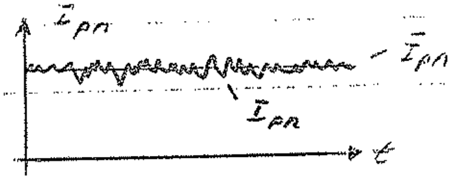
VIII Noise in Integrated Circuits

1. Sources of Noise

A) Shot Noise

1. Causes: Carriers must jump potential barrier between a p-n junction (Random process). Hence, the effective current flowing through the junction fluctuates around its average value in a random way.



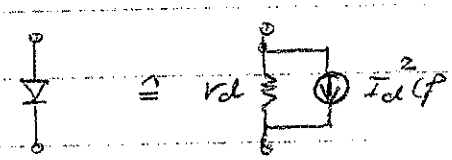
2. Spectral density:

$$\overline{i^2} = \lim \frac{1}{T}\int_0^T (I_{pn}-\bar{I}_{pn})^2 dt$$

$$= 2q\bar{I}_{pn}\Delta f \qquad f < \frac{1}{\tau}$$

q: electron charge
τ: Transit time of carriers
$\bar{I}_{pn}$: Average current
Δf: Bandwidth in Hz

e.g. Diode:



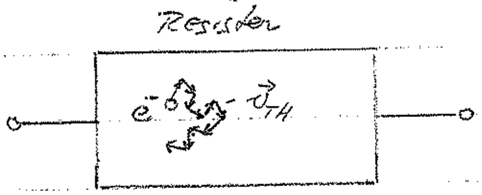
$r_d \cong \frac{kT}{qI_D}$ $\quad \overline{I_d^2}(f) \cong 2qI_D$

⇒ Frequency distribution: White
Amplitude variation: Gaussian ($\overline{i^2} = \sigma^2$)
i.e. noise signal is $< \pm 3\sigma$ in 99.7% of time

B) Thermal Noise

1. Causes: Random Thermal Motion of Electrons in any conductor

Note: Since $v_{diff} \ll v_{TH}$ Thermal noise is independent of current but proportional to T



$\bar{v}_{th} = \sqrt{\frac{3kT}{m_e^*}}$

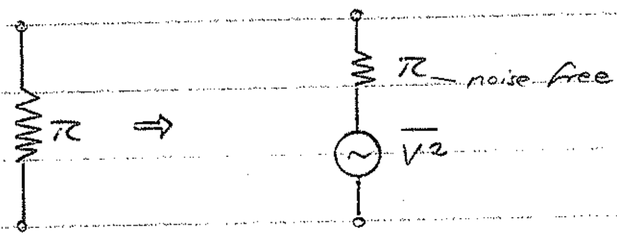
effective mass of e^- in crystal lattice

e.g. $T = 300°K$

$v_{th} \cong 10^5 \frac{m}{s}$

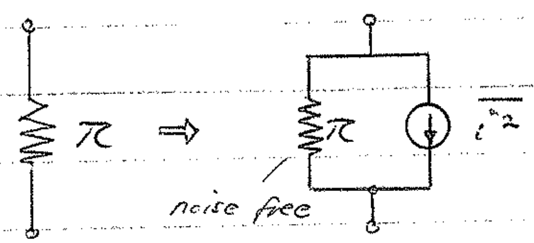
2. Spectral Density

represented by a voltage source



$$\underline{\overline{V^2} = 4kTR\Delta f}$$

represented by a current source



$$\underline{\overline{i^2} = \frac{4kT}{R}\Delta f}$$

⇒ Frequency distribution: White

Amplitude variation: Gaussian ($\overline{i^2} = \sigma^2$ / $\overline{V^2} = \sigma^2$)

C) Flicker Noise (1/F Noise)

1. Cause: Traps in a material capture and emit carriers in a Random Fashion

2. Spectral Density:

Empirical $$\overline{i^2} = K_1 \frac{I^a}{f^b} \Delta f$$

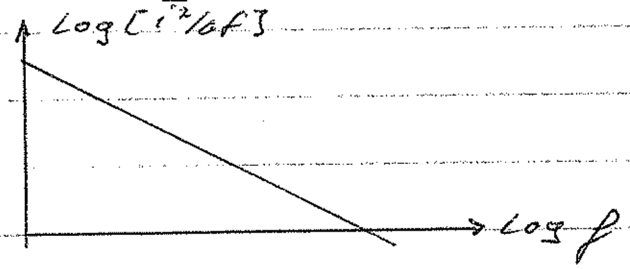
I: direct current
K_1: Constant
$a \approx 0.5$ to 2
$b \approx 1$

MOSFET

$$V_{i/p}^2 \approx \frac{k_v}{WLC_{ox}} \frac{1}{f} \Delta f$$

The time constants associated with the capture and release of charge carriers give rise to a noise signal with energy concentrated at low frequencies.

