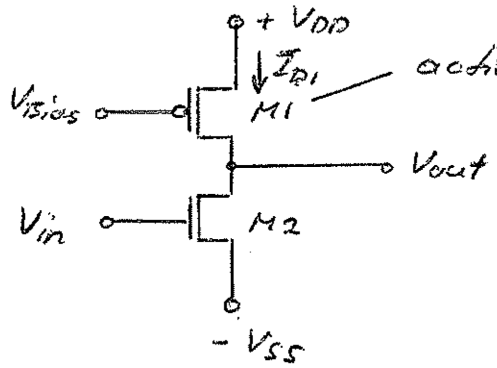
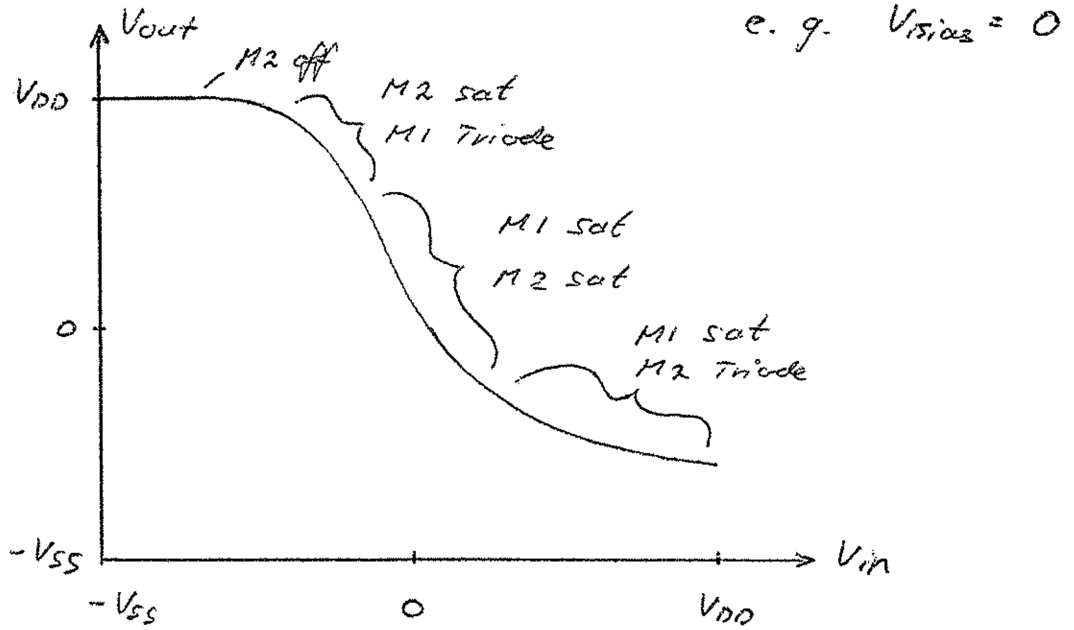
III MOS Sub Circuits (Building Blocks)

1. The MOS Inverter (Common Source Amp.)

Basic Configuration



a) Large Signal Response (DC Transfer char.)



DC Considerations

If $V_{DD} = V_{SS}$ and $V_{Bias} = V_{in\,DC} = V_{out\,DC}$ we can write:

$$I_{D_1} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p\left[-V_{DD} - V_{tp}\right]^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n\left[V_{SS} - V_{tn}\right]^2$$

$$\therefore \boxed{\mu_p\left(\frac{W}{L}\right)_p\left[-V_{DD} - V_{tp}\right]^2 = \mu_n\left(\frac{W}{L}\right)_n\left[V_{SS} - V_{tn}\right]^2}$$

or

$$\boxed{\frac{\mu_n\left(\frac{W}{L}\right)_n}{\mu_p\left(\frac{W}{L}\right)_p} = \frac{\left[-V_{DD} - V_{tp}\right]^2}{\left[V_{SS} - V_{tn}\right]^2}}$$

Examples $V_{DD} = V_{SS} = 2.5V$

$V_{tn} = 0.7V$ $\qquad C_{ox} = 2.5 \cdot 10^{-3}\ \frac{F}{m^2}$

$V_{tp} = -0.9V$

$\mu_n = 4 \cdot 10^{-2}\ \frac{m^2}{Vs}$

$\mu_p = 1.8 \cdot 10^{-2}\ \frac{m^2}{Vs}$

Find $\left(\frac{W}{L}\right)_n$ and $\left(\frac{W}{L}\right)_p$ such that $I_{D_1} = 100\mu A$

Solution

$$\left(\frac{W}{L}\right)_p = \frac{I_{D_1}}{\frac{1}{2}\mu_p C_{ox}\left[V_{DD} + V_{tp}\right]^2} \cong 1.74$$

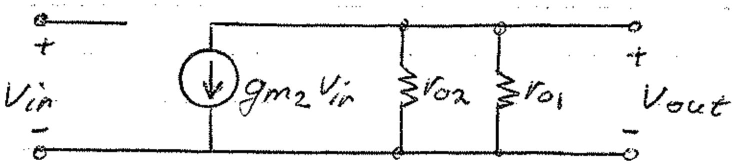
$$\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p \frac{\mu_p}{\mu_n}\frac{\left[-V_{DD} - V_{tp}\right]^2}{\left[V_{SS} - V_{tn}\right]^2} = 0.62$$

e.g. $W_p = 7$ $\quad W_n = 2.5$

$L_p = 4$ $\quad L_n = 4$

AC Considerations

Linear equivalent circuit



$$\boxed{A_V = \frac{V_{out}}{V_{in}} = -g_{m2}(r_{o1} \| r_{o2}) = -\frac{g_{m2}}{g_{o1}+g_{o2}}}$$

where

$$\boxed{\begin{aligned} g_{m2} &= \sqrt{2\left(\tfrac{W}{L}\right)_n \mu_n C_{ox} I_{D1}} \\ g_{o1} &= I_{D1}\lambda_p \\ g_{o2} &= I_{D1}\lambda_n \end{aligned}}$$

$$\therefore \quad \boxed{A_V = -\frac{\sqrt{2\left(\tfrac{W}{L}\right)_n \mu_n C_{ox}}}{\sqrt{I_{D1}}\,[\lambda_n + \lambda_p]}}$$

Recall: $\quad \lambda \cong \frac{1}{L}\sqrt{\frac{\varepsilon_s}{2qN_{sub}(\varphi_0 + V_{DS} - V_{eff})}} \approx \frac{9 \cdot 10^{-8}[m]}{L}$

Thus $\quad \boxed{|A_V| \propto \sqrt{\frac{W \cdot L \cdot \mu C_{ox} N_{sub}}{I_D}}}$

e.g. $L_n = L_p = 4\mu m$

$W_p = 7\mu m$

$W_n = 2.5\mu m$

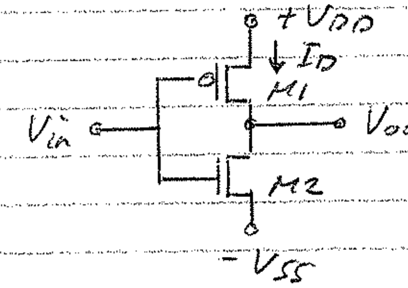
$I_D = 100\mu A$

$\mu_n C_{ox} = 10^{-4}\ \frac{A}{V^2}$

$\therefore \quad |A_V \cong -24.8|$

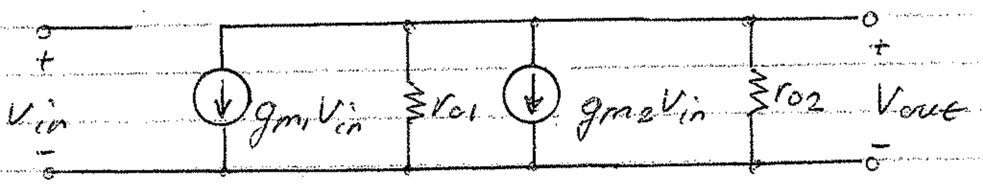
Special Case CMOS inverter

Self-biased inverter



$$\boxed{I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p\left[V_{inQ}-V_{DD}-V_{tp}\right]^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n\left[V_{inQ}-V_{SS}-V_{tn}\right]^2}$$

