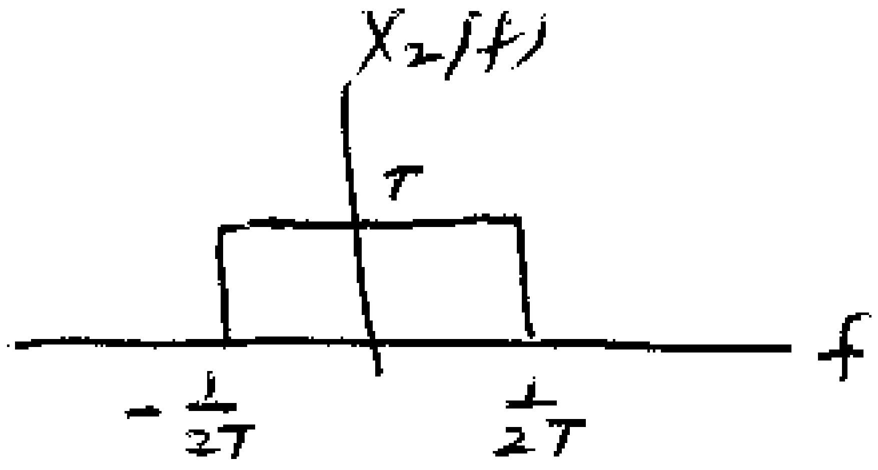
NYQUIST CONDITION: CHOOSE $x(t)$ SO THAT

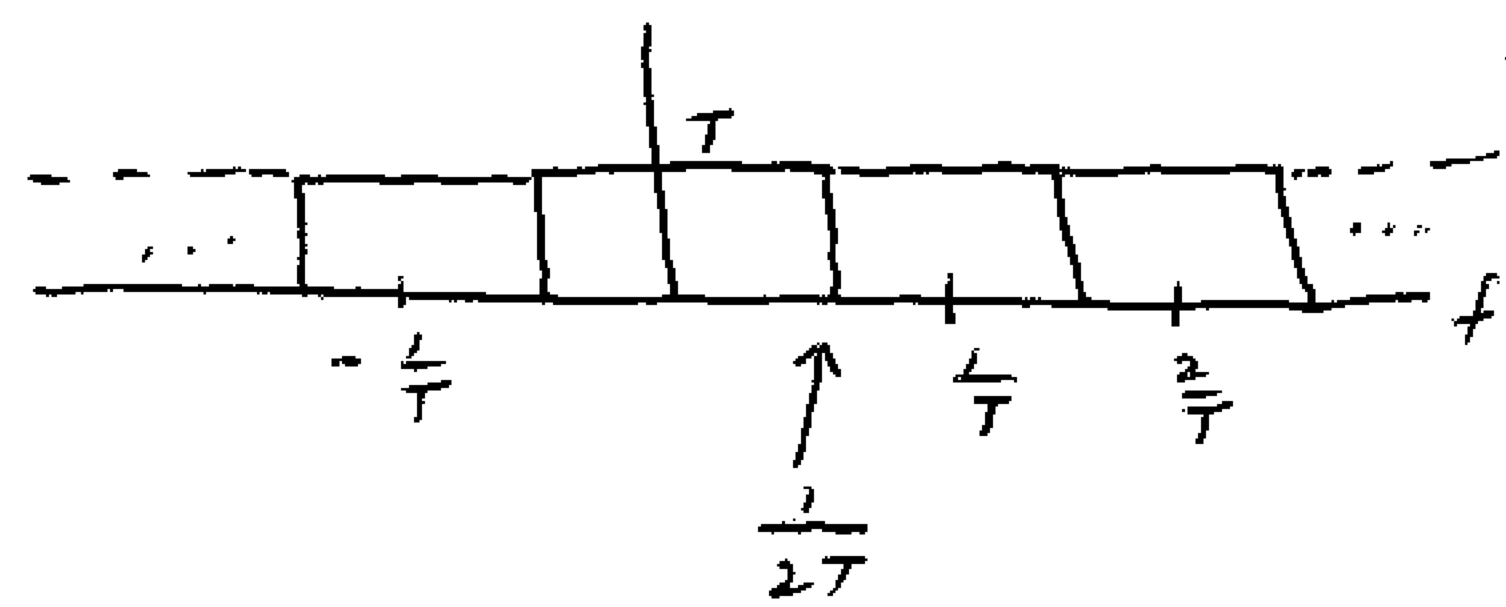
$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$$