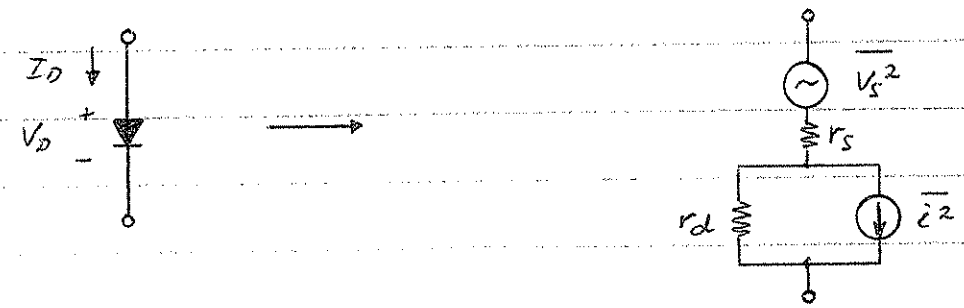
Frequency distribution



Amplitude variation: Often Not Gaussian

2. Noise Models for IC Components

A) Junction Diode



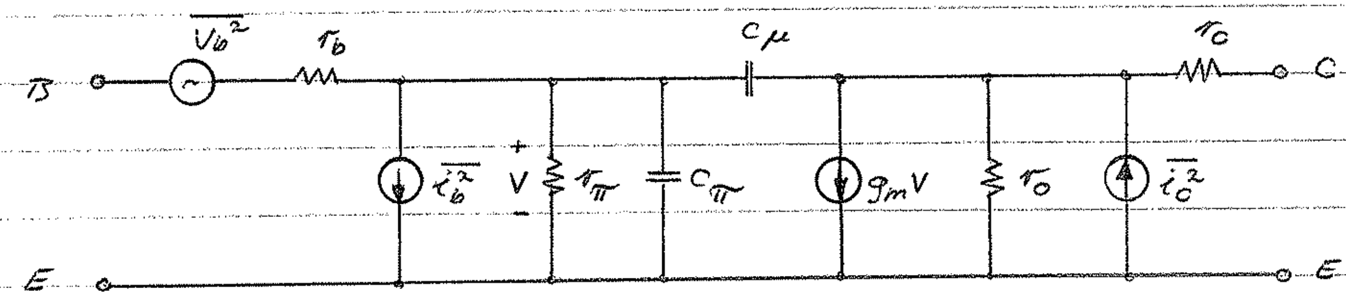
1. $r_d = \left[\frac{dI_D}{dV_D} \right]^{-1} = \frac{kT}{qI_D}$ Modelling resistor (produces no noise)

2. r_s physical series resistor (produces noise)

3. $\overline{v_s^2} = 4kTr_s\Delta f$: Thermal noise due to r_s

4. $\overline{i^2} = \underbrace{2qI_D\Delta f}_{\text{shot}} + \underbrace{K_1 \frac{I_D^{a}}{f}\Delta f}_{\text{fliche}}$ Shot noise and Flicker noise of pn junction and semicond. material

B) Bipolar Transistor



1. $\overline{V_b^2} = 4kT r_b \Delta f$ due to base series resistor r_b (Thermal)

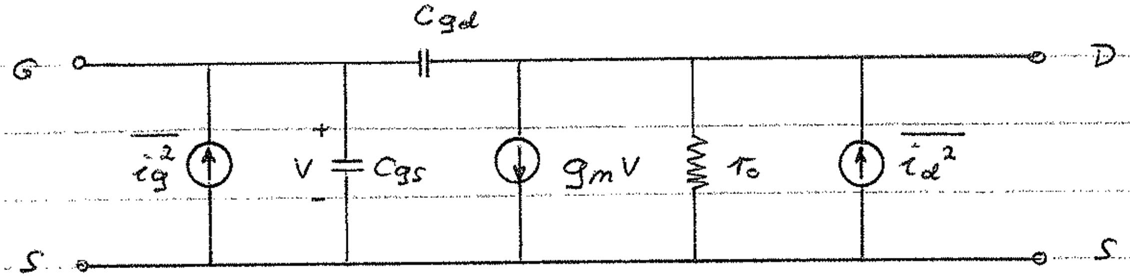
2. $\overline{i_b^2} = 2q I_B \Delta f + K_1 \dfrac{I_B^{\alpha}}{f} \Delta f$ Holes jumping into Emitter (Shot)
   Flicker Recombination Current in Base (Shot)

3. $\overline{i_c^2} = 2q I_C \Delta f$ El. jumping from Emitter (Shot)
   Recomb. Current in Base (Shot)

4. Neglect Noise source associated with r_c (small)

Note: If Recombination Current in Base is small, then all Noise Sources are statistically independent.

c) Field Effect Transistor



1. $\overline{i_g^2} = 2qI_G \Delta f$ Shot Noise due to reverse leakage in JFET

Note: $\overline{i_g^2} = 0$ for MOSFET

2. $$\overline{i_d^2} = \underbrace{4kT\left(\frac{1}{r_{CH}}\right)\Delta f}_{\text{Thermal Noise}} + \underbrace{K_1 \frac{I_D^{\alpha}}{f}\Delta f}_{\text{Flicker Noise}}$$

<u>channel resistance</u> r_{CH}:

<u>Recall:</u> $Q_T = \frac{2}{3} C_{ox} W \cdot L \, (V_{GS} - V_T)$ Total charge