Linear Equivalent Circuit



$$\therefore \boxed{A_V = \frac{V_{out}}{V_{in}} = -\frac{g_{m1}+g_{m2}}{g_{o1}+g_{o2}}}$$

If $\mu_n C_{ox}\left(\frac{W}{L}\right)_n = \mu_p C_{ox}\left(\frac{W}{L}\right)_p = \beta$

Then

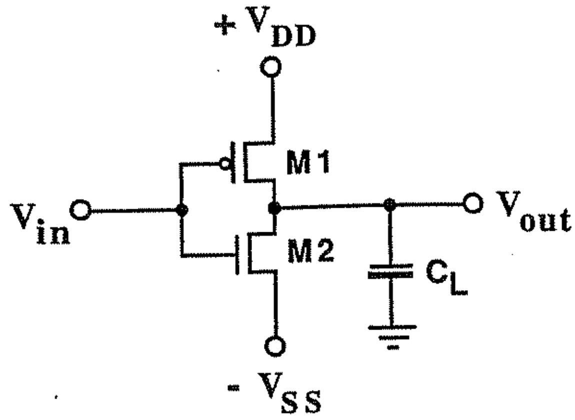
$$\boxed{A_V = -\frac{\sqrt{\beta}}{\sqrt{I_D}}\,\frac{2\sqrt{2}}{(\lambda_n+\lambda_p)}} \quad \therefore A_V \propto L_{eff}$$

e.g.

$$\beta = 2\cdot 10^{-4}\ \frac{A}{V^2}$$
$$I_D = 10^{-4}\,A$$
$$\lambda_n = \lambda_p = 0.05\ \frac{1}{V}$$

$$\therefore \boxed{A_V \cong -40}$$

The self-biased CMOS Inverter



Linear gain

Assumption:

$$\frac{1}{2}\mu_1\left(\frac{W}{L}\right)_1 C_{ox} = \frac{1}{2}\mu_2\left(\frac{W}{L}\right)_2 C_{ox} = k \qquad (1)$$

$$\left| \; A_v \cong -\frac{4}{[V_{SS}-V_t][\lambda_1+\lambda_2]} \; \right| \qquad (2)$$

where

$$\lambda \cong \frac{1}{L_{eff}}\sqrt{\frac{\varepsilon_S}{2\,q\,N_{Sub}\,(\phi_B+V_{DS}-V_{eff})}} \qquad (3)$$

and

$$L_{eff} \cong L_{Drawn} - x_{LD} - \sqrt{\frac{2\,\varepsilon_S\,(\phi_B+V_{DS}-V_{eff})}{q\,N_{Sub}}} \qquad (4)$$

Therefore

$$A_v \propto L_{eff} \qquad (5)$$

Settling behavior

A) Linear

$$\left| V_{out}(t) = \Delta V\, A_v \left[1 - e^{-\frac{t}{\tau}}\right] \right| \qquad t \geq 0 \quad \text{and} \quad \Delta V \leq \frac{V_t}{|A_v|} \qquad (6)$$

where

$$\tau \cong \frac{C_L}{k} \frac{1}{(\lambda_1+\lambda_2)\,[V_{SS}-V_t]^2} \qquad (7)$$

Consequently

$$\tau \propto \frac{L_{eff}}{k} \propto L_{eff}\left(\frac{L}{W}\right) \qquad (8)$$

B) Nonlinear

$$\left| t_{settl.} \cong \frac{C_L}{k} \frac{2\sqrt{\Delta V(V_{SS}-V_t)} + (V_t-\Delta V)}{2\Delta V(V_{SS}-V_t)} \right| \qquad \frac{V_t}{|A_v|} \leq \Delta V \leq V_t \qquad (9)$$

Case B:

$$\left| t_{settl.} \cong \frac{C_L}{k} \frac{2\sqrt{\Delta V(V_{SS}-V_t)} - (\Delta V-V_t)}{2\, V_{SS}\, \Delta V - \frac{1}{2}(\Delta V+V_t)^2} \right| \qquad V_t \leq \Delta V \leq (V_{SS}-V_t) \qquad (10)$$

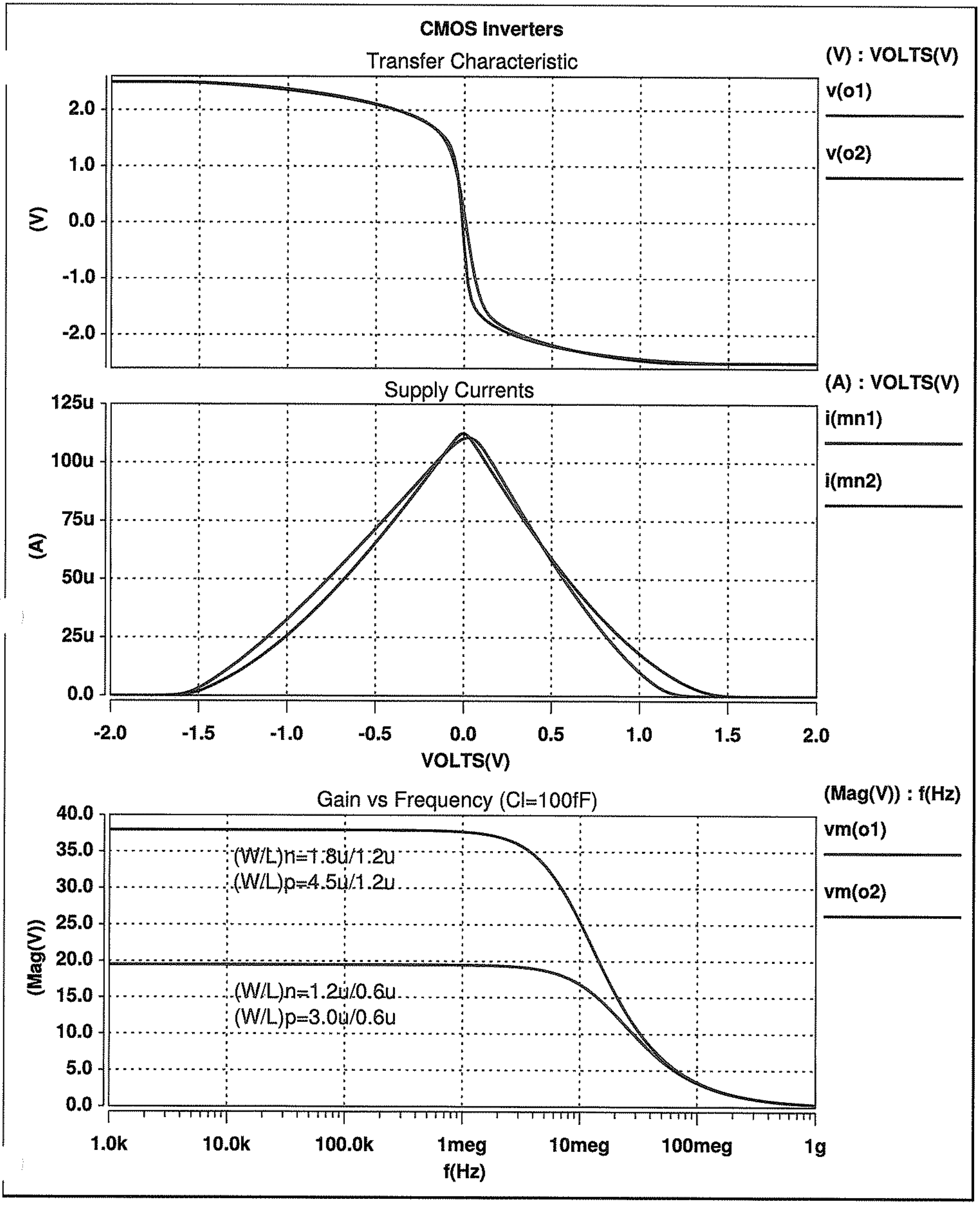
Case C:

$$\left| t_{settl.} \cong \frac{C_L}{k} \frac{1}{\frac{1}{2}V_{SS} + (\Delta V-V_t)} \right| \qquad (V_{SS}-V_t) \leq \Delta V \leq V_{SS} \qquad (11)$$

