Distributed Channel Probing for Efficient Transmission Scheduling in Wireless Networks

Bin Li and Atilla Eryilmaz

Abstract—**It is energy-consuming and operationally cumbersome for all users to continuously estimate the channel quality before each transmission decision in opportunistic scheduling over wireless fading channels. This observation motivates us to understand whether and how opportunistic gains can still be achieved with significant reductions in channel probing requirements and without centralized coordination amongst the competing users. To that end, we first study a basic scenario with symmetric arrivals to explicitly characterize the maximum achievable throughput as a function of the allowable probing rates in symmetric and independent ON-OFF fading channels. This result provides two insights: (i) almost the same opportunistic gains can be realized with significant reductions in probing rates when the number of users is large; (ii) a natural randomized strategy for distributed implementation cannot exploit the full opportunistic gains.**

These insights motivate us to consider the general fading scenario and develop probing and transmission schemes that are amenable to distributed implementation. After characterizing the maximum achievable throughput region under the probing constraints, we provide an optimal probing algorithm. Noting the difficulties in the implementation of the centralized solution, we develop a novel Sequential Greedy Probing (SGP) algorithm by using the maximum-minimums identity, which is naturally well-suited for physical implementation and distributed operation. We show that the SGP algorithm is optimal in the important scenario of symmetric and independent ON-OFF fading channels. Then, we study a variant of the SGP algorithm in general fading channels to obtain its efficiency ratio as an explicit function of the channel statistics and rates, and note its tightness in the symmetric and independent ON-OFF fading scenario. We further expand on the distributed implementation of these greedy solutions by using the Fast-CSMA technique.

I. Introduction

Opportunistic scheduling has long been observed (e.g., [12], [11]) to improve communication performance in wireless fading systems by selectively transmitting over channels that are in good condition. This presumes the knowledge of channel state information (CSI) at the outset of each transmission decision. However, in the presence of many contending users that utilize the time-varying channel, acquiring CSI per user is not only energy-consuming, but, more importantly, operationally difficult since it typically requires non-overlapping pilot training phases to obtain reliable channel quality estimates. Moreover, such persistent probing is likely unnecessary given that only few of them may be allowed to transmit due to the interference constraints. Yet, opportunistic gains from multi-user diversity cannot be realized if sufficient CSI is not present. This implies a natural tradeoff between exploring the multi-user diversity and energy consumption for channel acquisition, and raises a fundamental question on the design of opportunistic scheduling towards the determination of which subset of users to probe the channel given limited average probing rates.

Bin Li (celibin@gmail.com) and Atilla Eryilmaz (eryilmaz.2@osu.edu) are with the Department of Electrical and Computer Engineering at The Ohio State University, Columbus, Ohio 43210 USA.

The seminal works of Tassiulas and Ephremides (e.g., [22], [23] and [21]) have showed the throughput-optimality of the opportunistic scheduling, which prioritizes activation of links with the largest product of backlog awaiting service and corresponding channel rate given the full knowledge of CSI, also called Maximum Weight Scheduling (MWS). Recently, there has been an increasing understanding on efficient scheduling with limited CSI (e.g., [6], [10], [2], [17]). In [6], the authors propose a two-stage throughput-optimal MWS-type algorithm given partial CSI under the assumption that only users with known channel states can contend for the channel. However, they do not answer how to select a subset of users to probe the channel. In [10], the authors also develop a similar MWS-type algorithm that minimizes the energy consumption. However, the resulting decision space being exponentially increasing with the number of users appears to limit its applicability in multi-user environments. In fact, existing works in the design of joint probing and transmission strategies assume centralized controllers that utilize all state information, and hence are not suitable for distributed operation in large-scale networks. However, as we shall point out, the design for distributed probing strategies generates difficult challenges that require novel techniques beyond existing approaches discussed next.

In an exciting thread of work, it has been shown that Carrier Sense Multiple Access (CSMA) based distributed scheduling strategies (e.g., [7], [16], [5], [18]) can maximize long-term average throughput for general non-fading wireless topologies. Yet, the design of distributed schedulers in a fading environment has been observed to be much more difficult. Nevertheless, when CSI is available, a distributed Fast-CSMA (FCSMA) algorithm has also been developed [8] that guarantees throughput-optimal scheduling over wireless fading channels in a fully-connected network topology. Yet, to the best of our knowledge, there does not exist a distributed solution that also accounts for the energy and operational limitations in the CSI acquisition.

With this motivation, in this work, we address the problem of distributed joint probing and transmission scheduling when users have heterogeneous loads, probing rate constraints, and channel statistics. The following items list our main contributions along with references on where they appear in the text:

• In Section III, we study an important basic setup with many users sharing a common resource that motivates the rest of the work by illustrating that a small probing rate is sufficient to achieve almost the same performance as the case when all users continuously probe their channels. Yet, it is also observed that simplistic randomized solutions will under-perform, thus motivating more sophisticated distributed solutions.

• In Section IV, we first characterize the capacity region given the allowable probing rate for general fading channels. Then, we develop a throughput-optimal joint probing and transmission algorithm assuming a centralized controller. This algorithm, while impractical as is, forms the basis for the subsequent design of algorithms that are suitable for distributed operation.

• In Section V, based on the maximum-minimums identity [20], we first develop a novel Sequential Greedy Probing (SGP) algorithm where users probe the channel sequentially. Then, we show that the SGP algorithm can get the optimal probing schedule, leading to throughput-optimal performance over symmetric and independent ON-OFF fading channels.

• In Section VI, we introduce and analyze a Modified SGP (MSGP) algorithm that is adapted to general fading channels, and explicitly characterize the efficiency ratio that it achieves as an explicit function of the channel statistics and rates. The efficiency ratio is tight for symmetric and independent ON-OFF channels.

• In Section VII, we utilize the FCSMA strategy [8] to develop distributed implementations of proposed greedy algorithms, and analyze the performance of the resulting algorithm.

II. System Model

We consider a system where a set of N users contend for data transmission over wireless fading channels. We assume that the channel for each user has $M+1$ possible rates $c_0, c_1, c_2, ..., c_M$, where $c_0 < c_1 < c_2 < ... < c_M$ and $c_0 = 0$. Let $C_i[t]$ denote the maximum amount of service available in slot t if user i is scheduled. We assume that $\mathbf{C}[t] = (C_i[t])_{i=1}^N$ are independently and identically distributed (i.i.d.) over time, with $p_{ij} \triangleq \Pr\{C_i[t] = c_j\}, \forall i = 1, ..., N; j = 0, 1, ..., M$. Let $\mathcal{C}$ be the collection of possible global channel states. We reasonably assume that the channel for each user is unavailable with a strictly positive probability[1], that is, $p_{i0} > 0, \forall i$. In the rest of paper, we also use $\mathbf{C}$ to denote the fading channel.

In order to get CSI, each user needs to probe the channel by transmitting small control packets. Users cannot probe the channel at the same time due to the interference constraints. We denote the probing schedule as $\mathbf{X} = (X_i)_{i=1}^N$, where $X_i = 1$ if user i probes the channel and $X_i = 0$ otherwise. We also treat $\mathbf{X}$ as a set of probing users. Let $\mathcal{X}$ be the collection of probing schedules. Due to the interference constraints, at most one user can transmit in each slot. We call a schedule where at most one user is active in each slot as a *feasible schedule* and denote it as $\mathbf{S} = (S_i)_{i=1}^N$, where $S_i = 1$ if user i grabs the channel at slot t and $S_i = 0$ otherwise. We use $\mathcal{S}$ to denote the collection of feasible schedules.

[1] In practice, the probing packets and data packets are transmitted in low-rate (e.g., 1Mbps in IEEE 802.11b) and high-rate (e.g., 2/5.5/11Mbps in IEEE 802.11b) respectively, which implies that the transmission of probing packets requires lower signal-to-noise-ratio than that of data packets. Thus, it is reasonable to assume that when the channel is very poor, the user can still probe the channel but cannot transmit the data packets.

If the user does not probe the channel at the beginning of each time slot, it may underestimate the channel rate or may even fail to transmit due to a bad channel condition. Thus, it is reasonable to assume (as in [6]) that each user will not start a transmission if it does not observe the channel state at the beginning of each time slot. We denote the *allowable probing rate* for each user i as $m_i \in (0, 1], \forall i$, which puts an upper bound on the average number of probing operations that each user is allowed to make, i.e., $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[X_i[t]] \leq m_i$, $\forall i$. This bound, as noted in the introduction, may be due to energy or operational constraints associated with the channel estimation operation.

We assume that each user i serves its own traffic load and maintains them in a data queue with $Q_i[t]$ denoting its queue length at the beginning of slot t. Let $A_i[t]$ denote the number of packets arriving at user i in slot t that are i.i.d. over time with $\mathbb{E}[A_i[t]] = \lambda_i$, and $\mathbb{E}[A_i^2[t]] < A_{\max}$ for some $A_{\max} < \infty$. Then, the evolution of data queue i is described as follows.

$$Q_i[t+1] = (Q_i[t] + A_i[t] - X_i[t]S_i[t]C_i[t])^+, \forall i. \quad (1)$$

Our goal is to find an efficient joint probing and transmission schedule $\{\mathbf{X}[t], \mathbf{S}[t]\}_{t\geq 1}$ under the scheduling constraint that at most one user can be scheduled at each time slot and probing constraint that the average probing rate of each user should not be greater than its allowable probing rate. A key difficulty in the solution of this problem is that the information available at the transmission scheduling decision $\mathbf{S}[t]$ critically depends on the previously made probing decision $\mathbf{X}[t]$, which in turn must be performed distributively with only local information. We will address the problem of optimal centralized control, and then return to the distributiveness challenge.

We say that data queue i is *strongly stable* if it satisfies $\text{limsup}_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[Q_i[t]] < \infty$. The system is *stable* if all data queues are strongly stable. We define the *capacity region* as a maximum set of arrival rate vectors $\boldsymbol{\lambda} = (\lambda_i)_{i=1}^N$ for which the system is stable and the average probing rate of each user is no greater than its allowable probing rate under any policy. We call an algorithm *optimal* if it can make the system stable for any arrival rate vector that lies strictly inside the capacity region. An algorithm can achieve the *efficiency ratio* ρ if it can stabilize the system for any $\boldsymbol{\lambda}$ strictly within a fraction ρ of the capacity region. Next, we study a basic setup that motivates further investigations.

III. A Motivating Scenario

Here, we consider symmetric and independent ON-OFF fading channels with probability p of each channel being ON to support a unit rate in each time slot. Assume that each user has a uniform arrival rate λ and uniform allowable probing rate $m \in (0, 1]$. Thus, all users should be expected to have the same maximum achievable rate, which is denoted by $\lambda_{\max}(m)$. The next proposition explicitly characterizes $\lambda_{\max}(m)$ under any strategy with a long-term average as a piece-wise linear function of m.

Proposition 1: For the above setup, the maximum supportable arrival rate under any policy with a well-defined long term average is characterized as follows:

$$\lambda_{\max}(m) = mp, \text{if } 0 \le m \le \frac{1}{N};$$
$$\lambda_{\max}(m) = \frac{1}{N} + (m - \frac{i}{N})p(1-p)^i - \frac{1}{N}(1-p)^i,$$
$$\text{if } \frac{i}{N} \le m \le \frac{i+1}{N}, i = 1, ..., N-1.$$

Proof: See Appendix A for the proof. ■

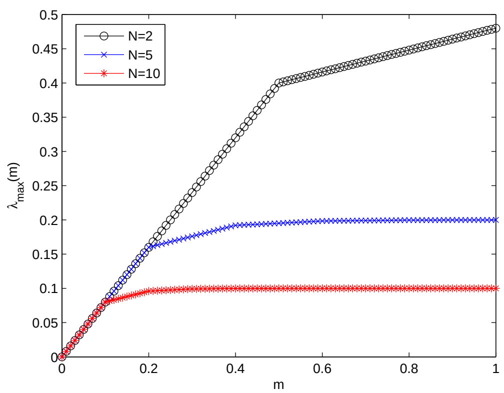


Fig. 1: Maximum throughput under different number of users

Figure 1 illustrates $\lambda_{max}(m)$ as a function of the allowable probing rate m for a range of the number of users, N, when $p = 0.8$. An interesting observation is that when the number of users increases a small probing rate appears enough to achieve almost the same maximum achievable rate as the case when all users always probe their channels, i.e., when $m = 1$. This observation can be accurately captured in the following corollary.