$1/r_S = \underbrace{\mu q \int_0^{x_j} n \, dx}_{Q_T \frac{1}{WL}}$ sheet resistance

$$\Rightarrow 1/r_S = \frac{2}{3}\mu C_{ox}(V_{GS} - V_T)$$

$$r_{CH} = \frac{L}{W} r_S = \frac{L}{\frac{2}{3}\mu C_{ox} W (V_{GS} - V_T)}$$

since $g_m = \mu C_{ox} \frac{W}{L}(V_{GS} - V_T)$

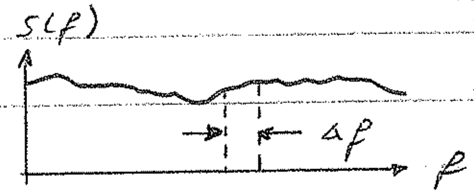
$$\Rightarrow \underline{\underline{r_{CH} = \frac{1}{\frac{2}{3} g_m}}} \qquad \Rightarrow \underline{\underline{\overline{i_d^2} = 4kT \frac{2}{3} g_m \Delta f + K_1 \frac{I_D^{\alpha}}{f}\Delta f}}$$

MOSFET $\left| \, \overline{i_d^2} = 4kT\frac{2}{3}g_m + \frac{K g_m^2}{WLC_{ox} f} \right|$

3. Circuit Noise Calculations

Given: Noise current source with spectral density

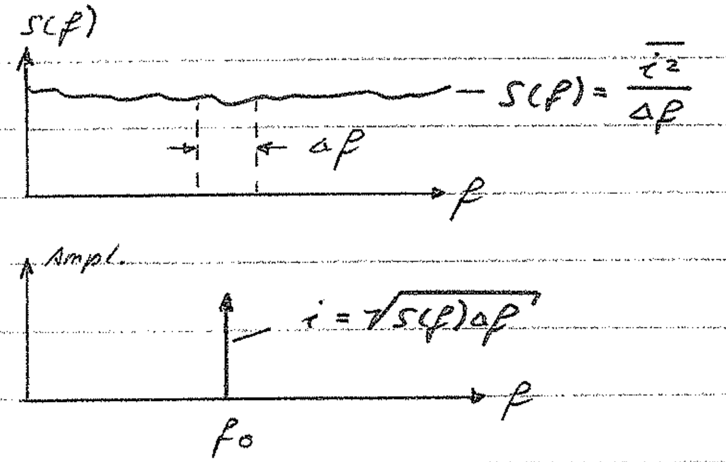
$$S(f) = \frac{\overline{i^2}}{\Delta f}$$

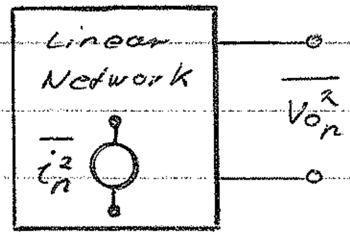


In a small bandwidth Δf, the mean-square value of the noise current is given by

$$i = \sqrt{S(f)\,\Delta f}$$

Thus, the noise current in bandwidth Δf can be represented approximately by a sinusoidal current generator with rms value i as shown above.
If the noise current in bandwidth Δf is now applied as an input signal to a circuit, its effect can be calculated by substituting the sinusoidal generator and performing circuit analysis in the usual fashion.
Thus network noise calculations reduce to familiar sinusoidal circuit analysis calculations.

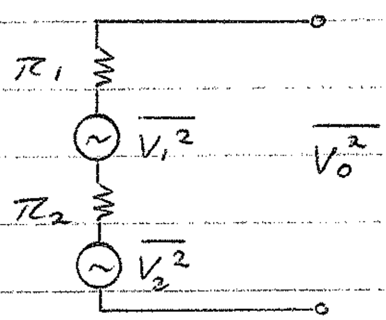




Procedure:

1. Replace $\overline{i_n^2}$ ($\overline{V_n^2}$) with sinusoidal source of magnitude i_n (V_n) and frequency f_0
2. Calculate $\overline{Vo_n(f_0)}$ due to $\overline{i_n(f_0)}$
3. $\overline{Vo_n^2}$ is output Power Spectrum Density at f_0
4. If more than 1 source and sources independent then add sum of squares $\overline{V_o^2} = \sum_n \overline{Vo_n^2}$

Example



Find $\overline{V_o^2}$

1. $\overline{V_1^2} = 4kTR_1 \Delta f$

$\overline{V_2^2} = 4kTR_2 \Delta f$

2. $V_o = V_1 + V_2 \quad \Rightarrow \quad \overline{V_o^2} = \overline{[V_1 + V_2]^2}$

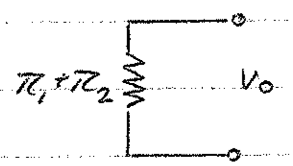
$= \overline{V_1^2} + \overline{V_2^2} + 2\,\overline{V_1 V_2}$

since V_1 is statistically independent of V_2

$\overline{V_1 V_2} = 0$ (orthogonality)

3. $\underline{\underline{\overline{V_o^2} = \overline{V_1^2} + \overline{V_2^2} = 4kT(R_1 + R_2)\Delta f}}$