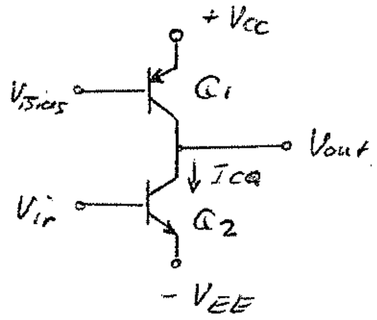
Numerical Example (90% settling times according to SPICE)

$V_{DD} = V_{SS} = 2.5V$
$V_t = 0.8V$
$C_L = 400fF$
$k = 100\mu A/V^2$

ΔV [V]	$t_{settl.}$ [ns]
0.2	10.4
0.3	7.6
0.4	6.0
0.6	4.4
0.9	3.1
1.2	2.5
1.6	2.0
2.0	1.6
2.5	1.4


CMOS Inverters
Transfer Characteristic
(V) : VOLTS(V)
v(o1)
v(o2)
(V)
2.0
1.0
0.0
-1.0
-2.0
Supply Currents
(A) : VOLTS(V)
i(mn1)
i(mn2)
(A)
125u
100u
75u
50u
25u
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
VOLTS(V)
Gain vs Frequency (Cl=100fF)
(Mag(V)) : f(Hz)
vm(o1)
vm(o2)
(Mag(V))
40.0
35.0
30.0
25.0
20.0
15.0
10.0
5.0
0.0
(W/L)n=1.8u/1.2u
(W/L)p=4.5u/1.2u
(W/L)n=1.2u/0.6u
(W/L)p=3.0u/0.6u
1.0k
10.0k
100.0k
1meg
10meg
100meg
1g
f(Hz)

BJT Inverter (CE amplifier)

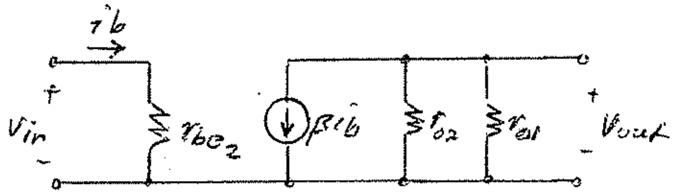


Q-point:

$$I_{CQ} \cong I_{S1}\, e^{\frac{(V_{CC} - V_{Bias})}{V_T}}$$

AC Performance:

linear eq. circuit



$$\left| \begin{array}{l} V_{out} = -\beta i_b (r_{o1} \| r_{o2}) \\ i_b = V_{in} \frac{1}{r_{be2}} \end{array} \right| \therefore \left| V_{out} = -V_{in} \frac{\beta}{r_{be2}} (r_{o1} \| r_{o2}) \right|$$

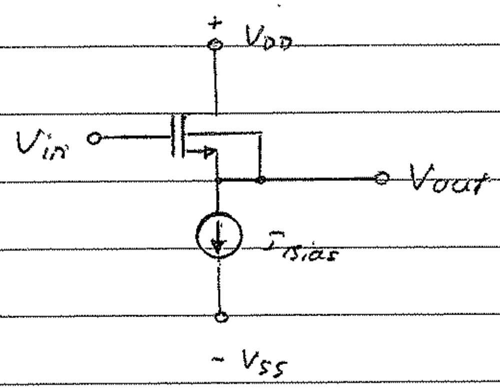
where $r_{be2} \cong \beta \frac{V_T}{I_{CQ}}$

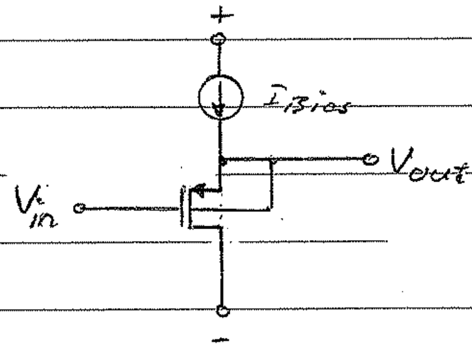
and $r_{o1} \cong V_{A1}/I_{CQ}$ $\quad r_{o2} = V_{A2}/I_{CQ}$

$$\therefore \left\| A_V = \frac{V_{out}}{V_{in}} \cong - \frac{V_{A1} \cdot V_{A2}}{(V_{A1} + V_{A2})} \frac{1}{V_T} \right\| \qquad V_T = \frac{kT}{q}$$

$$A_V \propto \frac{\overline{V_A}}{V_T}$$

2. Source Follower (Common Drain Amp.)





p-well process (no body effect) n-well process

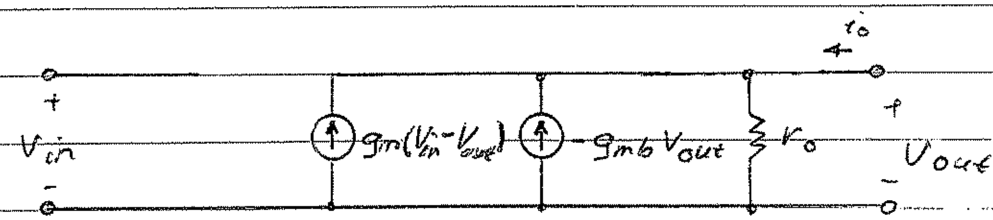
a) Large signal response

$$I_D = I_{Bias} \cong \frac{1}{2}\frac{W}{L}\mu C_{ox}(V_{in}-V_{out}-V_t)^2 \qquad \text{(sat.)}$$

$$\Rightarrow \boxed{V_{out} \cong (V_{in}-V_t) - \sqrt{2\frac{I_{Bias}\cdot L}{W\mu C_{ox}}}} \qquad \text{dc}$$

where $V_t = V_{t0} + \gamma\left[\sqrt{2\phi_F + \underbrace{V_{BS}}_{V_{out}-V_B}} - \sqrt{2\phi_F}\right]$

b) Small signal response ($V_{BS} \neq 0$)



$$\Rightarrow V_{out} = g_m r_o (V_{in}-V_{out}) - g_{mb} r_o V_{out}$$

$$V_{out}[1 + g_m r_o + g_{mb} r_o] = g_m r_o V_{in}$$

$$\Rightarrow \quad \underline{\underline{\frac{V_{out}}{V_{in}} = \frac{r_o g_m}{1 + r_o [g_m + g_{mb}]}}} \qquad g_{mb} = g_m \frac{1}{2} \gamma \frac{1}{\sqrt{2\phi_F + V_{SB}}}$$

$$g_m r_o = \frac{1}{\lambda} \sqrt{\frac{2k}{I_{Bias}}} \qquad \left(k = \frac{1}{2} \frac{W}{L} \mu C_{ox}\right)$$

$$\Rightarrow \quad \left\| \frac{V_{out}}{V_{in}} = \frac{1}{1 + \lambda \sqrt{\frac{I_{Bias}}{2k}} + \underbrace{\frac{1}{2} \gamma \frac{1}{\sqrt{2\phi_F + V_{SB}}}}_{\text{from Body effect}}} \right\|$$