EXAMPLE :









CLEARLY, $X(f)$ MUST HAVE FREQUENCY COMPONENTS TO AT LEAST $W = 1/2T = R_S/2$

PROOF : SAMPLING THEOREM PROOF
(SEE ALSO PROOF IN BOOK)

LET $x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT) = \delta(t)$

SINCE $x(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

$$\delta(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

FOURIER TRANSFORM BOTH SIDES

$$) = X(f) * \frac{1}{T}\sum_{m=-\infty}^{\infty} \delta(f - m/T)$$

$$= \frac{1}{T}\sum_{m=-\infty}^{\infty} X(f - m/T) = \frac{1}{T}\sum_{m=-\infty}^{\infty} X(f + m/T)$$

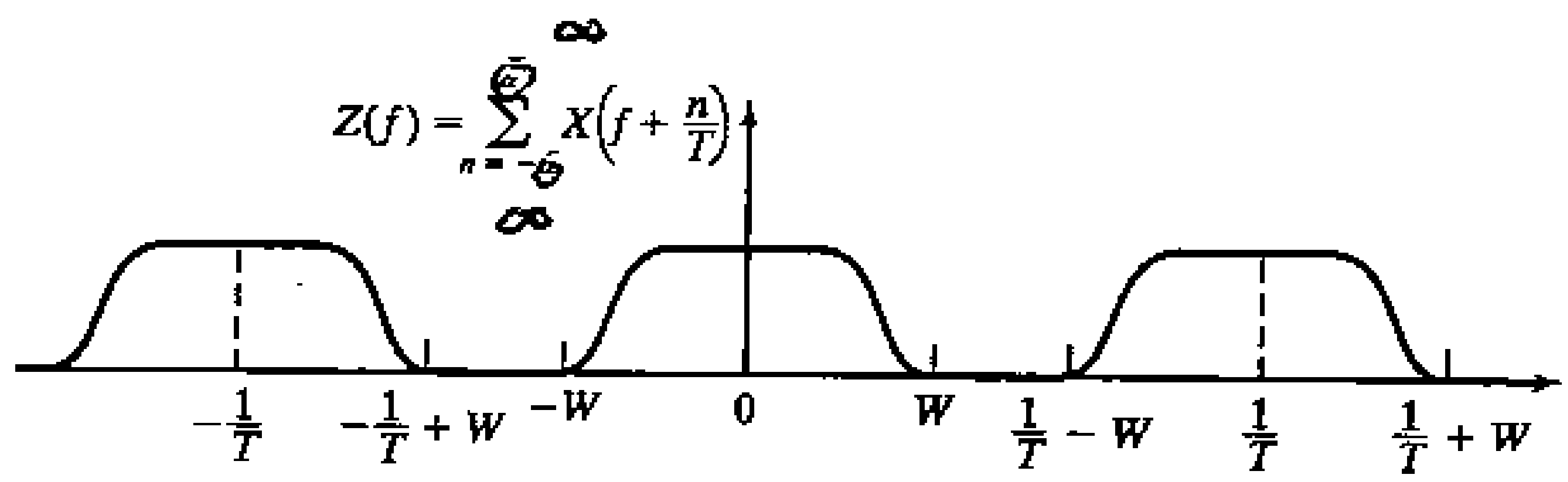


Figure 9.6 Plot of $Z(f)$ for the case $T < \frac{1}{2W}$. $\quad 1/T > 2W$

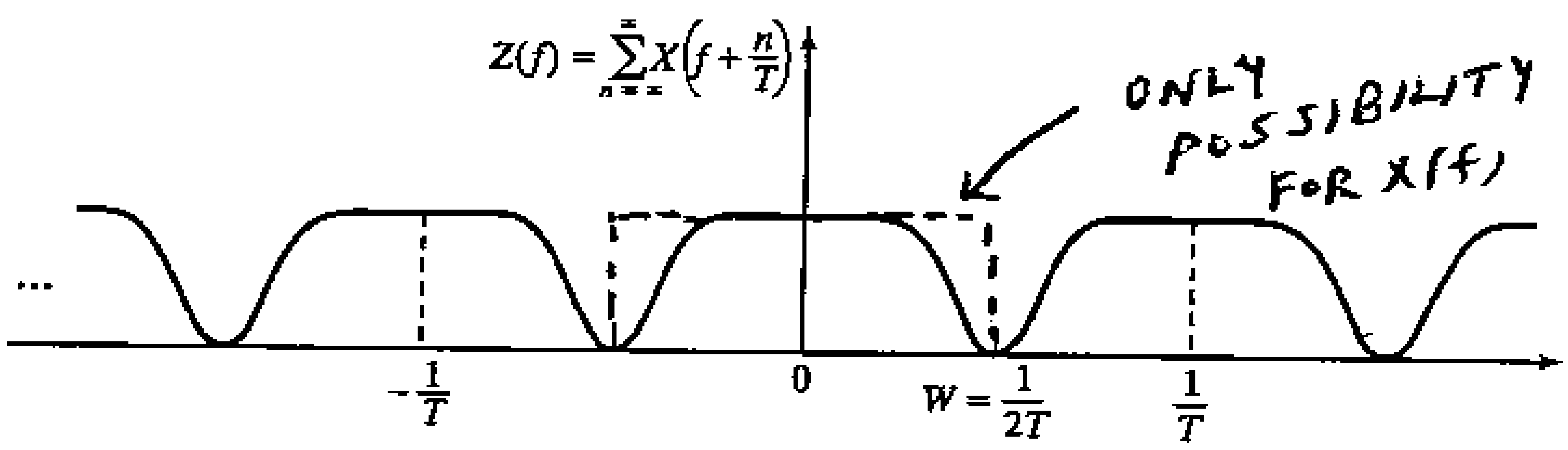
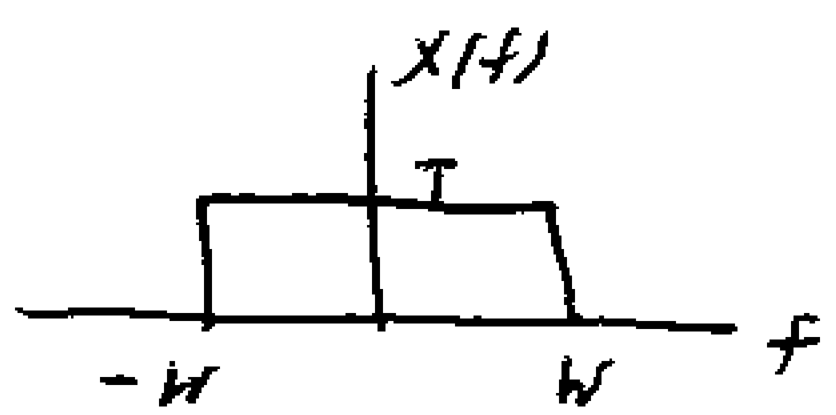