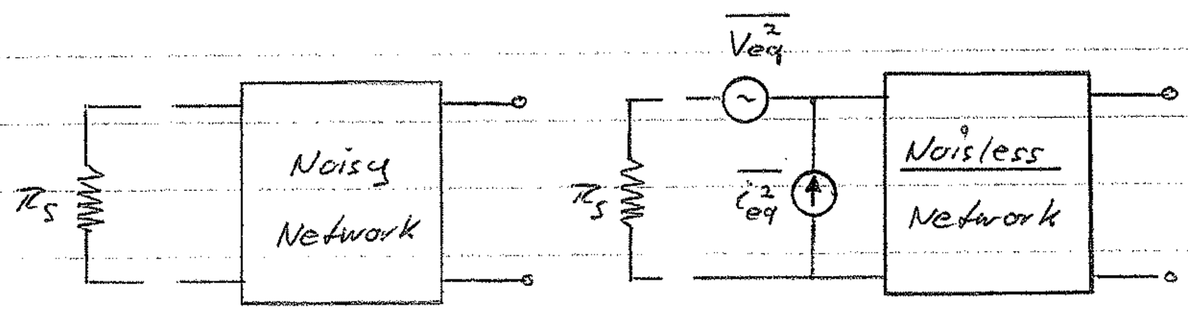
$\Rightarrow$ equivalent network:



$\Rightarrow V_o^2 = 4kT(R_1 + R_2)\Delta f$

4. <u>Equivalent Input Noise Generators</u>

<u>Idea:</u> Replace noisy network by a noisless network with equivalent input noise sources



1. If $Z_s = 0$ $\overline{i_{eq}^2}$ is shorted
$\overline{V_{eq}^2}$ is single noise source

2. If $Z_s \to \infty$ $\overline{V_{eq}^2}$ is in open-loop
$\overline{i_{eq}^2}$ is single noise source

3. Finite Z_s Both i_{eq} and V_{eq} contribute to output noise

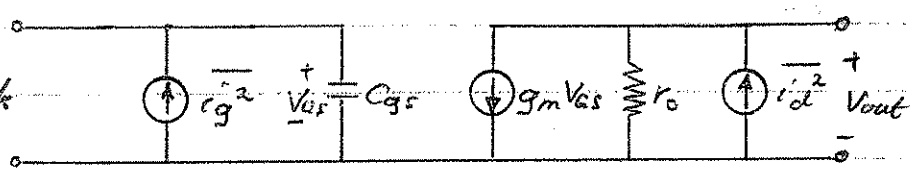
How to find $\overline{V_{eq}^2}$ and $\overline{i_{eq}^2}$?

a) Short input of both networks and equate output noise of both cases
Solve for $\overline{V_{eq}^2}$

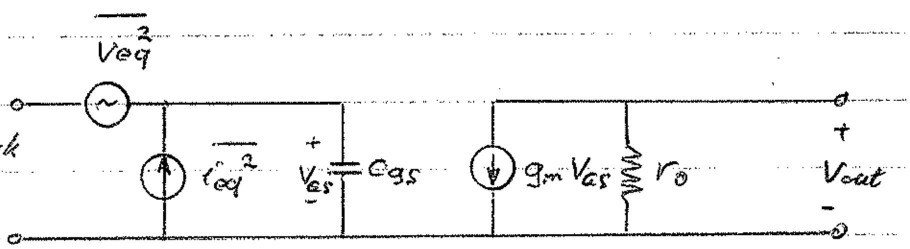
b) Open input of both networks and equate output noise of both cases
Solve for $\overline{i_{eq}^2}$

Example (1) JFET

noisy network



noiseless network with eq. input noise sources



A) Short inputs to find $\overline{v_{eq}^2}$

noisy NW　　　　　　noiseless NW

$|v_{out} = i_d r_o|$　　　　　$|v_{out} = -g_m v_{eq} r_o|$

$$\therefore |i_d = -g_m v_{eq}|$$

$$\left\| \overline{v_{eq}^2} = \frac{\overline{i_d^2}}{g_m^2} = 4kT\frac{2}{3}\frac{1}{g_m}\Delta f + K_1 \frac{I_D^a}{g_m^2 f}\Delta f \right\|$$

B) Open inputs to find $\overline{i_{eq}^2}$

noisy NW　　　　　　noiseless NW

$|v_{out} = (i_d - g_m \frac{i_g}{j\omega C_{gs}}) r_o|$　　$|v_{out} = -g_m \frac{i_{eq}}{j\omega C_{gs}} r_o|$

$$\therefore \left| i_d - g_m \frac{i_g}{j\omega C_{gs}} = -g_m \frac{i_{eq}}{j\omega C_{gs}} \right|$$

$$\left| i_{eq} = i_g - \frac{j\omega C_{gs}}{g_m} i_d \right|$$

since i_g and i_d are statistically independent

$$\overline{i_{eq}^2} = \overline{i_g^2} + \frac{\omega^2 C_{gs}^2}{g_m^2}\overline{i_d^2}$$

or

$$\left\| \overline{i_{eq}^2} = 2qI_G\Delta f + \omega^2 C_{gs}^2 \overline{v_{eq}^2}\Delta f \right\|$$

MOSFET　$I_G \cong 0$

$$\overline{i_{eq}^2}_{MOS} \cong \omega^2 C_{gs} \overline{v_{eq}^2}\Delta f$$

small at low to moderate frequencies

Comments:

1. V_{eq} is much higher than bipolar because g_m is lower for FETs
2. i_{eq} is much lower than bipolar at low to moderate frequencies ($\omega C_{gs} << 1$) since $I_G << I_B$

Note: MOSFET $I_G = 0$ ⇒ $i_{eq} \approx 0$ at low to mod. freq.