<u>Examples</u>: $\lambda = 4 \cdot 10^{-2}\ V^{-1}$

$I_{Bias} = 100\ \mu A$

$k = 200\ \frac{\mu A}{V^2}$

$\gamma = 0.5\ \sqrt{V}$

$\phi_F = 0.4 V$

a) $V_{SB} = 0 \quad (V_B = V_{OUT})$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + 4 \cdot 10^{-2} \sqrt{\frac{1}{4}}} \cong \underline{\underline{0.98}}$$

b) $V_{SB} \neq 0 \quad (V_B = 0)$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + 2 \cdot 10^{-2} + \frac{1}{4} \frac{\sqrt{V}}{\sqrt{0.8 V + V_{out}}}} \Bigg|_{V_{out} = 1V} = \underline{\underline{0.83}}$$

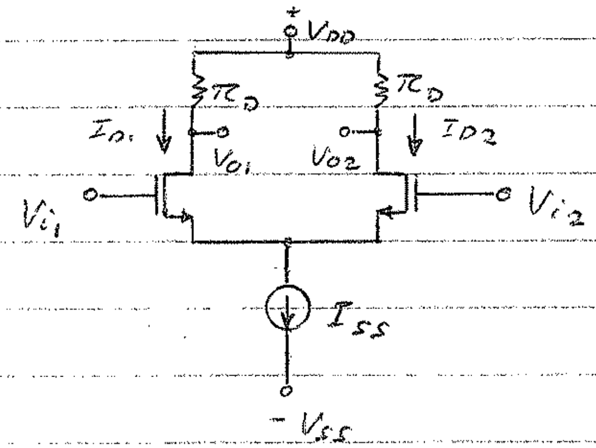
c) <u>output impedance of source follower</u>

from linear equivalent circuit:

$$V_o\Big|_{V_i=0} = [i_o - (g_m + g_{mb})V_o]\, r_o$$

$$\Rightarrow \frac{V_o}{i_o}\Big|_{V_i=0} = \frac{r_o}{1 + r_o(g_m + g_{mb})} \cong \underline{\underline{\frac{1}{g_m + g_{mb}}}} \qquad g_o \ll g_m$$

3. <u>Source Coupled pair</u>



a) <u>Large signal analysis</u>

$$I_{D1} = \frac{1}{2}\frac{W}{L}\mu C_{ox}[V_{GS1} - V_T]^2$$

$$I_{D2} = \frac{1}{2}\frac{W}{L}\mu C_{ox}[V_{GS2} - V_T]^2$$

$$V_{GS1} = V_T + \sqrt{2\frac{I_{D1}L}{W\mu C_{ox}}} = V_T + \sqrt{\frac{I_{D1}}{k_1}}$$

$$V_{GS2} = V_T + \sqrt{2\frac{I_{D2}L}{W\mu C_{ox}}} = V_T + \sqrt{\frac{I_{D2}}{k_2}}$$

we define:

$$\begin{array}{l|l} \Delta V_i = V_{i1} - V_{i2} = V_{GS1} - V_{GS2} & KVL \\ I_D = \frac{1}{2}(I_{D1} + I_{D2}) = \frac{1}{2} I_{SS} & KCL \\ \Delta I_D = I_{D1} - I_{D2} & \end{array}$$

$$I_{D1} = I_D + \frac{1}{2}\Delta I_D \qquad I_{D2} = I_D - \frac{1}{2}\Delta I_D$$ Differential Mode

$$\Rightarrow \Delta V_i = \sqrt{\frac{I_D + \frac{1}{2}\Delta I_D}{k}} - \sqrt{\frac{I_D - \frac{1}{2}\Delta I_D}{k}} \qquad k = \frac{1}{2}\frac{W}{L}\mu C_{ox}$$

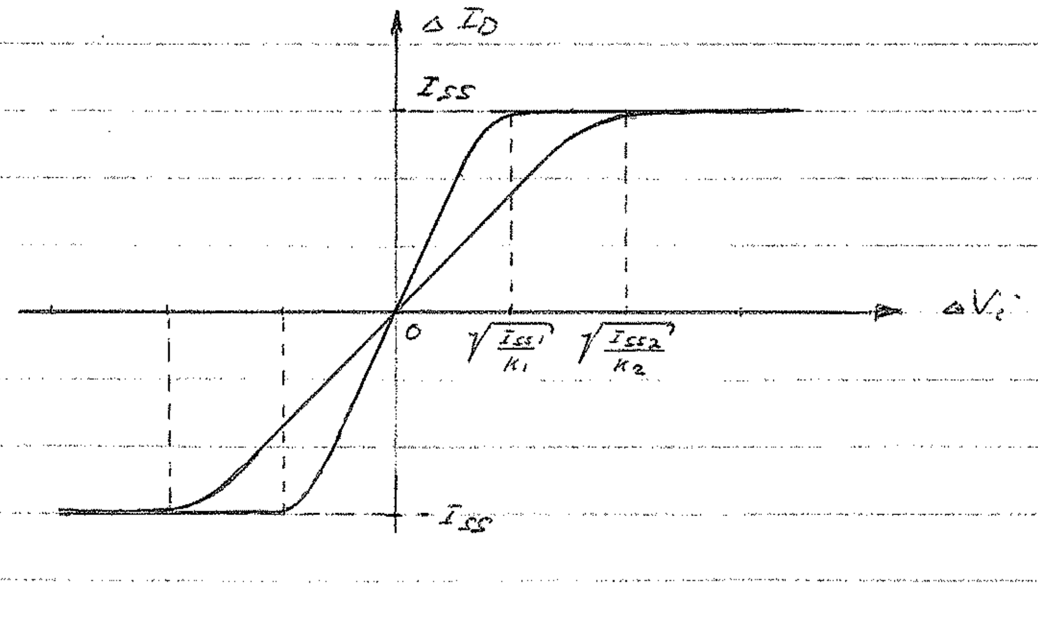
$$\Delta V_i^2 = \frac{1}{k}\left[(I_D + \cancel{\tfrac{1}{2}\Delta I_D}) + (I_D - \cancel{\tfrac{1}{2}\Delta I_D}) - 2\sqrt{I_D^2 - \frac{1}{4}\Delta I_D^2}\right]$$

$$\underset{I_{SS} = 2I_D}{=} \frac{1}{k}\left[I_{SS} - \sqrt{I_{SS}^2 - \Delta I_D^2}\right]$$

$$\Delta I_D^2 = I_{SS}^2 - (\Delta V_i^2 k - I_{SS})^2 = \Delta V_i^2 k^2 \left[\frac{2 I_{SS}}{k} - \Delta V_i^2\right]$$

$$\Rightarrow \underline{\underline{\Delta I_D = \Delta V_i k \sqrt{\frac{2 I_{SS}}{k} - \Delta V_i^2}}} \quad \text{for} \quad \underline{\Delta V_i^2 \leq \frac{I_{SS}}{k}}$$

<u>ΔI_D versus ΔV_i characteristics</u>



b) Small Signal analysis

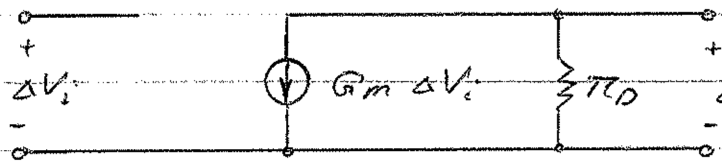
we define : $G_m = \left.\dfrac{d\Delta I_D}{d\Delta V_i}\right|_{\Delta V_i = 0}$

from large signal analysis :

$$\frac{d\Delta I_D}{d\Delta V_i} = \left.\frac{2 I_{SS} - 2k\Delta V_i^2}{\sqrt{\frac{2I_{SS}}{k} - \Delta V_i^2}}\right|_{\Delta V_i = 0} = \frac{2 I_{SS}}{\sqrt{\frac{2I_{SS}}{k}}}$$

$$\Rightarrow \quad \underline{G_m = \sqrt{2kI_{SS}} = \sqrt{\tfrac{W}{L}\mu C_{ox} I_{SS}} = g_{m1} = g_{m2}}$$

linear equivalent circuit (differential mode)



$$\Delta V_o = V_{o1} - V_{o2} = -\tfrac{1}{2}\Delta I_D r_D - (+\tfrac{1}{2}\Delta I_D r_D)$$