Corollary 1: The maximum achievable throughput $\lambda_{max}(m)$ approaches the upper limit $\lambda_{max}(1)$ asymptotically as N increases as long as the scaled probing rate mN diverges, however slowly. More explicitly, we have

$$\lim_{N\to\infty} \frac{\lambda_{\max}(\frac{\lfloor h(N)\rfloor}{N})}{\lambda_{\max}(1)} = 1, \tag{2}$$

where h is any non-negative and non-decreasing function with $h(x) \le x, \forall x$, and $\lim_{x\to\infty} h(x) = \infty$, and $\lfloor y \rfloor$ is the maximum integer that cannot be greater than y.

Proof: From Proposition 1, we get $\lambda_{\max}(\frac{\lfloor h(N)\rfloor}{N}) = \frac{1-(1-p)^{\lfloor h(N)\rfloor}}{N}$. Then, we have

$$\lim_{N\to\infty} \frac{\lambda_{\max}(\frac{\lfloor h(N)\rfloor}{N})}{\lambda_{\max}(1)} = \lim_{N\to\infty} \frac{1-(1-p)^{\lfloor h(N)\rfloor}}{1-(1-p)^N} = 1. \quad ■$$

Note that $h(x)$ can be $\log x$ or $\log\log x$. Thus, when the number of users is large, the probing rate $\frac{\lfloor h(N)\rfloor}{N}$, however small, is enough to guarantee the good performance. In practice, we are interested in the design of a distributed probing and scheduling algorithm that can support the maximum achievable rate. One may be inclined to suggest a natural Randomized Probing (RP) policy whereby each user independently probes the channel with probability m. From [23], the maximum achievable throughput of RP policy is given by $\frac{1}{N}\left(1-(1-mp)^N\right)$.

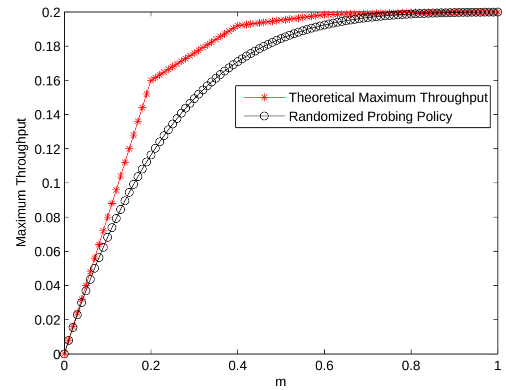


Fig. 2: The throughput performance of RP policy

Figure 2 compares this rate to the maximum achievable rate by any policy to demonstrate that the RP policy falls short of reaching the maximum achievable rate, especially for small allowable probing rates. This motivates us in the rest of the work to develop more sophisticated algorithms that can support the maximum achievable rates.

IV. Optimal Centralized Probing and Transmission

In this section, we first study the capacity region given the allowable probing rate in a general fading channel. Then, we propose a centralized probing and transmission algorithm that supports any throughput in it.

A. Characterization of the Capacity Region

The next lemma gives the capacity region $\Lambda(\mathbf{m}, \mathbf{C})$ under the allowable probing rate vector $\mathbf{m} = (m_i)_{i=1}^N$ in a general fading channel $\mathbf{C}$.

Lemma 1: The capacity region $\Lambda(\mathbf{m}, \mathbf{C})$ is a set of arrival rate vectors $\boldsymbol{\lambda} = (\lambda_i)_{i=1}^N$ such that there exist non-negative numbers $\alpha(\mathbf{x})$ and $\beta(\mathbf{x}, \mathbf{c}; \mathbf{s})$ satisfying

$$\lambda_i \le \sum_{\mathbf{x}\in\mathcal{X}} \alpha(\mathbf{x}) \sum_{\mathbf{c}\in\mathcal{C}} \Pr\{\mathbf{C}[t] = \mathbf{c}\} \sum_{\mathbf{s}\in\mathcal{S}} \beta(\mathbf{x}, \mathbf{c}; \mathbf{s}) x_i c_i s_i, \forall i, \tag{3}$$

$$\sum_{\mathbf{s}\in\mathcal{S}} \beta(\mathbf{x}, \mathbf{c}; \mathbf{s}) = 1, \forall \mathbf{x}, \mathbf{c}, \tag{4}$$

$$\sum_{\mathbf{x}\in\mathcal{X}} \alpha(\mathbf{x}) = 1, \tag{5}$$

$$\sum_{\mathbf{x}\in\mathcal{X}} \alpha(\mathbf{x}) x_i \le m_i, \forall i, \tag{6}$$

where $\alpha(\mathbf{x})$ and $\beta(\mathbf{x}, \mathbf{c}, \mathbf{s})$ denote the probability that selects the probing schedule $\mathbf{x}$ and the feasible schedule $\mathbf{s}$ given the probing schedule $\mathbf{x}$ and channel state $\mathbf{c}$, respectively.

Proof: See Appendix B for the proof. ■

In (3), the right-hand-side (RHS) is the total average service provided for each user and the left-hand-side (LHS) is just the average arrival rate. Thus, to stabilize the data queue, (3) should be satisfied. In (6), the LHS is the average probing rate for each user and the RHS is the allowable probing rate for each user. To meet the constraint of allowable probing rates, (6) should be satisfied.

Next, we characterize the equivalent capacity region for ON-OFF fading channels, which will be useful in the performance analysis of the algorithm proposed in the next section in general fading channels.

Lemma 2: For the case of ON-OFF fading channels, the capacity region $\Lambda(\mathbf{m},\mathbf{C})$ is equivalent to the following region $\Gamma(\mathbf{m},\mathbf{C})$ which is a set of arrival rate vectors $\boldsymbol{\lambda}$ such that there exist non-negative numbers $\alpha(\mathbf{x})$ satisfying: for any $\mathbf{A}\subseteq\mathbf{N}\triangleq\{1,2,\cdots,N\}$,

$$\sum_{i\in\mathbf{A}}\lambda_i \le 1-\sum_{\mathbf{x}}\alpha(\mathbf{x})\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\mathbb{1}_{\{x_ic_i=0,\forall i\in\mathbf{A}\}}, \quad (7)$$

$$\sum_{\mathbf{x}}\alpha(\mathbf{x})x_i \le m_i, \forall i, \quad (8)$$

$$\sum_{\mathbf{x}}\alpha(\mathbf{x}) = 1. \quad (9)$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function.

Remark: If a random probing schedule $\mathbf{X}=(X_i)_{i=1}^N$ has the probability distribution $\alpha(\mathbf{x})$, then

$$\sum_{\mathbf{x}}\alpha(\mathbf{x})\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\mathbb{1}_{\{x_ic_i=0,\forall i\in\mathbf{A}\}} = \Pr\{X_iC_i[t]=0,\forall i\in\mathbf{A}\} \quad (10)$$

In addition, since

$$\mathbb{E}[\max_{i\in\mathbf{A}} X_iC_i[t]] = 1-\Pr\{X_iC_i[t]=0,\forall i\in\mathbf{A}\}, \quad (11)$$

(7) is equivalent to

$$\sum_{i\in\mathbf{A}}\lambda_i \le \mathbb{E}[\max_{i\in\mathbf{A}} X_iC_i[t]], \quad \forall\mathbf{A}\subseteq\mathbf{N}. \quad (12)$$

Proof: See Appendix C for the proof. ■

B. An Optimal Joint Probing and Transmission Algorithm

To obtain the optimal centralized joint probing and transmission algorithm, we use the standard technique in [13] to introduce and guarantee stability of a virtual queue for each user that conveniently measures the degree of violation of the average probing constraint. Specifically, we let $U_i[t]$ denote the virtual queue length for user i at the beginning of slot t. The number of packets entering the virtual queue i at slot t is just $X_i[t]$. We use $I_i[t]$ to denote the service for virtual queue i at slot t that are i.i.d. over time with $\mathbb{E}[I_i[t]]=m_i$, and $\mathbb{E}[I_i^2[t]]\le I_{\max}$ for some $I_{\max}<\infty$. Then, the evolution of the virtual queue i is as follows:

$$U_i[t+1] = (U_i[t]+X_i[t]-I_i[t])^+, \forall i. \quad (13)$$

We say that virtual queue i is *mean rate stable* if it satisfies $\lim_{T\to\infty}\frac{\mathbb{E}[U_i[T]]}{T}=0$. If the virtual queue i is mean rate stable, then, by using Theorem 2.5 in [13], the average probing rate constraint of user i is automatically satisfied. Thus, we aim to design a joint probing and transmission policy that provides strong stability for data queues and mean rate stability for virtual queues under any arrival rate vector strictly within the capacity region $\Lambda(\mathbf{m},\mathbf{C})$.

Joint Probing and Transmission (JPT) Algorithm:

In each slot t, given $(\mathbf{Q}[t],\mathbf{U}[t])$, perform:

(1) *Probing Decision*: select the probing vector $\mathbf{X}^*[t]$ as

$$\mathbf{X}^*[t]\in\arg\max_{\mathbf{X}}\left(\mathbb{E}\left[\max_i Q_i[t]X_iC_i[t]\right]-\sum_{i=1}^N U_i[t]X_i\right), \quad (14)$$

(2) *Transmission Scheduling Decision*: After the channel states of the selected users are probed, schedule the transmission of user $i^*[t]$ that satisfies

$$i^*[t]\in\arg\max_i Q_i[t]X_i^*[t]C_i[t]. \quad (15)$$

Remark: Since at most one user can be scheduled at each time slot, we can also interpret i^* as the index such that $S^*_{i^*}[t]=1$, where

$$\mathbf{S}^*[t]\in\arg\max_{\mathbf{S}\in\mathcal{S}}\sum_{i=1}^N Q_i[t]X_i^*[t]C_i[t]S_i[t].$$

In the JPT algorithm, we first need to solve the optimization problem (14) to get the optimal probing schedule $\mathbf{X}^*[t]$ in the probing stage at slot t. Then, we need to solve the optimization problem (15) to get the optimal transmission schedule in the transmission stage given the optimal probing schedule $\mathbf{X}^*[t]$ and the observed channel states. Next, we will show that the JPT algorithm is optimal in the sense that it can stabilize the system for any arrival rate vector strictly within the capacity region. Let $\mathrm{Int}(R)$ denotes the set of interior points of the region R.

Proposition 2: The JPT algorithm is optimal, i.e., for any arrival rate $\lambda\in\mathrm{Int}(\Lambda(\mathbf{m},\mathbf{C}))$, the JPT algorithm stabilizes the system subject to the average probing rate constraints.

Proof: See Appendix D for the proof. ■

Even though the JPT algorithm is optimal, it cannot directly be applied in practice due to the complexity of computing an optimal probing schedule and the need of centralized coordination. In [8], the authors proposed a distributed FCSMA algorithm over a wireless fading channel in a fully-connected network topology. We can use a similar technique as in [8] to solve transmission scheduling component (15) of the JPT algorithm distributively if we know the optimal probing schedule. However, how to reduce the complexity of computing an efficient probing schedule and implement it in a distributed way still remains an open question. Next, we develop a sequential greedy algorithm that is well-suited for distributed computation of (14) and analyze its performance. From now on, we always use the well-known MWS algorithm or its distributed variants (e.g., the FCSMA algorithm) in the transmission stage.

V. Sequential Greedy Probing Policy and Analysis

In this section, we propose a sequential greedy algorithm for the probing component of the JPT algorithm, which can be implemented distributively as we will explain in Section VII. Then, we show that it can get an optimal probing schedule in a symmetric and independent ON-OFF fading channel.