⇒ Use FET when Input has high Source Impedance

Compare: equivalent noise sources of BJT

$$\boxed{\begin{aligned}\overline{V_{eq}^2} &= 4kT r_b \Delta f + 2kT \frac{1}{g_m} \Delta f \\ \overline{i_{eq}^2} &= 2qI_B \Delta f + K_1 \frac{I_B^a}{f} \Delta f + 2qI_C \frac{\Delta f}{|\beta(j\omega)|^2}\end{aligned}}$$

where $\beta(j\omega) = \dfrac{g_m r_\pi}{(1 + j\omega r_\pi C_\pi)}$ and $g_m = I_C \dfrac{q}{kT}$

Note: at low frequencies:

$$\overline{i_{eq}^2} \approx \overline{i_b^2}$$

$$\overline{V_{eq}^2} \approx \overline{V_b^2} + \frac{\overline{i_c^2}}{g_m^2}$$

independent sources

at high frequencies:

$$\overline{i_{eq}^2} \approx \overline{i_b^2} + \boxed{\frac{\overline{i_c^2}}{|\beta(j\omega)|^2}}$$

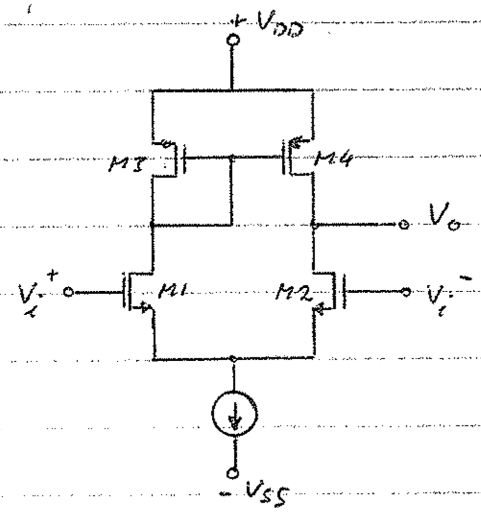
$$\overline{V_{eq}^2} \approx \overline{V_b^2} + \boxed{\frac{\overline{i_c^2}}{g_m^2}}$$

dependent sources

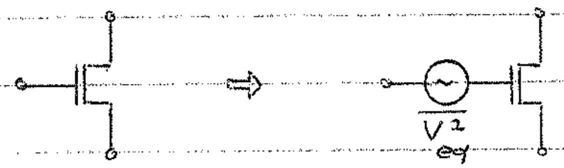
$$\overline{i_b^2} = 2qI_B \Delta f + K_1 \frac{I_B^a}{f} \Delta f$$

$$\overline{i_c^2} = 2qI_C \Delta f = 2kT g_m \Delta f$$

Example ② Differential Input Stage of CMOS Opamp



1. Replace circuit elements by noiseless elements and their equivalent noise sources.



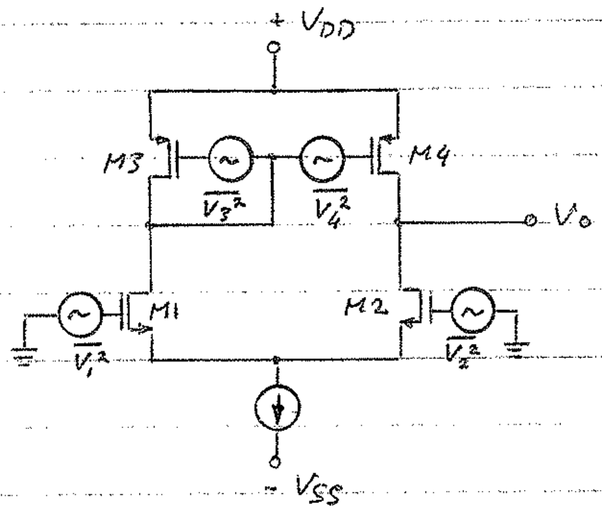
MOS

$$\frac{K}{WLC_{ox}f}$$

$$\overline{V_{eq}^2} = \left(\frac{8}{3}\frac{kT}{g_m} + K_1\frac{I_D^a}{f g_m^2}\right)\Delta f$$

$$\overline{i_{eq}^2} = \omega^2 C_{gs}^2 \overline{V_{eq}^2} \cong 0$$

2. Apply a small signal analysis to the circuit with the equivalent noise models and solve for the output noise power.



<u>output noise power:</u> (assume $\overline{V_1^2} = \overline{V_2^2}$ and $\overline{V_3^2} = \overline{V_4^2}$)