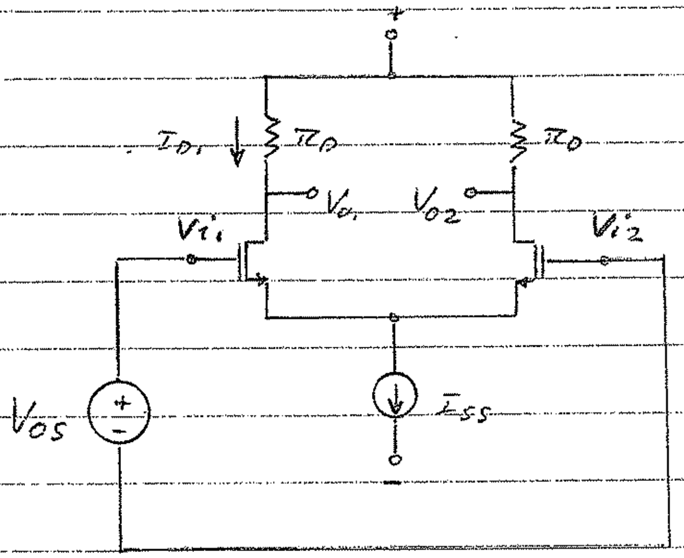
$$\Delta V_o = -\Delta V_i G_m r_D$$

$$\Rightarrow \quad \underline{\underline{\frac{\Delta V_o}{\Delta V_i} = -G_m r_D}} \qquad (G_m = g_{m1} = g_{m2})$$

Note:

$$\underline{\frac{V_{o1}}{\Delta V_i} = -\frac{1}{2} G_m r_D} \qquad \underline{\frac{V_{o2}}{\Delta V_i} = +\frac{1}{2} G_m r_D}$$

c) Offset Voltage



$$I_D = \frac{1}{2}\frac{W}{L}\mu C_{ox}\left[V_{GS} - V_t\right]^2$$

$$\Rightarrow \underline{V_{GS} = V_t + \sqrt{\frac{2 I_D}{\frac{W}{L}\mu C_{ox}}}}$$

Def $$\boxed{V_{OS} = \Delta V_i \Big|_{\Delta V_o = 0} = \Delta V_{GS}\Big|_{\Delta V_o = 0}}$$

$$\Rightarrow V_{OS} = \Delta V_t + \Delta\left\{\sqrt{\frac{2 I_D}{(\frac{W}{L})\mu C_{ox}}}\right\}$$

Random offset due to device mismatch

$$\frac{\Delta I_D}{I_D}\frac{1}{2}\sqrt{\frac{2 I_D}{(\frac{W}{L})\mu C_{ox}}} - \frac{\Delta(\frac{W}{L})}{(\frac{W}{L})}\frac{1}{2}\sqrt{\frac{2 I_D}{(\frac{W}{L})\mu C_{ox}}}$$

$$\Rightarrow \quad V_{OS} = \Delta V_t + \frac{1}{2}\sqrt{\frac{2 I_D}{\frac{W}{L}\mu C_{ox}}}\left[\frac{\Delta I_D}{I_D} - \frac{\Delta\left(\frac{W}{L}\right)}{\left(\frac{W}{L}\right)}\right]$$

$$= \Delta V_t + \frac{1}{2}\left[V_{GS} - V_t\right]\left[\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right]$$

Furthermore, $\frac{\Delta I_D}{I_D} = \frac{\Delta R_D}{R_D}$

$$\Rightarrow \quad \underline{\underline{V_{OS} = \Delta V_t + \frac{1}{2}\left[V_{GS} - V_t\right]\left[\frac{\Delta R_D}{R_D} - \frac{\Delta(W/L)}{(W/L)}\right]}}$$

or

$$\underline{\underline{V_{OS} = \Delta V_t + \frac{1}{2}\underbrace{\sqrt{\frac{I_D}{K}}}_{I_D/g_m}\left[\frac{\Delta R_D}{R_D} - \frac{\Delta(W/L)}{(W/L)}\right]}} \qquad K = \frac{1}{2}\frac{W}{L}\mu C_{ox}$$

<u>Compare:</u> In Bipolar

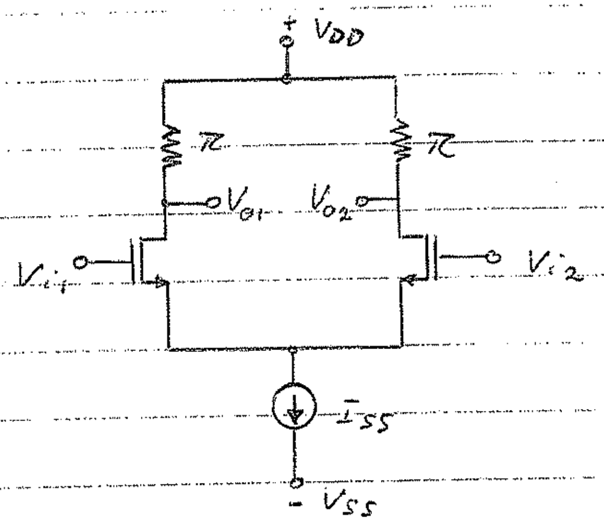
$$\underline{\underline{V_{OS} = \frac{kT}{q}\left[\overset{\text{Base-Q}}{\frac{\Delta Q_B}{Q_B}} - \overset{\text{Area}}{\frac{\Delta A}{A}} - \frac{\Delta R_C}{R_C}\right]}}$$

<u>Comments:</u> - MOS has additional offset component ΔV_t

- MOS offset voltage increases with I_D/g_m

$\Rightarrow$ we want g_m/I_D high

Summary   Source - Coupled pair



small signal analysis

$$\boxed{\frac{\Delta V_o}{\Delta V_i} = -G_m R}$$

$$G_m = \sqrt{\frac{W}{L}\mu C_{ox} I_{SS}}$$

Note: $\dfrac{V_{o1}}{\Delta V_i} = -\dfrac{1}{2} G_m R$ $\dfrac{V_{o2}}{\Delta V_i} = +\dfrac{1}{2} G_m R$

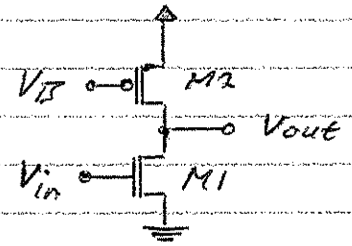
Offset Voltage

Def. $\left\| V_{os} = \Delta V_i \Big|_{\Delta V_o = 0} = \Delta V_{GS} \Big|_{\Delta V_o = 0} \right\|$

$$\boxed{V_{os} = \Delta V_T + \frac{1}{2}\left[V_{GS} - V_T\right]\left[\frac{\Delta R}{R} - \frac{\Delta (W/L)}{(W/L)}\right]}$$