Figure 9.7 Plot of $Z(f)$ for the case $T = \frac{1}{2W}$. $\quad 1/T = 2W$

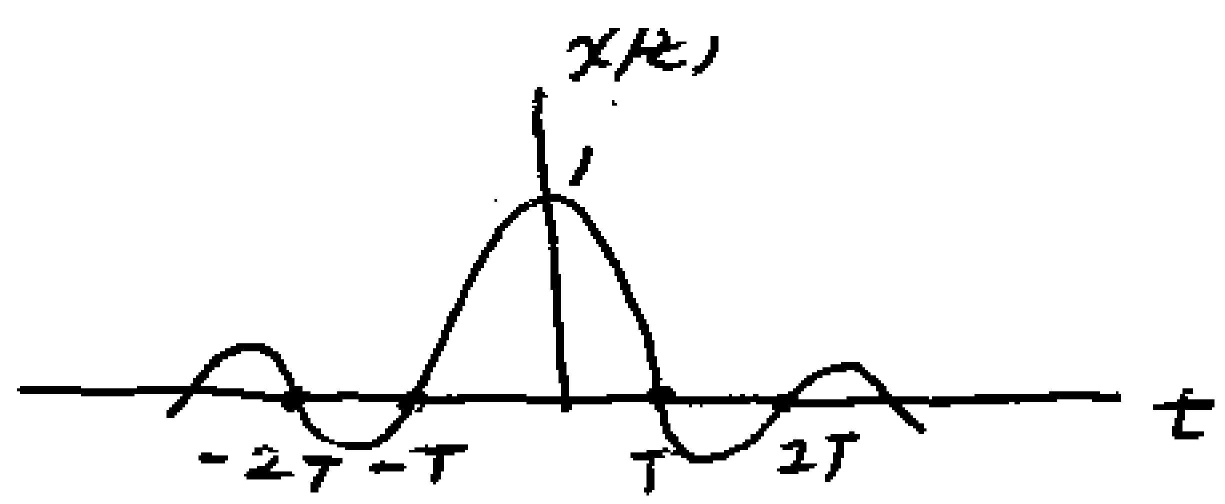
FOR $1/T > 2W \Rightarrow$ NO $X(f)$ EXISTS

FOR $1/T = 2W \Rightarrow$ ONLY SOLUTION IS



$$\Rightarrow x(t) = \frac{\sin 2\pi W t}{2\pi W t}$$

SEE SLIDE 18



$$x(t) = \frac{\sin \pi t/T}{\pi t/T}$$

PROBLEM WITH THIS PULSE IS THAT TAILS DIE OFF AS $1/t$. WHAT HAPPENS IF OUR SAMPLER IS IN ERROR (TIMING JITTER)?

EXAMPLE: SIGNAL AT SAMPLER INPUT IS

$$y(t) = \sum_{n=-\infty}^{\infty} a_n\, x(t-nT)$$

$$= \sum_{n=-\infty}^{\infty} a_n \frac{\sin \pi(t-nT)/T}{\pi(t-nT)/T}$$

$$y(0) = \sum_{n=-\infty}^{\infty} a_n \underbrace{\frac{\sin \pi nT/T}{\pi nT/T}}_{\frac{\sin n\pi}{n\pi} = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}}$$

$$= a_0$$

$$y(T/2) = \sum_{n=-\infty}^{\infty} a_n \frac{\sin \pi(T/2-nT)/T}{\pi(T/2-nT)/T}$$

$$= \frac{\sin(\pi/2 - n\pi)}{\pi(\frac{1}{2}-n)} = \frac{\sin \pi/2 \cos(-n\pi)}{-\pi(n-\frac{1}{2})}$$

$$= \frac{(-1)^n}{-\pi(n-\frac{1}{2})}$$

$$y(T/2) = \sum_{n=-\infty}^{\infty} a_n \frac{(-1)^n}{-\pi(n-\frac{1}{2})}$$

$$= -\frac{1}{\pi}\left(\dots + \frac{a_{-2}}{-5/2} + \frac{a_{-1}}{3/2} + \frac{a_0}{-\frac{1}{2}} + \frac{a_1}{-\frac{1}{2}} + \frac{a_2}{3/2} + \dots\right)$$

LET $a_{-2} = 1,\ a_{-1} = -1,\ a_0 = 1,\ a_1 = 1,\ a_2 = -1,\ \dots$

$$= \frac{1}{\pi}\left(\dots + \frac{2}{5} + \frac{2}{3} + 2 + 2 + \frac{2}{3} + \dots\right)$$

$$y(T/2) = \frac{1}{\pi}\sum_{n=-\infty}^{\infty} \frac{1}{|n-\frac{1}{2}|} \rightarrow \infty$$ "EYE" IS CLOSED

NEED $1/n^2$ DECAY OR AT LEAST $1/t^2$ FOR $x(t)$.