A. A Sequential Greedy Probing Algorithm

We need to establish some new notations to introduce our proposed algorithm. For any non-empty set $\mathbf{E} \subseteq \mathbf{N}$ (recall that $\mathbf{N} = \{1, 2, \cdots, N\}$), we define the function $f(\mathbf{E}, e)$ as follows:

$$f(\mathbf{E}, e) \triangleq \mathbb{E}[\max_{i \in \mathbf{E}} \min\{Q_i C_i, Q_e C_e\}], \tag{16}$$

where $e \notin \mathbf{E}$. Appendix E explores some properties of $f(\mathbf{E}, e)$ over a symmetric and independent ON-OFF fading channel. Here, it is worth noting that, by using the maximum-minimums identity [20], $f(\mathbf{E}, e)$ can be computed recursively.

Also, let $\phi_i \triangleq \mathbb{E}[Q_i C_i] - U_i, \forall i \in \mathbf{N}$, and consider a set $\mathbf{F} \subseteq \mathbf{N}$ of probing users and $r \in \mathbf{N} \backslash \mathbf{F}$. By using the maximum-minimums identity, we have the key relationship:

$$\begin{aligned} &\mathbb{E}\left[\max_{i \in \mathbf{F} \bigcup \{r\}} Q_i C_i\right] - \sum_{i \in \mathbf{F} \bigcup \{r\}} U_i \\ &= \left(\mathbb{E}[\max_{i \in \mathbf{F}} Q_i C_i] - \sum_{i \in \mathbf{F}} U_i\right) + \phi_r - f(\mathbf{F}, r). \end{aligned} \tag{17}$$

For the derivation of this identity, please see Appendix F for details. Based on the iterative equation (17), we can define a directed graph $\mathcal{G}$, where each probing schedule $\mathbf{X}$ denotes a node with an associated *value* of $\mathbb{E}[\max_{i \in \mathbf{X}} Q_i C_i] - \sum_{i \in \mathbf{X}} U_i$. Thus, $\mathcal{X}$ also represents the collection of all nodes. Since each node is a binary vector of N dimensions, we have $|\mathcal{X}| = 2^N$, where $|\cdot|$ denotes the cardinality of the set. For two nodes $\mathbf{X_1}$ and $\mathbf{X_2}$, there is a *directed link* from node $\mathbf{X_1}$ to node $\mathbf{X_2}$ if and only if $\mathbf{X_1}$ is a subset of $\mathbf{X_2}$ with the cardinality $|\mathbf{X_2}| - 1$. Let $q = \mathbf{X_2} \setminus \mathbf{X_1}$. We define the *weight* of a link from node $\mathbf{X_1}$ to node $\mathbf{X_2}$ as $\phi_q - f(\mathbf{X_1}, q)$. Let $\mathcal{E}$ be the collection of edges, and let node $\mathbf{X_0}$ denote the all-zero probing schedule where no user probes the channel, and thus the value of node $\mathbf{X_0}$ is 0. We say node $\mathbf{X}$ is in level $|\mathbf{X}|$ in the directed graph $\mathcal{G} = (\mathcal{X}, \mathcal{E})$. Finally, let $\mathbf{I} = \{i \in \mathbf{N} : \phi_i > 0\}$. Figure 3 shows the directed graph for $N = 3$.

Given the directed graph $\mathcal{G}$, the optimization problem (14) is equivalent to finding a path with the largest total weight emanating from node $\mathbf{X_0}$. By noting that the directed graph is acyclic, if we negate the weight of edges, the optimization problem (14) is also equivalent to finding a shortest path from node $\mathbf{X_0}$ in the directed graph, which can be solved by Bellman-Ford algorithm [3]. However, Bellman-Ford algorithm always goes back and forth to find a shortest path, which is not allowed in the probing problem since once a node probes its channel its energy is consumed. More importantly, the complexity of Bellman-Ford algorithm is $O(|\mathcal{X}||\mathcal{E}|)$ and thus increases exponentially with the number of users. Fortunately, the weights of edges are highly correlated with each other through the queue lengths. Thus, it is possible to design a sequential greedy probing algorithm as follows that can still yield good performance.

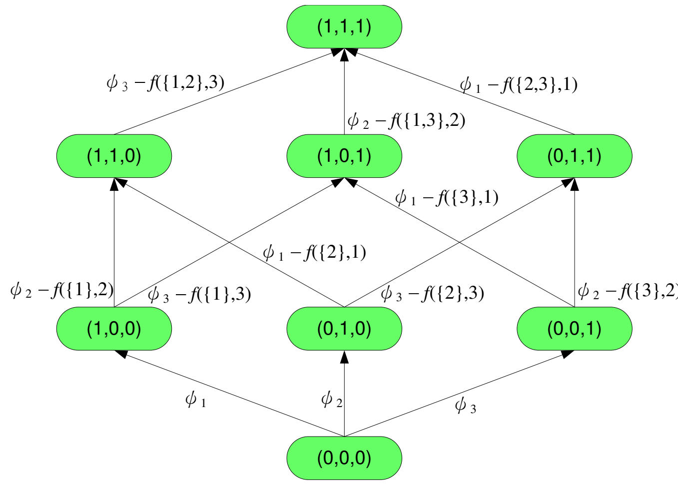


Fig. 3: The directed graph $\mathcal{G} = (\mathcal{X}, \mathcal{E})$ when $N = 3$

We first divide each time slot into a control slot and a data slot. The purpose of the control slot is to determine the probing schedule to get the channel state used for data transmission in the data slot. To achieve this goal, we further subdivide the control slot into N mini-slots.

Sequential Greedy Probing (SGP) Algorithm:

(1) In the first mini-slot, select user i_1 such that $i_1 \in \arg\max_{i \in \mathbf{I}} \phi_i$, where $\mathbf{I} = \{i \in \mathbf{N} : \phi_i > 0\}$ and we recall that $\phi_i \triangleq \mathbb{E}[Q_i C_i] - U_i, \forall i \in \mathbf{N}$. User i_1 probes the channel while also announcing its queue-length. If no users probe the channel, then all users keep silent in the rest of current slot and restarts in the next time slot.

(2) In the k^{th} $(1 < k \leq N)$ mini-slot, select user i_k such that

$$i_k \in \arg\max_{i \in \mathbf{I} \backslash \{i_1, ..., i_{k-1}\}} \left(\phi_i - f\left(\{i_1, ..., i_{k-1}\}, i\right)\right). \tag{18}$$

If $\phi_{i_k} > f\left(\{i_1, ..., i_{k-1}\}, i_k\right)$, then user i_k probes the channel while also announcing its queue length. Otherwise, all users stop probing and all probing users with non-zero channel states are candidates for transmission scheduling as dictated in (15).

Remark: In the SGP algorithm, we require that each probing user announces its queue-length information, which may cause the heavy message exchange overhead. Motivating by [24] that utilizes the delayed queue length information to provide the fair resource allocation, we may only allow the transmitting user to announce its queue-length information, and all users utilize this delayed queue length information to calculate the probing schedule. Our simulation results indicate that this modified version of the SGP algorithm does not degrade the system performance.

B. Optimality of the SGP Algorithm for Symmetric Channels

In this subsection, we will show that the SGP algorithm can achieve the optimal value of the maximization problem (14) for symmetric and independent ON-OFF fading channels. The next lemma and subsequent corollaries pave the path to this result by establishing a key property of the directed graph $\mathcal{G}$.

Lemma 3: For symmetric and independent ON-OFF fading channels with an ON probability p, if node $\mathbf{A}^*$ is the unique node with maximum value in level $|\mathbf{A}^*|$ in graph $\mathcal{G}$, then all nodes with maximum value in level $|\mathbf{A}^*|-1$ belong to a subset of nodes $\mathbf{A}^*$, where a subset of nodes $\mathbf{X}$ means a set of nodes with edge ending with node $\mathbf{X}$, and the value of node $\mathbf{X}$ is defined as $\mathbb{E}[\max_{i\in\mathbf{X}} Q_iC_i] - \sum_{i\in\mathbf{X}} U_i$.

Proof: Let $\mathcal{A}$ be the class of the nodes in level $|\mathbf{A}^*|$; $\mathcal{D}$ be the class of nodes in level $|\mathbf{A}^*|-1$; and $\mathcal{B}$ be the class of nodes that are a subset of node $\mathbf{A}^*$ in level $|\mathbf{A}^*|-1$. Thus, we need to show that $\exists \mathbf{B}^* \in \mathcal{B}$ such that

$$\mathbf{B}^* \in \arg\max_{\mathbf{D}\in\mathcal{D}} \left(\mathbb{E}[\max_{i\in\mathbf{D}} Q_iC_i] - \sum_{i\in\mathbf{D}} U_i \right). \tag{19}$$

We prove it by contradiction. Suppose there exists a $\mathbf{D}^* \in \mathcal{D}\backslash\mathcal{B}$ such that

$$\mathbf{D}^* \in \arg\max_{\mathbf{D}\in\mathcal{D}} \left(\mathbb{E}[\max_{i\in\mathbf{D}} Q_iC_i] - \sum_{i\in\mathbf{D}} U_i \right). \tag{20}$$

Let $d \in \arg\min_{i\in\mathbf{A}^*\backslash\mathbf{D}^*} Q_i$ and $\mathbf{B} \triangleq \mathbf{A}^* \setminus \{d\}$. Since $\mathbf{A}^*$ is the unique node with the maximum value in level $|\mathbf{A}^*|$, node $\mathbf{D}^* \bigcup\{d\} \in \mathcal{A}$ does not have the maximum value in level $|\mathbf{A}^*|$ and thus we have

$$\mathbb{E}[\max_{i\in\mathbf{D}^*\bigcup\{d\}} Q_iC_i] - \sum_{i\in\mathbf{D}^*\bigcup\{d\}} U_i < \mathbb{E}[\max_{i\in\mathbf{A}^*} Q_iC_i] - \sum_{i\in\mathbf{A}^*} U_i.$$

According to the iterative equation (17), we have

$$\begin{aligned} &\mathbb{E}[\max_{i\in\mathbf{D}^*} Q_iC_i] - \sum_{i\in\mathbf{D}^*} U_i + \phi_d - f(\mathbf{D}^*, d) \\ &\quad < \mathbb{E}[\max_{i\in\mathbf{B}} Q_iC_i] - \sum_{i\in\mathbf{B}} U_i + \phi_d - f(\mathbf{B}, d). \end{aligned} \tag{21}$$

Since $\mathbf{D}^*$ is one of the optimal solutions to (20), we have

$$\mathbb{E}[\max_{i\in\mathbf{D}^*} Q_iC_i] - \sum_{i\in\mathbf{D}^*} U_i \geq \mathbb{E}[\max_{i\in\mathbf{B}} Q_iC_i] - \sum_{i\in\mathbf{B}} U_i. \tag{22}$$

Hence, to let (21) hold, we should have $f(\mathbf{D}^*, d) > f(\mathbf{B}, d)$. To arrive at a contradiction, we need to show that $f(\mathbf{D}^*, d) \leq f(\mathbf{B}, d)$, which is not at all obvious and requires a challenging investigation. Please see Appendix G for details. ■

Corollary 2: For symmetric and independent ON-OFF fading channels, let $\mathbf{A}^*$ be one of nodes with maximum value in level $|\mathbf{A}^*|$ in the directed graph $\mathcal{G}$, then the node with maximum value in level $|\mathbf{A}^*|-1$ should be in the union of subsets of nodes with maximum value in level $|\mathbf{A}^*|$.

Proof: The proof is exactly the same as in the proof for Lemma 3 except that $\mathcal{B}$ denotes the class of nodes in level $|\mathbf{A}^*|-1$ that are the subset of all nodes with maximum value in level $|\mathbf{A}^*|$. ■

Corollary 3: For symmetric and independent ON-OFF fading channels, if node $\mathbf{A}^*$ has the maximum value in level $|\mathbf{A}^*|$, then there exists a node with maximum value in level $|\mathbf{A}^*|+1$ that is the superset of node $\mathbf{A}^*$.