Summary – MOS Gain Stages

1. Common Source Amplifier



$$A_V = -\frac{g_{m1}}{g_{o1}+g_{o2}}$$

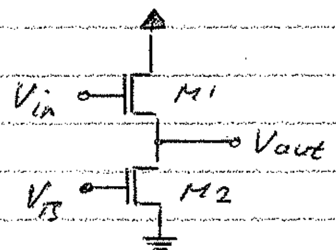
$$r_{in} \rightarrow \infty$$

$$r_{out} = \frac{1}{g_{o1}+g_{o2}}$$

$$g_m = \sqrt{2\frac{W}{L}\mu C_{ox} I_{DQ}}$$

$$g_o = \lambda I_{DQ}$$

2. Common Drain Amplifier (Source Follower)

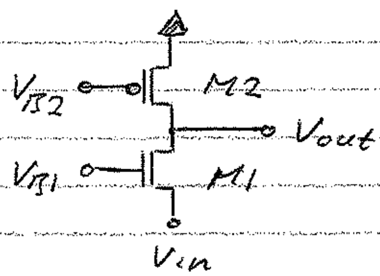


$$A_V = \frac{g_{m1}}{g_{m1}+g_{mb1}+g_{o1}+g_{o2}}$$

$$r_{in} \rightarrow \infty$$

$$r_{out} = \frac{1}{g_{m1}+g_{mb1}+g_{o1}+g_{o2}}$$

3. Common Gate Amplifier



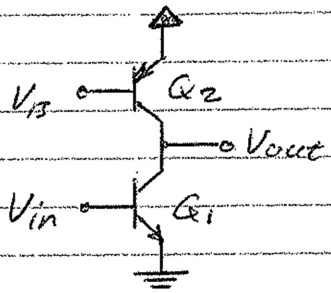
$$A_V = \frac{g_{m1}+g_{o1}}{g_{o1}+g_{o2}}$$

$$r_{in} = \frac{1+g_{o1}r_{o2}}{g_{m1}+g_{o1}}$$

$$r_{out} = \frac{1}{g_{o1}+g_{o2}}$$

Compare – BJT Gain Stages

Common Emitter Amplifier



$$A_V = -\beta \frac{r_{ce_1} \| r_{ce_2}}{r_{be}}$$

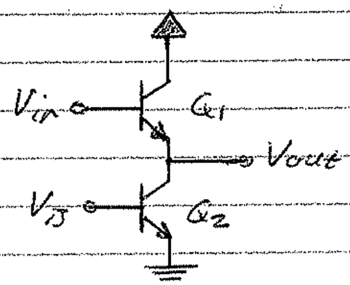
$$r_{in} = r_{be}$$

$$r_{out} = r_{ce_1} \| r_{ce_2}$$

$$r_{be} = \beta \frac{V_T}{I_{CQ}}$$

$$r_{ce} = \frac{V_A}{I_{CQ}}$$

Common Collector Amplifier (Emitter Follower)

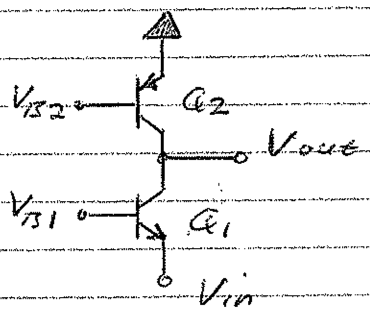


$$A_V = \frac{(1+\beta) r_{ce_1} \| r_{ce_2}}{r_{be_1} + (1+\beta) r_{ce_1} \| r_{ce_2}}$$

$$r_{in} = r_{be_1} + (1+\beta) r_{ce_1} \| r_{ce_2}$$

$$r_{out} = \frac{r_{be}}{(1+\beta) + \frac{r_{be}}{r_{ce_1} \| r_{ce_2}}}$$

Common Base Amplifier



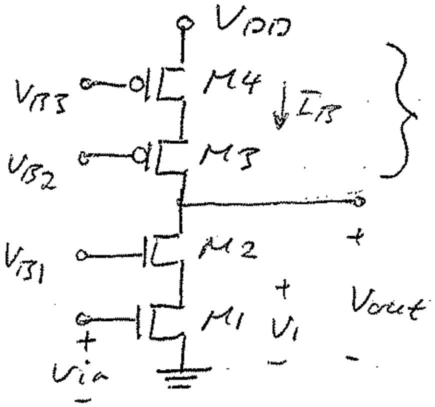
$$A_V = \beta \frac{r_{out}}{r_{be}}$$

$$r_{in} = \frac{1}{\beta} r_{be}$$

$$r_{out} = \beta r_{ce_1} \| r_{ce_2} \cong r_{ce_2}$$

4. Cascode Gain Stage (CS + CG Amp)

Basic Config.



Cascode current source as active load

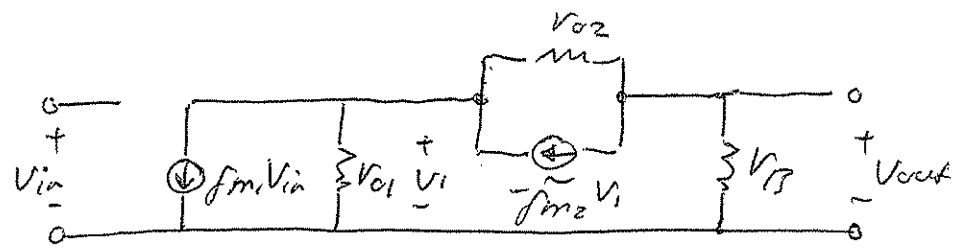
$$\boxed{r_B = \tilde{g}_{m3} r_{o3} r_{o4}}$$

$$\tilde{g}_m = g_m + g_{mb}$$

V_{B3} determines I_B

V_{B1} & V_{B2} input/output swing

AC equivalent Circuit



$$(1)\quad V_{out} = (V_1 - V_{out})\frac{r_B}{r_{o2}} + V_1 \tilde{g}_{m2} r_B$$

$$(2)\quad V_1 = -V_{in} g_{m1} r_{o1} - (V_1 - V_{out})\frac{r_{o1}}{r_{o2}} - \tilde{g}_{m2} r_{o1} V_1$$

from (1)

$$V_1 = V_{out}\frac{\left[1 + \frac{r_B}{r_{o2}}\right]}{\left[\frac{r_B}{r_{o2}} + \tilde{g}_{m2} r_B\right]} \qquad (3)$$

(3) into (2)

$$\therefore (4)\quad V_{out}\frac{\left[1+\frac{r_B}{r_{o2}}\right]\left[1+\frac{r_{o1}}{r_{o2}}+\tilde{g}_{m2} r_{o1}\right] - \frac{r_{o1}}{r_{o2}}\left[\frac{r_B}{r_{o2}}+\tilde{g}_{m2} r_B\right]}{\left[\frac{r_B}{r_{o2}}+\tilde{g}_{m2} r_B\right]} = -V_{in} g_{m1} r_{o1}$$

Rewriting (4) yields

$$\boxed{\frac{V_{out}}{V_{in}} = - \frac{g_{m1} r_{o1} r_{\Sigma}[1 + \tilde{g}_{m2} r_{o2}]}{r_{o2}[1 + \frac{r_{o1}}{r_{o2}} + \tilde{g}_{m2} r_{o1} + \frac{r_{\Sigma}}{r_{o2}}]}}$$

Since $g_m r_o \gg 1$

$$\boxed{\frac{V_{out}}{V_{in}} \cong - \frac{g_{m1} \tilde{g}_{m2} r_{o2} r_{o1} r_{\Sigma}}{\tilde{g}_{m2} r_{o2} r_{o1} + r_{\Sigma}}}$$

<u>Note:</u> $\tilde{g}_{m2} r_{o2} r_{o1}$ is the resistance from the output to ground (through M_1 & M_2) while r_{Σ} represents the resistance from the output to V_{DD} (through M_3 & M_4)

Thus $\boxed{r_{out} \cong \tilde{g}_{m2} r_{o2} r_{o1} \,//\, \tilde{g}_{m3} r_{o3} r_{o4}}$ large value!