MUST HAVE $1/T < 2W$

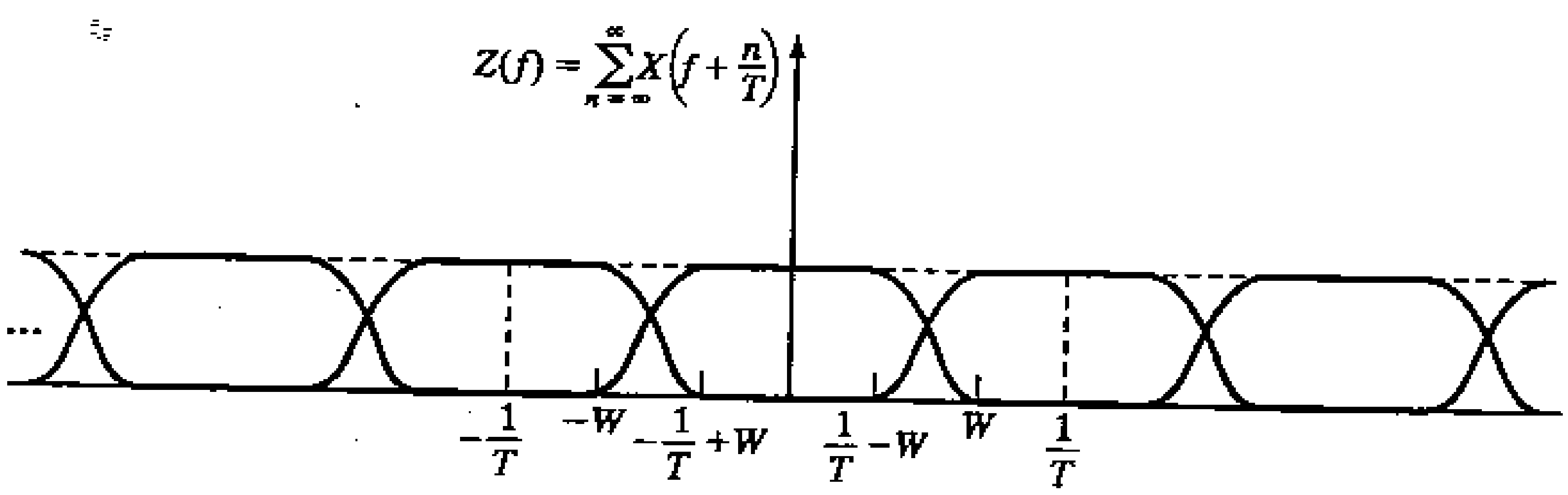


Figure 9.8 Plot of $Z(f)$ for the case $T > \frac{1}{2W}$.

IN PRACTICE, A RAISED COSINE FREQ RESPONSE IS USED

$$X_{rc}(f) = \begin{cases} T, & 0 \le |f| \le (1-\alpha)/2T \\ \frac{T}{2}\left[1 + \cos\left(\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right)\right], & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0, & |f| > \frac{1+\alpha}{2T} \end{cases}, \quad (9.2.20)$$

$0 \le \alpha \le 1$

$$W = \frac{1}{2T}(1+\alpha)$$

$$\frac{1}{2T} \le W \le \frac{1}{T}$$

↑ PREVIOUS CASE

↑ TWICE AS MUCH BANDWIDTH

$$x(t) = \frac{\sin \pi t/T}{\pi t/T} \quad \frac{\cos(\pi \alpha t/T)}{1 - 4\alpha^2 t^2/T^2}$$