Proof: If there is only one node with maximum value in level $|\mathbf{A}^*|+1$, then the result directly follows from Lemma 3. If there are multiple nodes with maximum value in level $|\mathbf{A}^*|+1$, then the result follows from Corollary 2. ■

It is important to note that Lemma 3 and its corollaries hold regardless of whether the edge weights are positive or negative valued. This property will be crucial in the proof of the following main result of this subsection.

Proposition 3: The SGP algorithm can achieve the optimal value of the maximization problem (14) in symmetric and independent ON-OFF fading channels.

Proof: If there are multiple nodes with optimal value in the directed graph $\mathcal{G}$, then we just consider the nodes with optimal value in the lowest level, say level K. Thus, for any node with the level lower than K, its value is strictly less than that of the nodes with optimal value in level K. Next, we first assume that the SGP algorithm can continue to work even when it picks an edge with a non-positive weight. Under this assumption, we can show that the SGP algorithm sequentially selects users $i_1, i_2, ..., i_K$ to get to the node $\mathbf{A}^* = \{i_1, i_2, ..., i_K\}$, which has the optimal value in the directed graph $\mathcal{G}$. Finally, we will show that all edges in a path leading to node $\mathbf{A}^*$ have a strictly positive weight and the SGP algorithm will stop at node $\mathbf{A}^*$.

Note that the proposed SGP algorithm first picks the user i_1, where the node $\{i_1\}$ has the maximum value in level 1. By corollary 3, there exists a node with maximum value in level 2 that is a superset of node $\{i_1\}$. Since the SGP algorithm picks an edge with maximum weight $\phi_{i_2} - f(\{i_1\}, i_2)$, the node $\{i_1, i_2\}$ has the maximum value in level 2. By using similar argument, we can see that the SGP algorithm sequentially selects users $i_1, i_2, ..., i_K$ to get to the node $\mathbf{A}^*$ in level K, where the node $\{i_1, ..., i_j\}$ has the maximum value in level j for each $j = 1, ..., K$. Since node $\mathbf{A}^*$ has the maximum value in level K and the node with optimal value is in level K, node $\mathbf{A}^*$ has the optimal value in the directed graph $\mathcal{G}$.

Let $\mathcal{G}(\mathbf{A}^*)$ be the subgraph of $\mathcal{G}$ that includes all subsets of the node $\mathbf{A}^*$ and their corresponding edges. Since node $\mathbf{A}^*$ has the optimal value, we have $\phi_i - f(\mathbf{A}^* \setminus \{i\}, i) > 0, \forall i \in \mathbf{A}^*$. Indeed, if $\phi_k - f(\mathbf{A}^* \setminus \{k\}, k) \leq 0$ for some $k \in \mathbf{A}^*$, then according to the iterative equation (17), we have

$$\mathbb{E}[\max_{j\in\mathbf{A}^*} Q_jC_j] - \sum_{j\in\mathbf{A}^*} U_j \leq \mathbb{E}[\max_{j\in\mathbf{A}^*\backslash\{k\}} Q_jC_j] - \sum_{j\in\mathbf{A}^*\backslash\{k\}} U_j,$$

which contradicts that the value of a node with the level less than K is strictly smaller than that of node $\mathbf{A}^*$. According to the definition of the function f (see equation (16)), it is easy to see that if $\mathbf{E} \subseteq \mathbf{F}$, then $f(\mathbf{E}, e) \leq f(\mathbf{F}, e)$, where $e \notin \mathbf{F}$. Thus, for any given $i \in \mathbf{A}^*$ and any $\mathbf{H} \subseteq \mathbf{A}^* \setminus \{i\}$, we have

$$\phi_i - f(\mathbf{H}, i) \geq \phi_i - f(\mathbf{A}^* \setminus \{i\}, i) > 0. \tag{23}$$

Thus, all edges in the subgraph $\mathcal{G}(\mathbf{A}^*)$ have the strictly positive weight. Hence, there always exists an edge with strictly positive weight from node $\{i_1, ..., i_k\}$ in level k to node $\{i_1, ..., i_k, i_{k+1}\}$ in level $k+1$ ($k = 1, 2, ..., K-1$).

In addition, there is no edge with strictly positive weight from node $\mathbf{A}^*$ in level K. Indeed, if there is an edge with strictly positive weight from node $\mathbf{A}^*$ in level K to a node in level $K+1$, say node $\mathbf{J}$, then node $\mathbf{J}$ should have the value larger than the optimal value, which contradicts that node $\mathbf{A}^*$ has the optimal value in the directed graph $\mathcal{G}$. Thus, when the SGP algorithm reaches node $\mathbf{A}^*$, it stops. ■

In a general wireless fading channel, the SGP algorithm cannot always find the optimal value of (14) as in the above symmetric setup, and thus its performance is unclear. Instead, we consider a Modified SGP (MSGP) algorithm in the next subsection to show that the MSGP algorithm combined with MWS algorithm in the transmission stage can at least achieve a constant efficiency ratio.

VI. The Modified SGP Policy and Analysis

In this section, we consider the more general fading channels and introduce a slightly modified version of the SGP algorithm studied in the previous section. Then, we explicitly characterize the efficiency ratio that this modified algorithm is guaranteed to achieve as a function of the channel statistics and rates.

We assume that the general fading channels satisfy the following assumption.

Assumption 1: The general fading channels are i.i.d. over time and the events that the channels have zero rate are independent, that is,

$$\Pr\{C_i[t]=0, \forall i \in \mathbf{A}\} = \prod_{i\in A} \Pr\{C_i[t]=0\}, \forall \mathbf{A} \subseteq \mathbf{N}. \quad (24)$$

Remark: If fading channels are independently over users, then condition (24) trivially holds.

To introduce the proposed algorithm, we first let $p_{\min} \triangleq 1-\max_j p_{j0}$ and $p_{\max} \triangleq 1-\min_j p_{j0}$ to denote the non-zero rate probability of the worst and the best channel, respectively. Then, we define two identical and independent ON-OFF fading channels $\mathbf{C}^{\min}[t]=(C_i^{\min}[t])_{i=1}^N$ and $\mathbf{C}^{\max}[t]=(C_i^{\max}[t])_{i=1}^N$ satisfying:

$$\Pr\{C_i^{\min}[t]=0\}=1-p_{\min}, \quad \Pr\{C_i^{\min}[t]=c_1\}=p_{\min}, \forall i;$$

$$\Pr\{C_i^{\max}[t]=0\}=1-p_{\max}, \quad \Pr\{C_i^{\max}[t]=c_M\}=p_{\max}, \forall i,$$

where we recall that c_1 and c_M are, respectively, the smallest and largest transmission rates achievable for any user.

Modified SGP (MSGP) Algorithm:

MSGP algorithm operates exactly the same as the SGP algorithm, except that steps are computed assuming the identical and independent ON-OFF fading channels $\mathbf{C}^{\min}$.

Remark: The MSGP algorithm differs from the SGP algorithm only in the assumed channel statistics and rates.

The following lemma gives the key relationship between the general fading channels under Assumption 1 and two constructed identical and independent ON-OFF fading channels.

Lemma 4: For general fading channels under Assumption 1, the following relationship

$$\mathbb{E}[\max_i a_i C_i^{\min}[t]] \le \mathbb{E}[\max_i a_i C_i[t]] \le \mathbb{E}[\max_i a_i C_i^{\max}[t]], \forall t, \quad (25)$$

holds for any constants $a_i \ge 0, \forall i$.

Proof: See Appendix H for the proof. ■

In the following lemma, we give the relationship of the capacity region for fading channels satisfying condition (25).

Lemma 5: Let $\mathbf{C}^I[t]=(C_i^I[t])_{i=1}^N$ and $\mathbf{C}^{II}[t]=(C_i^{II}[t])_{i=1}^N$ represent two fading channels. If

$$\mathbb{E}[\max_i a_i C_i^I[t]] \le \mathbb{E}[\max_i a_i C_i^{II}[t]], \forall t, \quad (26)$$

holds for any constants $a_i \ge 0, \forall i$, then, we have

$$\Lambda(\mathbf{m}, \mathbf{C}^I) \subseteq \Lambda(\mathbf{m}, \mathbf{C}^{II}). \quad (27)$$

Proof: See Appendix I for the proof. ■

The following lemma reveals an interesting monotonicity property of the mean of the maximum of a set of binary random variables, and will be used in the subsequent main result.

Lemma 6: Let $\mathbf{A} \subseteq \mathbf{N}$ and $\mathbf{X}=(X_1,...,X_{|\mathbf{A}|})$ be a zero-one random vector. $Z_i, \forall i=1,...,|\mathbf{A}|$, are independent and identical Bernoulli random variables with parameter p. Then, $h(p) \triangleq \frac{1}{p}\mathbb{E}[\max_{i\in\mathbf{A}} X_i Z_i]$ is a non-increasing function.

Proof: See Appendix J for the proof. ■

Proposition 4: The MSGP algorithm combined with the MWS algorithm in the transmission stage (see equation (15)) can at least achieve an efficiency ratio $\rho \triangleq \frac{p_{\min}}{p_{\max}}\frac{c_1}{c_M}$ in general fading channels under Assumption 1.

Proof: See Appendix K for the proof. ■

Remarks: (1) In symmetric and independent ON-OFF channels, the MSGP algorithm can achieve the full capacity region, which matches the result in Proposition 3.

(2) Even though the efficiency ratio is low in highly asymmetric fading channels, the MSGP algorithm still performs well in practice as we can see in the simulation section.

VII. Distributed Implementation with Fast CSMA

Here, we expand on the distributed implementation of the greedy sequential algorithms developed in the previous two sections by using the FCSMA technique developed in [8]. Since the MSGP algorithm has the same performance as the SGP Algorithm in the special case of symmetric ON-OFF channels, we focus on the distributed implementation of the MSGP Algorithm in the control slot.

Distributed MSGP (DMSGP) Algorithm:

In the first mini-slot, each user i with $\phi_i > 0$ independently generates an exponentially distributed random variable with rate $\exp(G\phi_i)$ ($G > 0$), and starts transmitting a small probing packet after this random duration unless it senses another transmission before. The user that grabs the channel transmits its probing packet until the end of the mini-slot. After probing, all other users know the queue length of the current probing user. If no users transmit the probing packet during this mini-slot, then all users keep silent in the rest of current slot and restarts in the next time slot.

In the k^{th} ($1 < k \leq N$) mini-slot, the remaining non-probing user i with $\phi_i - f(\{i_1, ..., i_{k-1}\}, i) > 0$ generates an exponential distributed random variable with rate $\exp(G(\phi_i - f(\{i_1, ..., i_{k-1}\}, i)))$ and uses the same produce as in the first mini-slot to probe the channel. If no users probe the channel in the current mini-slot or the control slot is over, then all the probing users with the available channel state start to contend for data transmission.

Remark: Here, we assume that the sensing is instantaneous and the backoff time is continuous, which excludes the possible collisions. Yet, in practice, the sensing time is non-zero and the backoff time is typically a multiple of time units, where a time unit is equal to the time required to detect the transmission from other links. Thus, we should use the discrete-time version of the FCSMA algorithm, whose performance is close to its continuous counterpart as shown in [8].

The above procedure leads to a probing schedule $\mathbf{X}^{DMSGP}$ by the end of the control slot, where each selected probing user i knows its channel state C_i. Then, to determine the one that transmits the data packet each probing user i distributively runs the FCSMA algorithm as described in [8] with parameter $\exp(Q_i C_i)$. This is known to solve the transmission decision (15) if the queue-lengths are large enough. In order to establish the performance of such a distributed probing and transmission algorithm, we need an additional assumption.

Assumption 2: The channel rates and their corresponding probability for each user, i.e., $c_j, \forall j = 1, ..., M$ and $p_{ij}, \forall i = 1, ..., N, j = 0, ..., M$, are rational numbers.