<u>Numerical Example</u>

$I_B = 50\mu A \qquad \mu_n C_{ox}\left(\frac{W}{L}\right)_n = \mu_p C_{ox}\left(\frac{W}{L}\right)_p = 1 mA/V^2$

$\therefore g_{mn} = g_{mp} = 320 \mu S \qquad \tilde{g}_m \approx 400 \mu S$

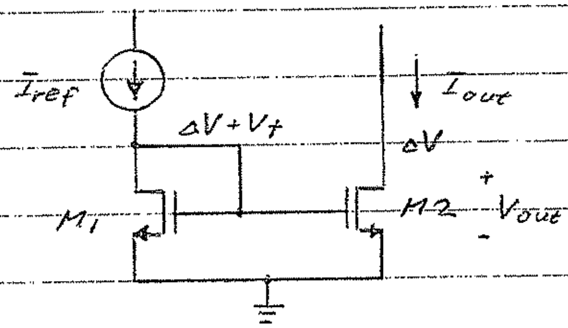
$r_{o1} \cong r_{o2} \cong r_{o3} \cong r_{o4} = 400 k\Omega$

$\therefore g_m r_o = 160 \qquad \tilde{g}_{m2} r_{o2} r_{o1} \cong \tilde{g}_{m3} r_{o3} r_{o4} = 64 M\Omega$

$\therefore \boxed{\frac{V_{out}}{V_{in}} \cong -10,000} \quad \boxed{r_{out} \cong 32 M\Omega}$

5. Current Sources

a) Simple Current Mirror



Let $\Delta V = V_{GS} - V_t \approx \sqrt{\frac{2 I_{ref}}{(W/L)_1 \mu C_{ox}}}$

For saturation: $V_{out} \geq \Delta V$

$(\Delta V \geq 0)$

Large Signal response

$$V_{GS_1} = V_{GS_2} \cong V_t + \sqrt{\frac{2 I_{ref}}{(W/L)_1 \mu C_{ox}}}$$

$$I_{out} = \frac{1}{2}\left(\frac{W}{L}\right)_2 \mu C_{ox} [V_{GS_2} - V_t]^2$$

$$= \frac{1}{2}\left(\frac{W}{L}\right)_2 \mu C_{ox} \frac{2 I_{ref}}{(W/L)_1 \mu C_{ox}} = \frac{(W/L)_2}{(W/L)_1} I_{ref}$$

Note: If you want to scale the current ratio, do it by means of $\frac{W_2}{W_1}$ (not $\frac{L_1}{L_2}$) (equal undercut yields constant ratio $\frac{L_1}{L_2}$ only if $L_1 = L_2$.)

$$I_{ref} = k_1 [V_{GS} - V_t]^2 (1 + \lambda V_t)$$

$$I_{out} = k_2 [V_{GS} - V_t]^2 (1 + \lambda [V_{out} - V_{GS} + V_t])$$

including channel length modulation

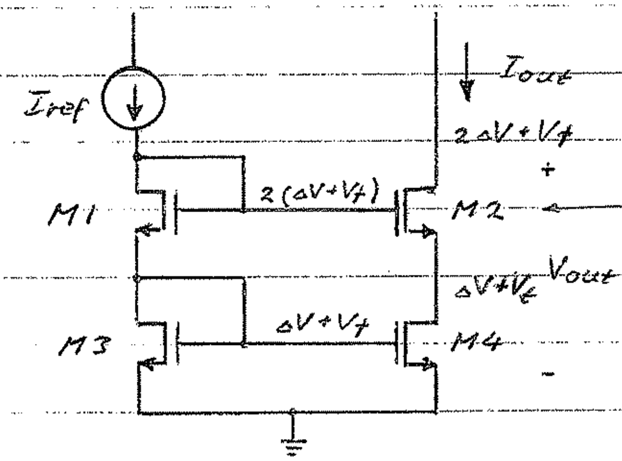
$$\therefore \quad I_{out} = I_{ref} \frac{k_2}{k_1} \cdot \frac{1 + \lambda [V_{out} - V_{GS} + V_t]}{1 + \lambda V_t}$$

$$\frac{k_1}{k_2} = \frac{(W/L)_1}{(W/L)_2}$$

Output resistance

$$r_o = r_{o2} = \frac{1}{\lambda_2 I_{out}} = \frac{L_{eff2}}{I_{out}}\sqrt{\frac{2qN_{sub}(\phi_0 + V_{DS} - V_{eff})}{\varepsilon_S}}$$

b) Cascode current source



Common-Gate Stage

Let $\Delta V = V_{GS} - V_t$ at $I_D = I_{ref}$

$(\Delta V \geq 0)$

$V_{out} = V_{out_2} + V_{out_4}$

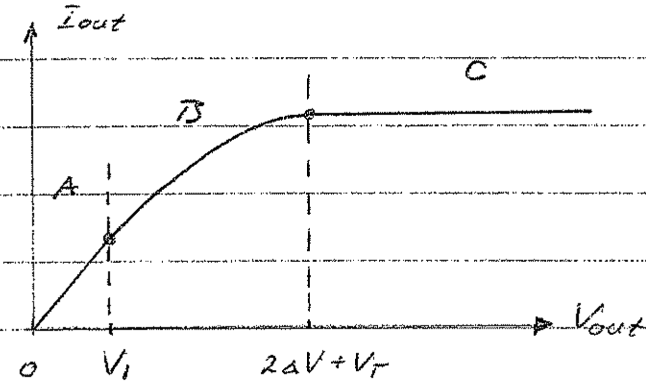
For saturation: $V_{out_4} \geq \Delta V$

$V_{out_2} \geq 2(\Delta V + V_t) - \Delta V - V_t = \underline{\Delta V + V_t}$

$$\Rightarrow V_{out} \geq 2\Delta V + V_t$$

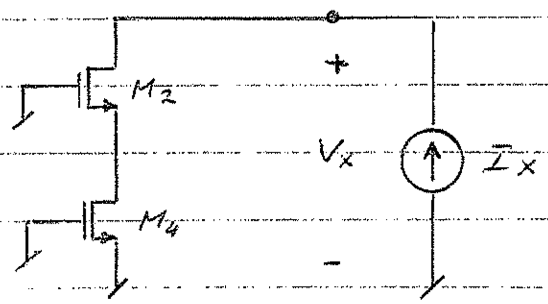
Compared to the simple current mirror, the voltage swing of the cascode current source is reduced by $\underline{V_t + \Delta V}$

output characteristics



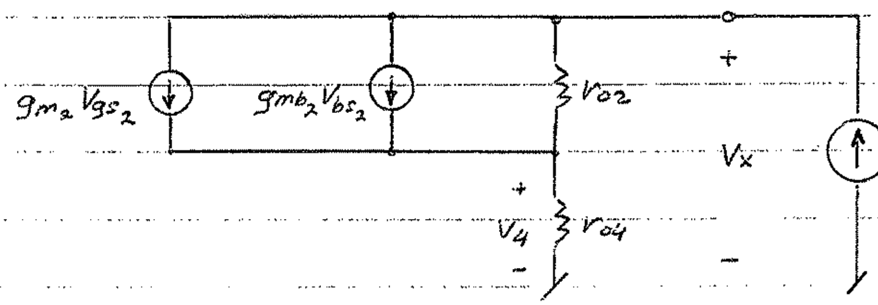
A: $M_2 + M_4$ Triode region	$V_{out} \leq V_1 = \Delta V\left[1 + \frac{\Delta V}{2(V_T + \Delta V)}\right]$
B: M_4 sat., M_2 Triode	$V_1 < V_{out} < V_T + 2\Delta V$
C: $M_2 + M_4$ sat.	$V_{out} \geq V_T + 2\Delta V$

output resistance



$$r_{out} = \frac{V_x}{I_x}$$

linear equivalent circuit (for output resistance)