$x(0) = 1$ AND $x(t)$ DECAYS AS $1/t^3$

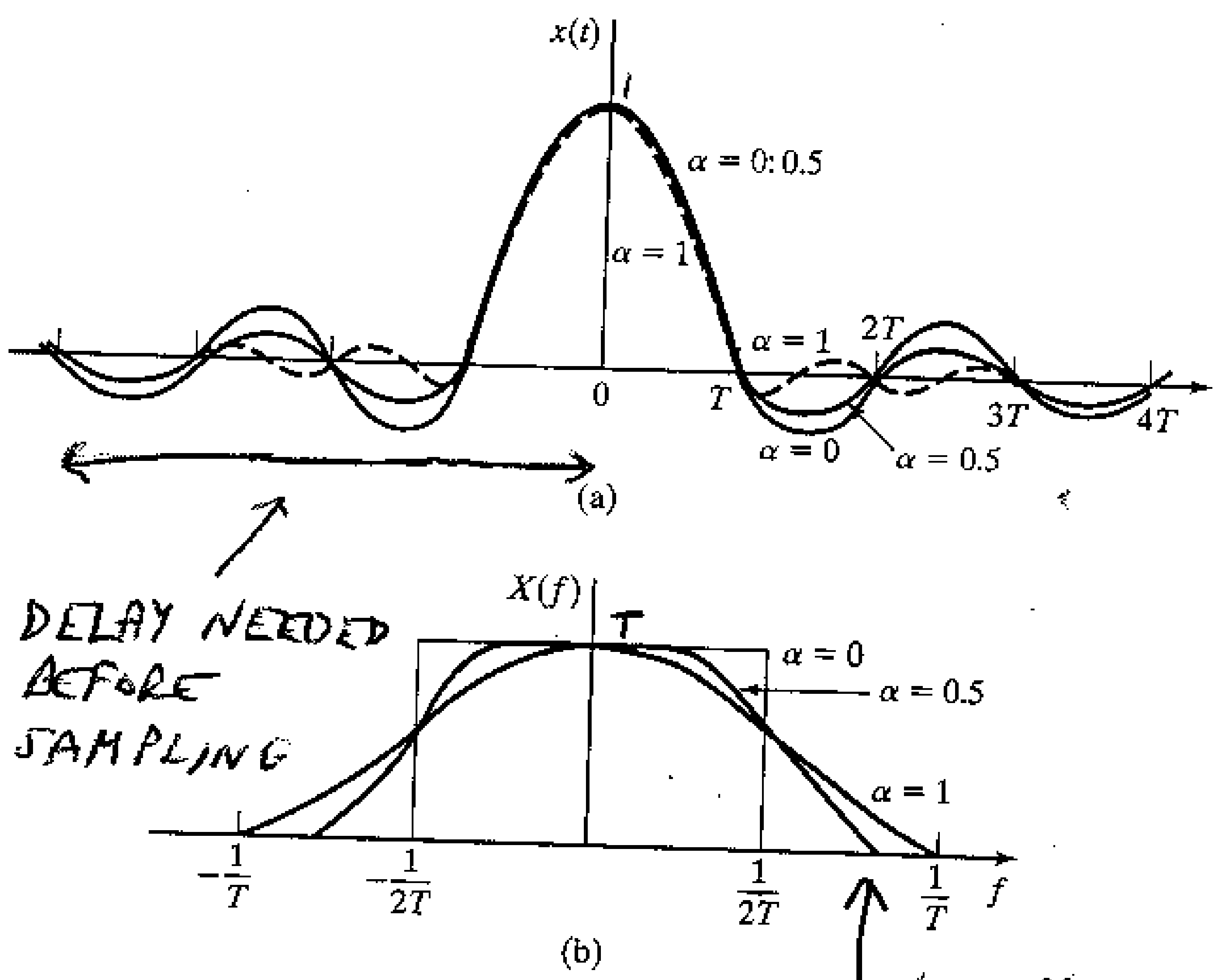


Figure 9.9 Pulses having a raised cosine spectrum.

FOR AN IDEAL CHANNEL $C(f) = 1 \quad |f| \le W$

$$X(f) = G_T(f)\, C(f)\, G_R(f) = G_T(f)\, G_R(f)$$

AND $G_R(f)$ A MATCHED FILTER $\Rightarrow$

$$G_R(f) = G_T^*(f)$$

$$X(f) = |G_T(f)|^2$$