Proposition 5: For any $\zeta > 0$ and arrival rate vector $\boldsymbol{\lambda}$ satisfying $\boldsymbol{\lambda} + \zeta \in \rho \text{Int}(\Lambda(\mathbf{m}, \mathbf{C}))$, with the efficiency ratio ρ given in Proposition 4, there exists a design parameter $G > 0$ such that the DMSGP algorithm, combined with the FCSMA algorithm in the transmission stage, can support $\boldsymbol{\lambda}$ subject to the given probing rate constraints $\mathbf{m}$ in general fading channels under Assumptions 1 and 2.

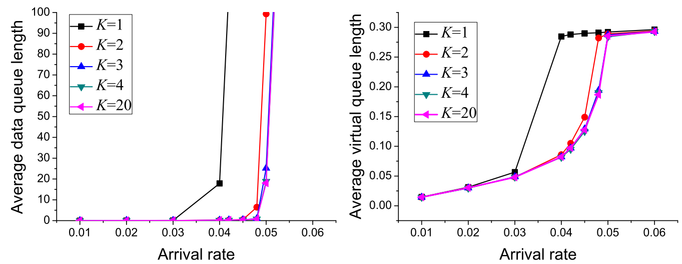
Proof: See Appendix L for the proof. ■

VIII. Simulation Results

In this section, we first study the impact of iterative steps and using the delayed queue length information (i.e., only the transmitting user broadcasts its queue length information) on the performance of the SGP algorithm. Then, we compare the performance between the SGP algorithm and the MSGP algorithm in asymmetric ON-OFF fading channels and symmetric general fading channels. In the simulation, we consider three different fading models that are i.i.d. over time and independently distributed over users: symmetric and independent ON-OFF channels with probability $p = 0.8$ that the channel is available in each time slot; asymmetric ON-OFF channels that one user has channel availability probability of 0.1 and all others have probability of 0.9 and symmetric general fading channels available to each user with rates $0, 1, 10$ and corresponding probability $0.1, 0.2, 0.7$. All users have the same arrival rate and require that the allowable probing rate cannot exceed $m = 0.4$. Without loss of generality, we use arrival process where the number of arrivals in each slot follows Bernoulli distribution and Poisson distribution when we consider ON-OFF fading channels and general fading channels respectively.

A. The impact of iterative steps

In this subsection, we study the impact of iterative steps on the performance of the SGP algorithm. We consider $N = 20$ users over a symmetric and independent ON-OFF fading channel. Under this setup, we can use Proposition 1 to get the capacity region $\Gamma = \{\lambda : \lambda < 0.05\}$. We use K to denote the maximum allowable number of iterative steps.



(a) Average data queue length (b) Average virtual queue length

Fig. 4: Impact of iterative steps

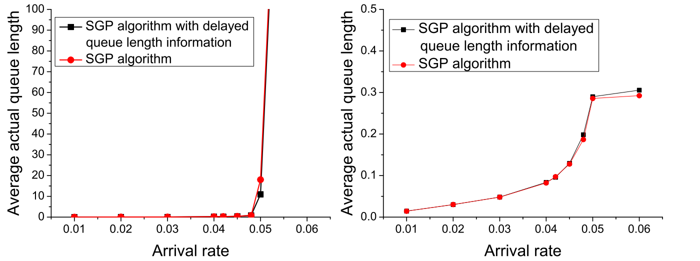
From Figure 4a and 4b, we observe that the SGP algorithm with unlimited iterative steps can achieve full capacity. In addition, as K increases, the performance of the SGP algorithm improves. Especially, we can see that four iterative steps are enough to reach almost optimal performance. This implies that while the original algorithm may be defined over more steps, in practice, we can limit the iterative steps to a small number virtually without hurting the throughput.

B. The impact of using delayed queue length information

In this subsection, we study the impact of using the delayed queue length information (i.e., each user only have the queue length information of the transmitting user) on the performance of the SGP algorithm. Figure 5a and 5a compare the performance between the SGP algorithm and the SGP algorithm using the delayed queue length information in the network of $N = 5$ users over symmetric ON-OFF fading channels. We can observe that using the delayed queue length information does not affect the system performance of the SGP algorithm. This promising property allows us to significantly reduce the overhead of exchanging queue length information under the SGP algorithm.

C. The performance of the greedy probing algorithms

In this subsection, we compare the performance among the SGP algorithm, the MSGP algorithm and the JPT algorithm. We consider $N = 5$ users. Figure 6 and Figure 7 compare the performance among the SGP algorithm, the MSGP algorithm and the JPT algorithm under an asymmetric ON-OFF channel and a symmetric general fading channel, respectively. From

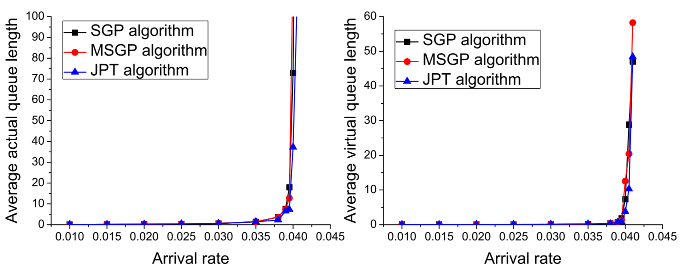


(a) Average data queue length (b) Average virtual queue length

Fig. 5: Impact of using delayed queue length information

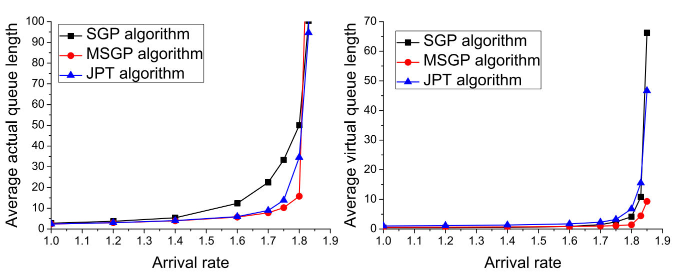
Figure 6 and 7, we can see that these algorithms have almost the same throughput performance. Noting that the JPT algorithm is throughput-optimal, both SGP and MSGP algorithm are probably throughput-optimal in general fading channels. We will investigate whether these greedy algorithms can achieve maximum throughput in general setups.

In addition, we can observe from Figure 6 that the SGP algorithm is insensitive to the channel statistics. Furthermore, from Figure 7, we can observe that the MSGP algorithm has the smallest average actual queue length and virtual queue length. Thus, while the throughput performance of the SGP algorithm is not sensitive to the channel rates, its delay performance may be significantly affected by the channel rates.



(a) Average data queue length (b) Average virtual queue length

Fig. 6: Impact of asymmetric channel statistics



(a) Average data queue length (b) Average virtual queue length

Fig. 7: Impact of asymmetric channel rates

IX. Conclusion

In this paper, we considered the distributed channel probing for opportunistic scheduling under heterogeneous allowable probing rate constraints. We first analyzed a basic scenario with symmetric arrivals and uniform allowable probing rate to express the maximum achievable throughput as a function of the allowable probing rate in symmetric and independent ON-OFF fading channels. This result not only indicates that almost the same opportunistic gains can be achieved with significant reductions in probing rates when the number of users is relatively large, but also points out that a simplistic randomized policy cannot achieve the full opportunistic gains.

Then, we characterized the capacity region under the heterogeneous probing constraints and provided the centralized throughput-optimal JPT algorithm. Realizing the operational difficulty of centralized solution, we put effort in developing a novel SGP algorithm based on the maximum-minimums identity, which is easy for distributed implementation. Also, we showed that the SGP algorithm is optimal in the crucial scenario of symmetric and independent ON-OFF fading channels. In the case of more general fading channels, we analyzed a more tractable variant of the SGP algorithm to obtain its efficient ratio as an explicit function of the channel statistics and rates and show that this ratio is tight in the symmetric and independent ON-OFF fading scenario. Finally, we discussed the distributed implementation of these greedy probing algorithms by using the FCSMA technique.

Appendix A
Proof for Proposition 1

Proof: To characterize the capacity region, similar to [23], [21], [14], [10], it is enough to consider a class of stationary randomized policies (see Lemma 1), where the probing decision in each slot is made randomly. Let R_i and θ_j be the rate that i^{th} user can achieve and the probability that j users probe the channel, respectively, where $i = 1, 2, ..., N$ and $j = 0, 1, ..., N$. Then, we can get the average probing rate as follows:

$$\frac{1}{N}\mathbb{E}\left[\sum_{i=1}^{N} X_i\right] = \frac{1}{N}\sum_{i=1}^{N} i\theta_i, \tag{28}$$

where we use the fact that $\sum_{i=1}^{N} X_i = j$ with probability of θ_j.

When j users probe the channel, by recalling our assumption that only probing users are allowed to transmit, we have $\sum_{i=1}^{N} R_i = 1 - (1-p)^j$. Thus, the average achievable rate can be expressed as follows:

$$\frac{1}{N}\mathbb{E}\left[\sum_{i=1}^{N} R_i\right] = \frac{1}{N}\sum_{i=1}^{N} \theta_i \left(1 - (1-p)^i\right). \tag{29}$$

We want to select a probability distribution $\{\theta_i\}_{i=0}^{N}$ such that the average achievable rate is maximized.

$$\max_{\theta=(\theta_i)_{i=1}^N} \quad \frac{1}{N}\sum_{i=1}^{N}\theta_i\left(1-(1-p)^i\right) \tag{30}$$

$$\text{Subject to} \quad \sum_{i=1}^{N}\theta_i \le 1 \tag{31}$$

$$\sum_{i=1}^{N} i\theta_i \le Nm \tag{32}$$

$$\theta_i \ge 0, \forall i = 1, ..., N, \tag{33}$$

where (31) is true since $\sum_{i=0}^{N}\theta_i = 1$ and $\theta_0 \ge 0$, and (32) holds since the average probing rate is not greater than m. By associating Lagrangian Multipliers $\mu_1 \ge 0$ and $\mu_2 \ge 0$ with constraints (31) and (32) respectively, we get the following partial Lagrangian function $L(\theta, \mu_1, \mu_2)$:

$$\begin{aligned} & L(\theta,\mu_1,\mu_2) \\ =& \frac{1}{N}\sum_{i=1}^{N}\theta_i\left(1-(1-p)^i\right) - \mu_1\left(\sum_{i=1}^{N}\theta_i - 1\right) - \mu_2\left(\sum_{i=1}^{N} i\theta_i - Nm\right) \\ =& \sum_{i=1}^{N}\left(\frac{1}{N}\left(1-(1-p)^i\right) - \mu_1 - \mu_2 i\right)\theta_i + \mu_1 + \mu_2 Nm. \end{aligned}$$

Then, the dual function $q(\mu_1, \mu_2)$ can be expressed as follows:

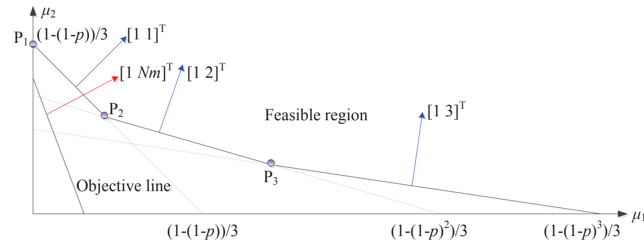
$$\begin{aligned} q(\mu_1,\mu_2) &= \sup_{\theta\ge 0} L(\theta,\mu_1,\mu_2) \\ &= \begin{cases} \mu_1 + \mu_2 Nm & \text{, if } \frac{1}{N}\left(1-(1-p)^i\right) \le \mu_1 + \mu_2 i \\ & \quad \forall i = 1, ..., N; \\ +\infty & \text{, otherwise.} \end{cases} \end{aligned}$$

Since the original optimization problem is just a linear programming, there is no duality gap and thus it is equivalent to solve the following dual problem:

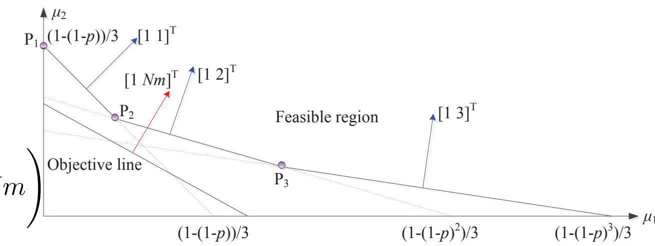
$$\min_{\mu_1\ge 0, \mu_2 \ge 0} \quad \mu_1 + \mu_2 Nm \tag{34}$$

$$\text{Subject to} \quad \mu_1 + \mu_2 i \ge \frac{1}{N}\left(1-(1-p)^i\right), \forall i = 1, ..., N.$$