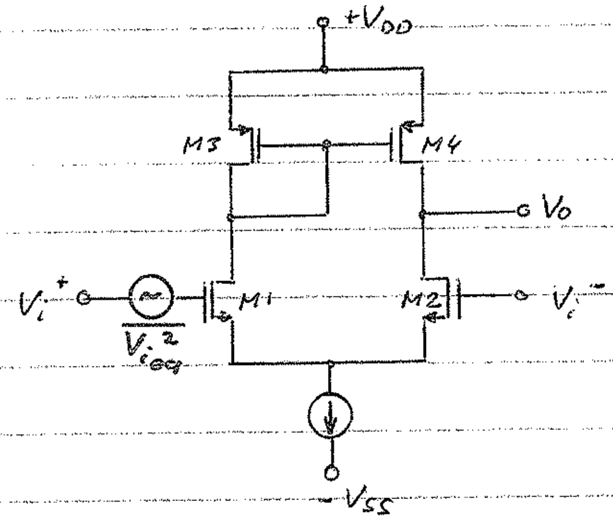
$$\overline{V_o^2} = \left(\frac{g_{m1}}{g_{o1}+g_{o3}}\right)^2 \left(2\overline{V_1^2} + 2\left(\frac{g_{m3}}{g_{m1}}\right)^2 \overline{V_3^2}\right)$$

<u>Note:</u> $\left(\frac{g_{m1}}{g_{o1}+g_{o3}}\right)$ = Gain of differential stage

3. Calculate equivalent input noise source that produces the same output noise power

$$\overline{V_{ieq}^2} = \frac{\overline{V_o^2}}{A_d^2} \qquad \text{where } A_d = \left(\frac{g_{m1}}{g_{o1}+g_{o3}}\right)$$

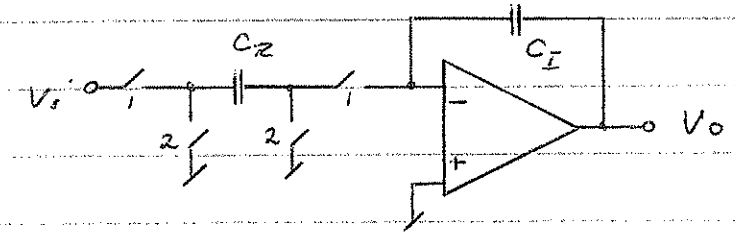
$$\Rightarrow \boxed{\overline{V_{ieq}^2} = 2\overline{V_1^2} + 2\left(\frac{g_{m3}}{g_{m1}}\right)^2 \overline{V_3^2}}$$



<u>Note:</u> $g_{m3} << g_{m1}$ (for high gain-bandwidth product)
Thus noise contribution of <u>M1 and M2 is dominant</u>

5. Sampled Noise (J. Fischer: Journ. SSC, pp. 742-752, Aug. 1982)

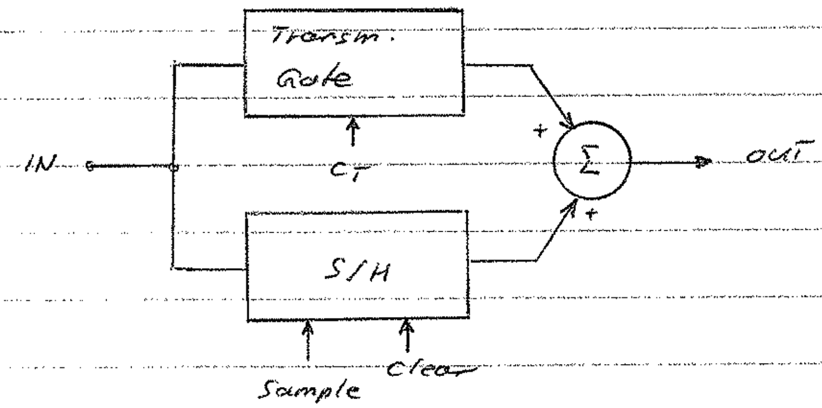
Example inverting SC integrator

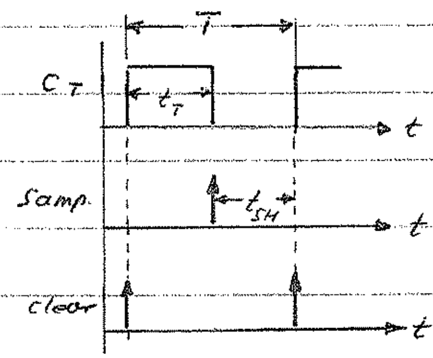


What is noise response of this system?

This circuit performs a track and hold function (tracking during phase 1 and holding during phase 2)

Track and hold equivalent circuit





We assume input noise is white with equivalent band-width BW_n and power density η_n

note: equivalent noise bandwidth

$$\int_{-\infty}^{\infty} S_i(f)\,df = \eta_{n_i} \cdot BW_{n_i}$$

a) Sample-and-Hold Noise Model

If we assume an ideal S/H operation with holding time $T_H = \tau_{SH} \cdot T = \frac{\tau_{SH}}{f_S}$ ($\tau_{SH} = \frac{t_{SH}}{T}$), we obtain the following output power density

$$\eta_o(f) \leq \eta_n \, \tau_{SH}^2 \, \mathrm{sinc}^2\left(\frac{\tau_{SH} f}{f_S}\right) \qquad BW_n \leq \frac{1}{2} f_S$$

$$\eta_o(f) \leq \eta_n \, 2 \frac{BW_n}{f_S} \tau_{SH}^2 \, \mathrm{sinc}^2\left(\frac{\tau_{SH} f}{f_S}\right) \qquad BW_n \geq \frac{1}{2} f_S$$