$$V_{gs_2} = V_{bs_2} = -V_4 \qquad (1)$$

$$V_4 = i_x r_{o4} \qquad (2)$$

$$V_x = (i_x - g_{m2} V_{gs2} - g_{mb2} V_{bs2})\, r_{o2} + V_4 \qquad (3)$$

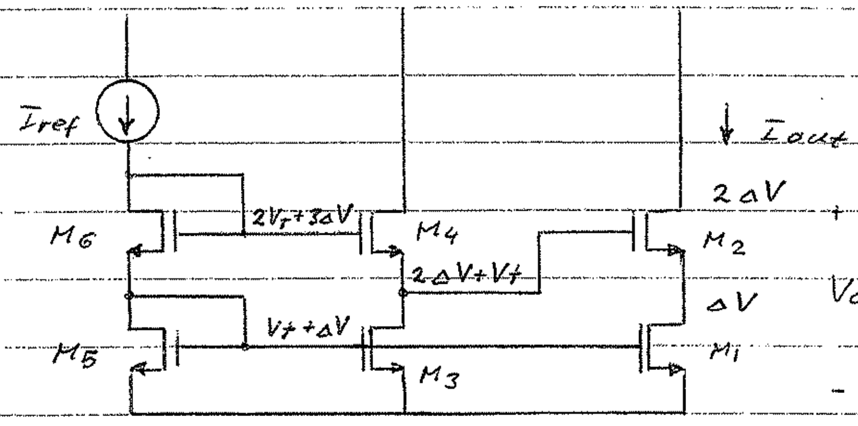
(1) + (2) in (3) $\Rightarrow V_x = i_x (1 + g_{m2} r_{o4} + g_{mb2} r_{o4})\, r_{o2} + i_x r_{o4}$

$$\Rightarrow r_{out} = \frac{V_x}{i_x} = r_{o2} + r_{o4}(1 + \underbrace{g_{m2} r_{o2} + g_{mb2} r_{o2}}_{= A_2 \gg 1})$$

$$r_{out} \cong r_{o4}(1 + A_2)$$

The source impedance of the cascode configuration is increased by the voltage gain of the additional common-gate stage (i.e. A_2)

c) Improved Cascode current source (Better dynamic range)



$$I_{ref} = I_6 = k'\left(\frac{W}{L}\right)_6 [V_{GS6} - V_T]^2 \qquad V_{GS6} = V_T + 2\Delta V$$

$$I_{ref} = I_5 = k'\left(\frac{W}{L}\right)_5 [V_{GS5} - V_T]^2 \qquad V_{GS5} = V_T + \Delta V$$

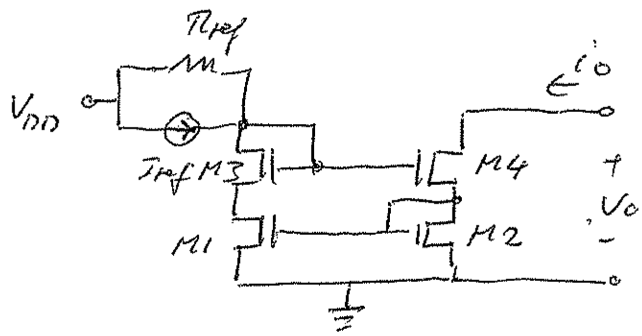
$$\Rightarrow \left(\frac{W}{L}\right)_6 4\Delta V^2 = \left(\frac{W}{L}\right)_5 \Delta V^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_6 = \frac{1}{4}\left(\frac{W}{L}\right)_5 \qquad \text{But } \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_3$$

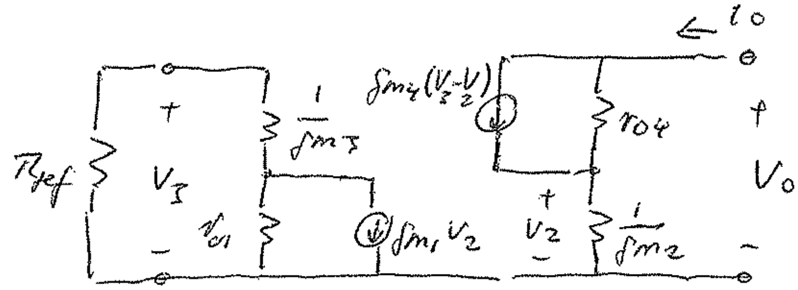
$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1$$

The voltage swing of the improved cascode current source is improved by V_T by inserting a voltage-level-shifting stage in series between the gate of M_6 and M_2.

Wilson Current Mirror



ac equivalent circuit



$$(1)\quad V_3 = -\frac{V_3}{R_{ref}}\left[\frac{1}{g_{m3}} + r_{o1}\right] - g_{m1} r_{o1} V_2$$

$$(2)\quad V_2 = i_o \frac{1}{g_{m2}}$$

$$(3)\quad V_o = i_o\left[r_{o4} + \frac{1}{g_{m2}}\right] - g_{m4} r_{o4} (V_3 - V_2)$$

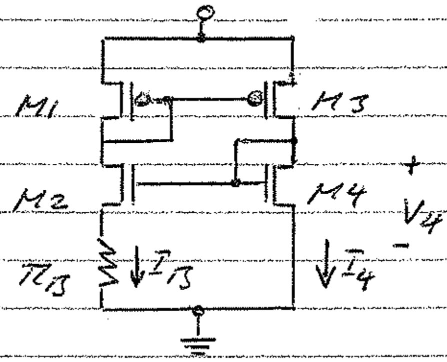
$$(1)+(2) \;\therefore\; V_3\left[1 + \frac{1}{g_{m3} R_{ref}} + \frac{r_{o1}}{R_{ref}}\right] = -\frac{g_{m1} r_{o1}}{g_{m2}} i_o \qquad (4)$$

$$(3)+(2)+(4) \;\therefore\; V_o = i_o\left[r_{o4} + \cancel{\frac{1}{g_{m2}}} + \frac{g_{m4}}{g_{m2}} r_{o4} + g_{m4} r_{o4} \frac{g_{m1} r_{o1}}{g_{m2}} \frac{R_{ref}}{\left[R_{ref} + \cancel{\frac{1}{g_{m3}}} + r_{o1}\right]}\right]$$

$$\therefore\; r_o = \frac{V_o}{i_o} \cong r_{o4}\left[1 + \frac{g_{m4}}{g_{m2}}\right] + g_{m4} r_{o4} \frac{g_{m1}}{g_{m2}} R_{ref} \| r_{o1}$$

Supply-Independent Current Source

Basic Configuration



assume $M1 \equiv M3$

<u>Note:</u> This circuit may require a start-up procedure if $V_4(t=0) < V_t$

Since $M1 \equiv M3$ $\therefore$ $I_4 = I_B$

If we neglect the body effect in $M2$, we can write the following device equation

$$\left| \beta_2 \cdot [V_4 - I_B R_B - V_t]^2 = \beta_4 [V_4 - V_t]^2 \right| \qquad \beta = \frac{1}{2}\mu C_{ox}\frac{W}{L}$$

$$V_4 - I_B R_B - V_t = \sqrt{\frac{\beta_4}{\beta_2}}\,[V_4 - V_t] \qquad \beta_4 < \beta_2$$

$$\therefore \left\| I_B = \frac{1}{R_B}[V_4 - V_t]\left[1 - \sqrt{\frac{\beta_4}{\beta_2}}\right] \right\|$$

$$\text{If } L_2 = L_4 \quad \therefore \sqrt{\frac{\beta_4}{\beta_2}} \cong \sqrt{\frac{W_4}{W_2}} \;\Big|$$

<u>Note:</u> If body effect is included, the current I_B will be reduced by $\left| \frac{\Delta V_{t2}}{R_B} = \frac{\gamma}{R_B}\left[\sqrt{2\phi_F + I_B R_B} - \sqrt{2\phi_F}\right] \right|$

iterative solution required