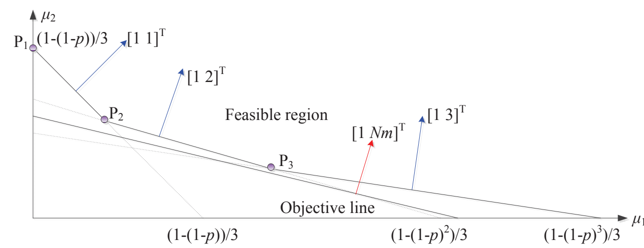
Since the objective function and constraint function are linear functions representing lines in $\mathbb{R}^2$, we call the objective function and constraint function as the objective line and constraint line respectively. Note that the normal vector of the objective line is $[1, Nm]^T$ and the normal vector of the constraint line i is $[1, i]^T$, where the notation $\mathbf{a} = [a_1, a_2]$ represents a vector with the first and second components being a_1 and a_2, respectively, and $\mathbf{a}^T$ denotes the transpose of the vector $\mathbf{a}$. If $0 \le Nm \le 1$, by the optimality condition, the optimal objective line should pass the point $(0, \frac{p}{N})$, and thus the maximum achievable rate is $0 + \frac{p}{N}Nm = mp$, see figure 8a for the case when $N = 3$; if $i \le Nm \le i+1$ ($i = 1, ..., N-1$), the optimal objective line should pass the intersection point of two constraint lines $\mu_1 + \mu_2 i = \frac{1}{N}\left(1-(1-p)^i\right)$ and $\mu_1 + \mu_2(i+1) = \frac{1}{N}\left(1-(1-p)^{i+1}\right)$, which is $\left(\frac{1-(1+ip)(1-p)^i}{N}, \frac{p(1-p)^i}{N}\right)$, and hence the maximum achievable rate $i\frac{1-(1+ip)(1-p)^i}{N} + \frac{p(1-p)^i}{N}Nm = \frac{1}{N} + \left(m - \frac{i}{N}\right)p(1-p)^i - \frac{1}{N}(1-p)^i$, see figures 8b and 8c for the case when $N = 3$. ■



(a) $0 \le Nm \le 1$: the optimal objective line should pass point P_1



(b) $1 \le Nm \le 2$: the optimal objective line should pass point P_2



(c) $2 \le Nm \le 3$: the optimal objective line should pass point P_3

Fig. 8: The optimal solution when $N = 3$

Appendix B
Proof for Lemma 1

Proof: (1) (Necessity) Suppose all data queues are strongly stable and each user satisfies its allowable probing rate constraint under some policy Φ which determines the probing schedule $\mathbf{X}[t]$ and the transmission schedule $\mathbf{S}[t]$ in every slot t. For some positive integer number M, we define $\overline{\mu}_i(M) \triangleq \frac{1}{M}\sum_{\tau=1}^{M} X_i(\tau)C_i(\tau)S_i(\tau)$ and $\overline{p}_i(M) \triangleq \frac{1}{M}X_i(\tau)$ as the empirical average service rate and probing rate for user i, respectively.

Let $\mathbf{T}_M^{\mathbf{x}}$ be the set of slots in $[1, M]$ in which the probing schedule is $\mathbf{x}$, and $\mathbf{T}_M^{\mathbf{x}}(\mathbf{c})$ be the set of slots in $\mathbf{T}_M^{\mathbf{x}}$ in which the channel state vector is $\mathbf{c}$. First, we consider the empirical average service rate $\overline{\mu}_i(M)$.

$$\begin{aligned}\overline{\mu}_i(M) =&\frac{1}{M}\sum_{\tau=1}^{M} X_i(\tau)C_i(\tau)S_i(\tau)\\ =&\frac{1}{M}\sum_{\mathbf{x}\in\mathcal{X}}\sum_{\mathbf{c}}\sum_{\tau\in\mathbf{T}_M^{\mathbf{x}}(\mathbf{c})} x_i c_i S_i(\tau)\\ =&\sum_{\mathbf{x}\in\mathcal{X}}\frac{|\mathbf{T}_M^{\mathbf{x}}|}{M}\sum_{\mathbf{c}}\frac{|\mathbf{T}_M^{\mathbf{x}}(\mathbf{c})|}{|\mathbf{T}_M^{\mathbf{x}}|}\cdot\frac{1}{|\mathbf{T}_M^{\mathbf{x}}(\mathbf{c})|}\sum_{\tau\in\mathbf{T}_M^{\mathbf{x}}(\mathbf{c})} x_i c_i S_i(\tau)\\ =&\sum_{\mathbf{x}\in\mathcal{X}}\alpha_M(\mathbf{x})\sum_{\mathbf{c}}\sigma_M(\mathbf{c})y_M(\mathbf{x},\mathbf{c}),\end{aligned} \tag{35}$$

where $\alpha_M(\mathbf{x}) \triangleq \frac{|\mathbf{T}_M^{\mathbf{x}}|}{M}$, $\sigma_M(\mathbf{c}) \triangleq \frac{|\mathbf{T}_M^{\mathbf{x}}(\mathbf{c})|}{|\mathbf{T}_M^{\mathbf{x}}|}$ and $y_M(\mathbf{x},\mathbf{c}) \triangleq \frac{1}{|\mathbf{T}_M^{\mathbf{x}}(\mathbf{c})|}\sum_{\tau\in\mathbf{T}_M^{\mathbf{x}}(\mathbf{c})} x_i c_i S_i(\tau)$.

Observe that $y_M(\mathbf{x},\mathbf{c})$ is a convex combination of the set $\{0, x_i c_i\}$. By Caratheodory's theorem, there exists a non-negative real sequence $\{\beta_M(\mathbf{x},\mathbf{c};\mathbf{s})\}_{\mathbf{s}\in\mathcal{S}}$ with $\sum_{\mathbf{s}\in\mathcal{S}}\beta_M(\mathbf{x},\mathbf{c};\mathbf{s}) = 1$, such that $y_M(\mathbf{x},\mathbf{c})$ can be rewritten as

$$y_M(\mathbf{x},\mathbf{c}) = \sum_{\mathbf{s}\in\mathcal{S}}\beta_M(\mathbf{x},\mathbf{c};\mathbf{s})x_i c_i s_i. \tag{36}$$

Hence, we have

$$\overline{\mu}_i(M) = \sum_{\mathbf{x}\in\mathcal{X}}\alpha_M(\mathbf{x})\sum_{\mathbf{c}}\sigma_M(\mathbf{c})\sum_{\mathbf{s}\in\mathcal{S}}\beta_M(\mathbf{x},\mathbf{c};\mathbf{s})x_i c_i s_i. \tag{37}$$

Next, we consider the empirical average probing rate $\overline{p}_i(M)$.

$$\begin{aligned}\overline{p}_i(M) =&\frac{1}{M}\sum_{\tau=1}^{M} X_i(\tau)\\ =&\frac{1}{M}\sum_{\mathbf{x}\in\mathcal{X}}\sum_{\tau\in\mathbf{T}_M^{\mathbf{x}}} x_i\\ =&\sum_{\mathbf{x}\in\mathcal{X}}\frac{|\mathbf{T}_M^{\mathbf{x}}|}{M}x_i\\ =&\sum_{\mathbf{x}\in\mathcal{X}}\alpha_M(\mathbf{x})x_i.\end{aligned} \tag{38}$$

For each positive integer number M, the number of $\alpha_M(\mathbf{x})$ and $\beta_M(\mathbf{x},\mathbf{c};\mathbf{s})$ is bounded. By compactness, we can find a subsequence of integers $\{M_k\}$ such that $M_k \to \infty$, and such that there exist limiting probabilities $\alpha(\mathbf{x})$ and $\beta(\mathbf{x},\mathbf{c};\mathbf{s})$ satisfying:

$$\alpha_{M_k}(\mathbf{x}) \to \alpha(\mathbf{x}), \forall\mathbf{x}\in\mathcal{X},$$
$$\beta_{M_k}(\mathbf{x},\mathbf{c};\mathbf{s}) \to \beta(\mathbf{x},\mathbf{c},\mathbf{s}), \forall\mathbf{x}\in\mathcal{X},\mathbf{c}.$$

In addition, channel states are i.i.d. over time, we have

$$\sigma_{M_k}(\mathbf{c}) \to \Pr\{\mathbf{C}[t] = \mathbf{c}\}. \tag{39}$$

Hence, the sequences $\{\overline{\mu}_i(M_k)\}$ and $\{\overline{p}_i(M_k)\}$ converge to

$$\sum_{\mathbf{x}\in\mathcal{X}}\alpha(\mathbf{x})\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\sum_{\mathbf{s}\in\mathcal{S}}x_i c_i s_i$$

and $\sum_{\mathbf{x}\in\mathcal{X}}\alpha(\mathbf{x})x_i$, respectively.

Since the policy Φ makes all data queues strongly stable, by Lemma 1 in [15], the arrival rate to each queue should be no greater than its service rate, i.e.,

$$\lambda_i \le \sum_{\mathbf{x}\in\mathcal{X}}\alpha(\mathbf{x})\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\sum_{\mathbf{s}\in\mathcal{S}}x_i c_i s_i. \tag{40}$$

Also, each user satisfies the allowable probing rate constraint under policy Φ, which implies that

$$\sum_{\mathbf{x}\in\mathcal{X}}\alpha(\mathbf{x})x_i \le m_i. \tag{41}$$

(2) (Sufficiency) We will show that any arrival rate vector λ strictly inside $\Lambda(\mathbf{m},\mathbf{C})$ can be supported by a simple randomized probing and transmission policy that selects probing schedule $\mathbf{X}$ with probability $\alpha(\mathbf{X})$ and chooses transmission schedule $\mathbf{S}$ with probability $\beta(\mathbf{X},\mathbf{C};\mathbf{S})$ at each slot. First, we should note that the average probing rate of each user under this policy is not greater than its allowable probing rate, since $\sum_{\mathbf{x}}\alpha(\mathbf{x})x_i \le m_i, \forall i$. Next, we will show that all data queues are strongly stable under this policy.