Foldover effect due to undersampling of input noise

b) Track-and-Hold Noise Model

Since the Track-and-Hold operation occupies the entire sampling period, we can denote the tracking time $T_T = \tau_T \cdot T$ by $(1 - \tau_{SH}) T$. The output power density is then equal to

$$\eta_o(f) \leq \eta_n \Big[\underbrace{\tau_{SH}^2 \, \mathrm{sinc}^2\left(\frac{\tau_{SH} f}{f_S}\right)}_{S/H} + \underbrace{(1 - \tau_{SH})}_{Track} \Big] \qquad BW_n < \frac{1}{2} f_S$$

$$\eta_o(f) \leq \eta_n \Big[\underbrace{2 \frac{BW_n}{f_S} \tau_{SH}^2 \, \mathrm{sinc}^2\left(\frac{\tau_{SH} f}{f_S}\right)}_{S/H} + \underbrace{(1 - \tau_{SH})}_{Track} \Big] \qquad BW_n \geq \frac{1}{2} f_S$$

Examples: a) $\tau_{SH} = 0$ (fully tracking → no S/H term)

$$\underline{\eta_o(f) = \eta_n} \quad \forall f$$

b) $\tau_{SH} = 1$ (ideal S/H → no tracking term)

$$\eta_o(f) \leq \eta_n \,\text{sinc}^2\left(\frac{f}{f_S}\right) \qquad \beta W_n < \tfrac{1}{2} f_S$$

$$\eta_o(f) \leq \eta_n \, 2\,\frac{\beta W_n}{f_S}\,\text{sinc}^2\left(\frac{f}{f_S}\right) \qquad \beta W_n \geq \tfrac{1}{2} f_S$$

Output noise spectrum exhibits familiar sin x / x envelope

c) $\tau_{SH} = \frac{1}{2}$

$$\eta_o(f) \leq \eta_n \,\frac{1}{2}\left[\frac{1}{2}\,\text{sinc}^2\left(\frac{f}{2f_S}\right) + 1\right] \qquad \beta W_n < \tfrac{1}{2} f_S$$

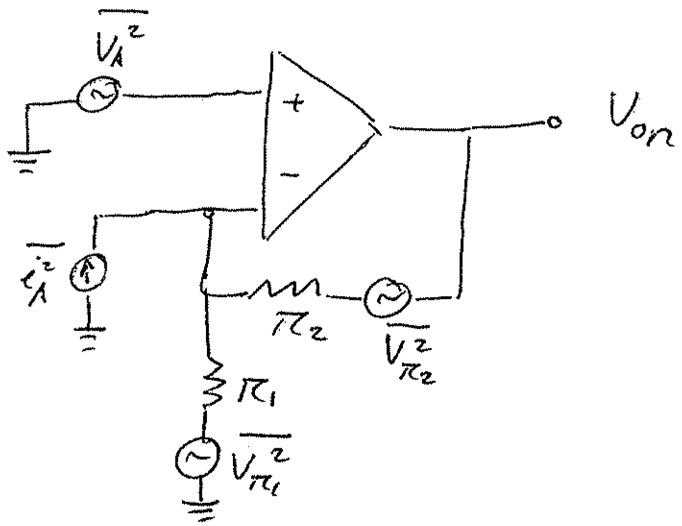
$$\eta_o(f) \leq \eta_n \,\frac{1}{2}\left[\frac{\beta W_n}{f_S}\,\text{sinc}^2\left(\frac{f}{2f_S}\right) + 1\right] \qquad \beta W_n \geq \tfrac{1}{2} f_S$$

In case of undersampled noise ($f_S \ll \beta W_n$) the low frequency noise is dominated by the S/H term ($\eta_o(f \to 0) \approx \frac{1}{2}\eta_n \frac{\beta W_n}{f_S}$) while the transmission gate term sets the high-frequency noise floor ($\eta_o(f \to \infty) = \frac{1}{2}\eta_n$)

Concept of Equivalent Input Noise

Example Noninverting amplifier

Equivalent Circuit for Noise Analysis



$$\overline{V_{on}^2} = \overline{V_A^2}\left(1+\frac{R_2}{R_1}\right)^2 + \overline{i_A^2}\,R_2^2 + \overline{V_{R_1}^2}\,\frac{R_2^2}{R_1^2} + \overline{V_{R_2}^2}$$

where $\overline{V_A^2} \cong 4kT\,\frac{2}{3}\,\frac{1}{g_{m_{in}}} + K_n\,\frac{1}{(W\cdot L\,C_{ox})_{in}\,f}$

Amp. input pair

$$\overline{i_A^2} \cong \overline{V_A^2}\,\omega^2 C_{gs_{in}}^2$$

$$\overline{V_R^2} = 4kTR$$

$$\boxed{\overline{V_{ieq}^2} = \frac{V_{on}^2}{Gain^2} = \overline{V_A^2} + \overline{i_A^2}\,\frac{R_2^2}{(1+R_2/R_1)^2} + 4kT\,\frac{R_2}{(1+R_2/R_1)}}$$

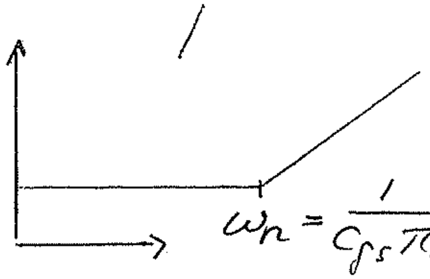
Inserting the actual amplifier noise sources yields:

$$\boxed{\overline{V_{ieq}^2} \cong \overline{V_A^2}\left[1+\frac{\omega^2 C_{gs}^2 R_2^2}{(1+R_2/R_1)^2}\right] + 4kTR_2\,\frac{1}{(1+R_2/R_1)}}$$

Opamp Contrib. External Res.