Consider the Lyapunov function $V[t] \triangleq V(\mathbf{Q}[t]) = \frac{1}{2}\sum_{i=1}^{N}Q_i^2[t]$. Then, we have

$$\begin{aligned}\Delta V \triangleq& \mathbb{E}[V[t+1]-V[t]|\mathbf{Q}[t]=\mathbf{Q}]\\ \le&\frac{1}{2}\sum_{i=1}^{N}\mathbb{E}[(Q_i[t]+A_i[t]-X_i[t]S_i[t]C_i[t])^2 - Q_i^2[t]|\mathbf{Q}[t]]\\ =&\sum_{i=1}^{N}\mathbb{E}\left[Q_i[t](A_i[t]-X_i[t]S_i[t]C_i[t])|\mathbf{Q}[t]\right]+B_1,\end{aligned} \tag{42}$$

where

$$\begin{aligned}B_1 \triangleq&\frac{1}{2}\sum_{i=1}^{N}\mathbb{E}[(A_i[t]-X_i[t]S_i[t]C_i[t])^2|\mathbf{Q}[t]]\\ \le&\frac{1}{2}\sum_{i=1}^{N}\mathbb{E}[A_i^2[t]+C_i^2[t]]\\ \le&\frac{1}{2}N(A_{\max}+c_M^2)\triangleq B_{1,\max}.\end{aligned}$$

Thus, we have

$$\Delta V \le B_{1,\max} + \sum_{i=1}^{N}Q_i\left(\lambda_i - \sum_{\mathbf{x}}\alpha(\mathbf{x})\sum_{\mathbf{c}}P(\mathbf{C}[t]=\mathbf{c})\sum_{\mathbf{s}\in\mathbf{S}}\beta(\mathbf{x},\mathbf{c};\mathbf{s})x_i c_i s_i\right).$$

Since $\boldsymbol{\lambda}$ is strictly within $\Lambda(\mathbf{m},\mathbf{C})$, there exists a $\epsilon > 0$ such that

$$\lambda_i \le \sum_{\mathbf{x}}\alpha(\mathbf{x})\sum_{\mathbf{c}}P(\mathbf{C}[t]=\mathbf{c})\sum_{\mathbf{s}\in\mathbf{S}}\beta(\mathbf{x},\mathbf{c};\mathbf{s})x_i c_i s_i - \epsilon, \forall i.$$

Then, by using the above inequality, ΔV becomes

$$\Delta V \le -\epsilon\sum_{i=1}^{N}Q_i + B_{1,\max}. \tag{43}$$

By using Theorem 4.1 in [13], all data queues are strongly stable. ■

APPENDIX C
PROOF FOR LEMMA 2

(1) (Necessity) For any $\lambda \in \Lambda(\mathbf{m}, \mathbf{C})$, there exist non-negative numbers $\alpha(\mathbf{x})$ and $\beta(\mathbf{x}, \mathbf{c}; \mathbf{s})$ satisfying (3), (4), (5) and (6). Thus, for any set of users $\mathbf{A} \subseteq \mathbf{N}$, we have

$$\sum_{i\in\mathbf{A}} \lambda_i \leq \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} P(\mathbf{C}[t]=\mathbf{c}) \sum_{\mathbf{s}\in\mathbf{S}} \beta(\mathbf{x},\mathbf{c};\mathbf{s}) \sum_{i\in\mathbf{A}} x_i c_i s_i. \quad (44)$$

For any given $\mathbf{x}$ and $\mathbf{c}$, since at most one user can be scheduled at each slot, we have

$$\sum_{\mathbf{s}\in\mathbf{S}} \beta(\mathbf{x},\mathbf{c};\mathbf{s}) \sum_{i\in\mathbf{A}} x_i c_i s_i \begin{cases} =0, & x_i c_i = 0, \forall i \in \mathbf{A}; \\ \leq 1, & \text{otherwise.} \end{cases} \quad (45)$$

By substituting (45) into (44), we get (7).

(2) (Sufficiency) Since $\lambda \in \Gamma(\mathbf{m}, \mathbf{C})$, there exists non-negative numbers $\alpha(\mathbf{x})$ satisfying (7), (8) and (9). Consider the following policy: in each slot t, during the probing stage, it selects the probing schedule $\mathbf{X}^R[t]$ with probability $\alpha(\mathbf{X}^R[t])$; during the transmission stage, it selects user i^* satisfying $i^* \in \arg\max_i Q_i[t] X_i^R[t] C_i[t]$. We first note that each user satisfies the average probing constraint under this policy. Next, we will show that this policy can makes all data queues strongly stable for any arrival rate vector $\boldsymbol{\lambda}$ strictly inside $\Gamma(\mathbf{m}, \mathbf{C})$.

The following proof is similar to that in [23]. By choosing the same Lyapunov function and following the same argument as in Lemma 1, we have

$$\Delta V \leq \sum_{i=1}^{N} \lambda_i Q_i - \mathbb{E}\left[\sum_{i=1}^{N} Q_i[t] X_i^R[t] S_i^*[t] C_i[t] | \mathbf{Q}[t]\right] + B_{1,\max}.$$

Next, let's focus on the term $\mathbb{E}\left[\sum_{i=1}^{N} Q_i[t] X_i^R[t] S_i^*[t] C_i[t] | \mathbf{Q}[t]\right]$. Consider a permutation e_i, $i = 1, ..., N$ of the integers 1 to N which is such that $Q_{e_i} \geq Q_{e_{i-1}}$, for $i = 2, ..., N$, and if $Q_{e_i} = Q_{e_{i-1}}$ then $e_i > e_{i-1}$. For any given $\mathbf{x}$ and $\mathbf{c}$, we define the following sets:

$$\mathcal{R}_0 \triangleq \{x_i c_i = 0, \forall i = 1, 2, ..., N\}; \quad (46)$$

$$\mathcal{R}_i \triangleq \{x_{e_i} c_{e_i} = 1, x_{e_j} c_{e_j} = 0 \text{ for } N \geq j > i\} \text{ for } i = 1, 2, ..., N-1; \quad (47)$$

$$\mathcal{R}_N \triangleq \{x_{e_N} c_{e_N} = 1\}; \quad (48)$$

$$\mathcal{T}_i \triangleq \{x_{e_j} c_{e_j} = 0, \text{ for } N \geq j \geq i\}, \forall i = 1, 2, ..., N. \quad (49)$$

Thus, we have

$$\begin{aligned} &\mathbb{E}\left[\sum_{i=1}^{N} Q_i[t] X_i^R[t] S_i^*[t] C_i[t] | \mathbf{Q}[t] = \mathbf{Q}\right] \\ =& \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t]=\mathbf{c}\} \sum_{i=1}^{N} x_i c_i Q_i S_i^*[t] \\ =& \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t]=\mathbf{c}\} \sum_{j=1}^{N} \sum_{i=1}^{N} x_i c_i Q_i S_i^*[t] \mathbb{1}_{\mathcal{R}_j}. \quad (50) \end{aligned}$$

Since

$$\sum_{i=1}^{N} x_i c_i Q_i S_i^*[t] \mathbb{1}_{\mathcal{R}_j} = Q_{e_j}, \quad (51)$$

we have

$$\begin{aligned} &\mathbb{E}\left[\sum_{i=1}^{N} Q_i[t] X_i^R[t] S_i^*[t] C_i[t] | \mathbf{Q}[t]\right] \\ =& \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t]=\mathbf{c}\} \sum_{i=1}^{N} Q_{e_i} \mathbb{1}_{\mathcal{R}_i}. \quad (52) \end{aligned}$$

Observe that

$$\sum_{i=1}^{N} Q_{e_i} \mathbb{1}_{\mathcal{R}_i} = Q_{e_1}(1-\mathbb{1}_{\mathcal{T}_1}) + \sum_{j=2}^{N} (Q_{e_j} - Q_{e_{j-1}})(1-\mathbb{1}_{\mathcal{T}_j}),$$

where we use facts that $\mathbb{1}_{\mathcal{T}_{i+1}} = \mathbb{1}_{\mathcal{T}_i} + \mathbb{1}_{\mathcal{R}_i}, \forall i = 1, 2, ..., N-1$, and $1 = \mathbb{1}_{\mathcal{T}_N} + \mathbb{1}_{\mathcal{R}_N}$. Thus, we have

$$\begin{aligned} &\mathbb{E}\left[\sum_{i=1}^{N} Q_i[t] X_i^R[t] S_i^*[t] C_i[t] | \mathbf{Q}[t]\right] \\ =& Q_{e_1}\left(1 - \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t]=\mathbf{c}\} \mathbb{1}_{\mathcal{T}_1}\right) \\ &+ \sum_{j=2}^{N} (Q_{e_j} - Q_{e_{j-1}}) \left(1 - \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t]=\mathbf{c}\} \mathbb{1}_{\mathcal{T}_j}\right). \end{aligned}$$

Since

$$\sum_{i=1}^{N} \lambda_i Q_i = \sum_{i=1}^{N} \lambda_{e_i} Q_{e_i} = Q_{e_1} \sum_{i=1}^{N} \lambda_{e_i} + \sum_{j=2}^{N} (Q_{e_j} - Q_{e_{j-1}}) \sum_{i=j}^{N} \lambda_{e_i},$$

ΔV becomes

$$\begin{aligned} \Delta V \leq& B_{1,\max} + Q_{e_1}\left(\sum_{i=1}^{N} \lambda_{e_i} - 1 + \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t]=\mathbf{c}\} \mathbb{1}_{\mathcal{T}_1}\right) \\ &+ \sum_{j=2}^{N} (Q_{e_j} - Q_{e_{j-1}}) \left(\sum_{i=j}^{N} \lambda_{e_i} - 1 + \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t]=\mathbf{c}\} \mathbb{1}_{\mathcal{T}_j}\right) \end{aligned}$$

We define

$$\zeta \triangleq \min_{\mathbf{A} \subseteq \mathbf{N}} \left\{1 - \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t]=\mathbf{c}\} \mathbb{1}_{\{x_i c_i = 0, \forall i \in \mathbf{A}\}} - \sum_{i\in\mathbf{A}} \lambda_i\right\}.$$

Since the arrival rate vector $\boldsymbol{\lambda}$ is strictly inside the region $\Gamma(\mathbf{m}, \mathbf{C})$, we have $\zeta > 0$. Thus, we have

$$\begin{aligned} \Delta V &\leq B_{1,\max} - Q_{e_1}\zeta - \zeta \sum_{j=2}^{N} (Q_{e_j} - Q_{e_{j-1}}) \\ &= B_{1,\max} - \zeta Q_{e_N} \\ &\leq B_{1,\max} - \frac{\zeta}{N} \sum_{i=1}^{N} Q_i. \end{aligned}$$

By using Theorem 4.1 in [13], all data queues are strongly stable.

Appendix D
Proof for Proposition 2

Proof: Consider the Lyapunov function

$$L[t] \triangleq L(\mathbf{Q}[t], \mathbf{U}[t]) = \frac{1}{2}\sum_{i=1}^{N}(Q_i^2[t] + U_i^2[t]). \quad (53)$$

Then, we have

$$\begin{aligned}
&\Delta L \triangleq \mathbb{E}[L[t+1] - L[t] | \mathbf{Q}[t] = \mathbf{Q}, \mathbf{U}[t] = \mathbf{U}] \\
\leq & \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}[(Q_i[t] + A_i[t] - X_i^*[t]S_i^*[t]C_i[t])^2 - Q_i^2[t] | \mathbf{Q}[t], \mathbf{U}[t]] \\
&+ \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}[(U_i[t] + X_i^*[t] - I_i[t])^2 - U_i^2[t] | \mathbf{Q}[t], \mathbf{U}[t]] \\
= & \sum_{i=1}^{N}\mathbb{E}\left[Q_i[t](A_i[t] - X_i^*[t]S_i^*[t]C_i[t]) | \mathbf{Q}[t], \mathbf{U}[t]\right] \\
&+ \sum_{i=1}^{N}\mathbb{E}\left[U_i[t](X_i^*[t] - I_i[t]) | \mathbf{Q}[t], \mathbf{U}[t]\right] + B_2, \quad (54)
\end{aligned}$$

where

$$\begin{aligned}
B_2 \triangleq & \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}[(A_i[t] - X_i^*[t]S_i^*[t]C_i[t])^2 | \mathbf{Q}[t], \mathbf{U}[t]] \\
&+ \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}[(X_i^*[t] - I_i[t])^2 | \mathbf{Q}[t], \mathbf{U}[t]] \\
\leq & \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}[A_i^2[t] + C_i^2[t] + I_i^2[t] + 1] \\
\leq & \frac{1}{2}N(A_{\max} + I_{\max} + c_M^2 + 1) \triangleq B_{2,\max}.
\end{aligned}$$

Hence, we have

$$\begin{aligned}
\Delta L \leq & \sum_{i=1}^{N}\lambda_i Q_i - \sum_{i=1}^{N} m_i U_i + B_{2,\max} \\
&- \sum_{i=1}^{N}\mathbb{E}\left[Q_i[t]X_i^*[t]S_i^*[t]C_i[t] - U_i[t]X_i^*[t] | \mathbf{Q}[t], \mathbf{U}[t]\right] \\
= & \sum_{i=1}^{N}\lambda_i Q_i - \sum_{i=1}^{N} m_i U_i + B_{2,\max} \\
&- \mathbb{E}\left[\max_i Q_i[t]X_i^*[t]C_i[t] - \sum_{i=1}^{N} U_i[t]X_i^*[t] | \mathbf{Q}[t], \mathbf{U}[t]\right].
\end{aligned}$$