If $R_2 \gg R_1$

$$\boxed{\overline{V_{ieq}^2} \cong \overline{V_A^2}\left[1+\omega^2 C_{gs}^2 R_1^2\right] + 4kTR_1}$$



$\omega_p = \frac{1}{C_{gs}R_1}$

e.g. $C_{gs} = 100fF$ $\quad \therefore \ \omega_p = 2\pi \cdot 1.6\,GHz$
$R_1 = 1k\Omega$

$$\therefore \ \boxed{\overline{V_{ieq}^2} \approx \overline{V_A^2} + 4kTR_1}$$

Note: Opamp gain roll-off prevents high-frequency noise increase

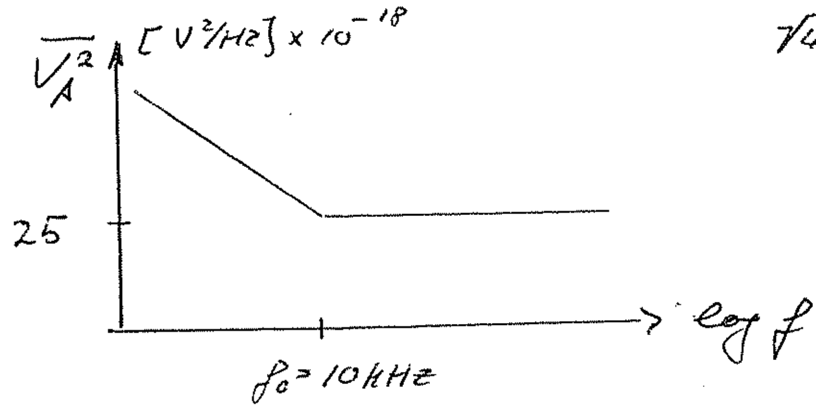
Total Noise Power

Example Noninverting amplifier

$$\left| \overline{V_{ieq}^2} \approx \overline{V_A^2} + 4kTR_1 \right|$$

$R_1 = 1k\Omega$ $\qquad$ $4kTR_1 = 16.6 \times 10^{-18} \frac{V^2}{Hz}$

$\sqrt{4kTR_1} = 4.07 \frac{nV}{\sqrt{Hz}}$



Question: What is the total noise power in the frequency band from 10Hz to 100kHz?

$$V_{niTot}^2 = \int_{B_1}^{B_2} V_{ieq}^2(f)\,df \qquad B_1 = 10Hz,\quad B_2 = 10^5 Hz$$

$$= \int_{B_1}^{B_2} (25+16.6)\times 10^{-18}\,df + \int_{B_1}^{B_2} 25\times 10^{-18} \frac{f_0}{f}\,df$$

$$V_{niTot}^2 \cong 41.6\times 10^{-13}[V^2] + 25\times 10^{-14}\cdot 4\cdot \ln(10)[V^2]$$

$$\left\| V_{niTot}^2 \cong \underbrace{41.6\times 10^{-13}[V^2]}_{\text{Thermal Contr.}} + \underbrace{23\times 10^{-13}[V^2]}_{1/f \text{ contribution}} \right\|$$

$$\therefore \left\| V_{niTot} \cong 2.54\mu V \right\|$$

Low Noise CMOS Opamp

Design Guidelines

A) kTR Noise

$$\overline{V^2_{nkTRi}} \propto \frac{1}{\sqrt{(W/L)_{in}}}\left[1+\mu_r\sqrt{\frac{(W/L)_{Ld}}{(W/L)_{in}}}\right]\frac{1}{\sqrt{I_{Diff}}} \qquad (1)$$

B) 1/f Noise

$$\overline{V^2_{n1/fi}} \propto \frac{1}{(W\cdot L)_{in}}\left[1+\mu_r\frac{L_{in}^2}{L_{Ld}^2}\right] \qquad (2)$$

where $\mu_r = \begin{cases} \frac{K_p\mu_p}{K_n\mu_n} & \text{n-channel input stage} \\ \frac{K_n\mu_n}{K_p\mu_p} & \text{p-channel input stage} \end{cases}$

n-channel input $\mu_{rn_{Si}} \approx 0.15-0.35$

p-channel input $\mu_{rp_{Si}} \approx 3-7$

To keep 1/f contribution of load stage within reason, one should select $L_{Ld} > \sqrt{2\mu_r}\,L_{in}$

e.g. n-channel input: $L_{Ld} > 0.7\,L_{in}$

p-channel input: $L_{Ld} > 3.2\,L_{in}$