Since $\boldsymbol{\lambda} \in \text{Int}(\Lambda(\mathbf{m}, \mathbf{C}))$, there exists a $\epsilon > 0$ such that

$$\lambda_i \leq \sum_{\mathbf{x}}\alpha(\mathbf{x})\sum_{\mathbf{c}}P(\mathbf{C}[t] = \mathbf{c})\sum_{\mathbf{s}\in\mathbf{S}}\beta(\mathbf{x}, \mathbf{c}; \mathbf{s})x_i c_i s_i - \epsilon, \forall i.$$

Then, by using the above inequality, ΔL becomes

$$\begin{aligned}
\Delta L \leq & -\epsilon\sum_{i=1}^{N}Q_i + \sum_{i=1}^{N}U_i\left(\sum_{\mathbf{x}}\alpha(\mathbf{x})x_i - m_i\right) + B_{2,\max} \\
&+ \sum_{\mathbf{x}}\alpha(\mathbf{x})\left(\sum_{\mathbf{c}}P(\mathbf{C}[t] = \mathbf{c})\sum_{\mathbf{s}\in\mathbf{S}}\beta(\mathbf{x}, \mathbf{c}; \mathbf{s})\sum_{i=1}^{N}x_i c_i s_i Q_i - \sum_{i=1}^{N}U_i x_i\right) \\
&- \mathbb{E}\left[\max_i Q_i[t]X_i^*[t]C_i[t] - \sum_{i=1}^{N}U_i[t]X_i^*[t] | \mathbf{Q}[t], \mathbf{U}[t]\right]. \quad (55)
\end{aligned}$$

By using (6), we have

$$\begin{aligned}
\Delta L \leq & -\epsilon\sum_{i=1}^{N}Q_i + B_{2,\max} \\
&+ \sum_{\mathbf{x}}\alpha(\mathbf{x})\left(\sum_{\mathbf{c}}P(\mathbf{C}[t] = \mathbf{c})\max_i x_i c_i Q_i - \sum_{i=1}^{N}U_i x_i\right) \\
&- \mathbb{E}\left[\max_i Q_i[t]X_i^*[t]C_i[t] - \sum_{i=1}^{N}U_i[t]X_i^*[t] | \mathbf{Q}[t], \mathbf{U}[t]\right] \\
= & -\epsilon\sum_{i=1}^{N}Q_i + B_{2,\max} \\
&+ \sum_{\mathbf{x}}\alpha(\mathbf{x})\mathbb{E}\left[\max_i x_i C_i[t]Q_i[t] - \sum_{i=1}^{N}U_i[t]x_i | \mathbf{Q}[t], \mathbf{U}[t]\right] \\
&- \sum_{\mathbf{x}}\alpha(\mathbf{x})\mathbb{E}\left[\max_i Q_i[t]X_i^*[t]C_i[t] - \sum_{i=1}^{N}U_i[t]X_i^*[t] | \mathbf{Q}[t], \mathbf{U}[t]\right] \\
\leq & -\epsilon\sum_{i=1}^{N}Q_i + B_{2,\max}. \quad (56)
\end{aligned}$$

By using Theorem 4.1 in [13], all data queues are strongly stable and all virtual queues are mean rate stable. ∎

Appendix E
Some properties for function $f(\mathbf{E}, e)$

Lemma 7:

$$\max_{1\leq i\leq n}\{\min\{x_i, y\}\} = \min\{\max_{1\leq i\leq n} x_i, y\}. \quad (57)$$

Proof: (i) If $n = 1$, LHS $= \min\{x_1, y\} =$ RHS.
(ii) Assume (57) is true for all $n \leq k$. Then for $n = k+1$, we have

$$\begin{aligned}
&\max_{1\leq i\leq k+1}\{\min\{x_i, y\}\} \\
=& \max\{\max_{1\leq i\leq k}\{\min\{x_i, y\}\}, \min\{x_{k+1}, y\}\} \\
=& \max\{\min\{\max_{1\leq i\leq k} x_i, y\}, \min\{x_{k+1}, y\}\} (\text{by assumption}(n = k)) \\
=& \min\{\max_{1\leq i\leq k+1} x_i, y\} (\text{by assumption}(n = 2)). \quad (58)
\end{aligned}$$

∎

Remarks: We can use similar argument to show that $\min_{1\leq i\leq n}\{\max\{x_i, y\}\} = \max\{\min_{1\leq i\leq n} x_i, y\}$.

Consider a set $\mathbf{E}$ of users and $e \notin \mathbf{E}$ over a symmetric ON-OFF fading channel with $\Pr\{C_i = 1\} = p, \forall i$. We assume that there are K users in $\mathbf{E}$ whose queue lengths are less than or equal to Q_e. Without loss of generality, we assume that $Q_1 \leq Q_2 \leq ... \leq Q_K \leq Q_e \leq Q_{K+1} \leq ... \leq Q_{|\mathbf{E}|}$. We denote $\mathbf{E}_1 \triangleq \{1,2,...,K\}$ and $\mathbf{E}_2 \triangleq \{K+1, K+2, ..., |\mathbf{E}|\}$. Let $\mathcal{H}$ be the event that at least one users in $\mathbf{E_2}$ have the available channel. Let $\mathcal{I}_i$ be the event that $C_i = 1, C_j = 0$ for $K \geq j > i$, $i = 1,2,...,K-1$, and $\mathcal{I}_K$ be the event that $C_K = 1$.

Lemma 8:

$$\max_{l\in\mathbf{E}} \min\{Q_l C_l, Q_e\} = \begin{cases} Q_e & \text{, if } \mathcal{H} \text{ happens;} \\ Q_i & \text{, if } \mathcal{H}^c \bigcap \mathcal{I}_i \text{ happens,} \\ & \text{ for } i = 1,2,...,K. \end{cases} \tag{59}$$

Proof:

$$\begin{aligned} &\max_{l\in\mathbf{E}} \min\{Q_l C_l, Q_e\} \\ =& \max\left(\max_{l\in\mathbf{E_1}} \min\{Q_l C_l, Q_e\}, \max_{l\in\mathbf{E_2}} \min\{Q_l C_l, Q_e\}\right) \\ =& \max_{l\in\mathbf{E_1}} \min\{Q_l C_l, Q_e\} + \max_{l\in\mathbf{E_2}} \min\{Q_l C_l, Q_e\} \\ &- \min\left(\max_{l\in\mathbf{E_1}} \min\{Q_l C_l, Q_e\}, \max_{l\in\mathbf{E_2}} \min\{Q_l C_l, Q_e\}\right) \end{aligned} \tag{60}$$

Note that

$$\max_{l\in\mathbf{E_2}} \min\{Q_l C_l, Q_e\} = \begin{cases} Q_e & \text{, if event } \mathcal{H} \text{ happens;} \\ 0 & \text{, otherwise.} \end{cases} \tag{61}$$

Thus,

$$\begin{aligned} &\min\left(\max_{l\in\mathbf{E_1}} \min\{Q_l C_l, Q_e\}, \max_{l\in\mathbf{E_2}} \min\{Q_l C_l, Q_e\}\right) \\ =& \begin{cases} \min\left(\max_{l\in\mathbf{E_1}} \min\{Q_l C_l, Q_e\}, Q_e\right) & \text{, if event } \mathcal{H} \text{ happens;} \\ 0 & \text{, otherwise} \end{cases} \\ =& \begin{cases} \max_{l\in\mathbf{E_1}} \min\{Q_l C_l, Q_e\} & \text{, if event } \mathcal{H} \text{ happens;} \\ 0 & \text{, otherwise.} \end{cases} \end{aligned} \tag{62}$$

where we use Lemma 7. In addition, if event $\mathcal{I}_i$ ($i = 1,2,...,K$) happens, we have

$$\max_{l\in\mathbf{E_1}} \min\{Q_l C_l, Q_e\} = Q_i \tag{63}$$

By substituting (61), (62) and (63) into (60), we have (59). ■

Corollary 4:

$$f(\mathbf{E}, e) = \sum_{k=1}^{K} p^2 (1-p)^{|\mathbf{E}|-k} Q_k + p(1-(1-p)^{|\mathbf{E}|-K}) Q_e \tag{64}$$

Proof: $\Pr\{\mathcal{H}\} = 1-(1-p)^{|\mathbf{E}|-K}$ and $\Pr\{\mathcal{H}^c \bigcap \mathcal{I}_i\} = p(1-p)^{|\mathbf{E}|-i}$ for $i = 1,2,...,K$. Thus,

$$\begin{aligned} f(\mathbf{E}, e) \triangleq& \mathbb{E}[\max_{l\in\mathbf{E}} \min\{Q_l C_l, Q_e C_e\}] \\ =& p\mathbb{E}[\max_{l\in\mathbf{E}} \min\{Q_l C_l, Q_e\}] \\ =& \sum_{k=1}^{K} p^2 (1-p)^{|\mathbf{E}|-k} Q_k + p(1-(1-p)^{|\mathbf{E}|-K}) Q_e, \end{aligned}$$

where we use Lemma 8. ■

Appendix F
Proof for basic iterative equation

According to the maximum-minimums identity, we have

$$\begin{aligned} &\max_{i\in\mathbf{F}\bigcup\{r\}} Q_i C_i = \max\{\max_{i\in\mathbf{F}} Q_i C_i, Q_r C_r\} \\ &= \max_{i\in\mathbf{F}} Q_i C_i + Q_r C_r - \min\{\max_{i\in\mathbf{F}} Q_i C_i, Q_r C_r\} \\ &= \max_{i\in\mathbf{F}} Q_i C_i + Q_r C_r - \max_{i\in\mathbf{F}} \min\{Q_i C_i, Q_r C_r\}, \end{aligned} \tag{65}$$

where we use Lemma 7. By taking expectation and subtracting the term $\sum_{i\in\mathbf{F}\bigcup\{r\}} U_i$ on both sides of (65), we get (17).

Appendix G
Proof for $f(\mathbf{D}^*, d) \leq f(\mathbf{B}, d)$ in Lemma 3

Proof: (1) If $\mathbf{A}^* \bigcap \mathbf{D}^* = \emptyset$ or $Q_d \leq \min_{i\in\mathbf{B}} Q_i$, then, by Corollary 4, we have

$$f(\mathbf{B}, d) = p\left(1-(1-p)^{|\mathbf{B}|}\right) Q_d. \tag{66}$$

Without loss of generality, we assume there are K_1 users in $\mathbf{D}^*$ whose queue lengths are less than or equal to Q_d, that is, $Q_{j_1} \leq Q_{j_2} \leq ... \leq Q_{j_{K_1}} \leq Q_d \leq Q_{j_{K_1+1}} \leq Q_{j_{|\mathbf{D}^*|}}$. Then, by Corollary 4, we have

$$f(\mathbf{D}^*, d) = \sum_{k=1}^{K_1} p^2 (1-p)^{|\mathbf{D}^*|-k} Q_{j_k} + p\left(1-(1-p)^{|\mathbf{D}^*|-K_1}\right) Q_d.$$

Hence, by noting that $|\mathbf{D}^*| = |\mathbf{B}|$, we have

$$\begin{aligned} &f(\mathbf{D}^*, d) - f(\mathbf{B}, d) \\ =& \sum_{k=1}^{K_1} p^2 (1-p)^{|\mathbf{D}^*|-k} Q_{j_k} + p\left((1-p)^{|\mathbf{D}^*|} - (1-p)^{|\mathbf{D}^*|-K_1}\right) Q_d. \end{aligned}$$

Since

$$-\sum_{k=1}^{K_1} p^2 (1-p)^{|\mathbf{D}^*|-k} = p\left((1-p)^{|\mathbf{D}^*|} - (1-p)^{|\mathbf{D}^*|-K_1}\right),$$

we have

$$\begin{aligned} &f(\mathbf{D}^*, d) - f(\mathbf{B}, d) \\ =& \sum_{k=1}^{K_1} p^2 (1-p)^{|\mathbf{D}^*|-k} \left(Q_{j_k} - Q_d\right) \leq 0. \end{aligned} \tag{67}$$

Thus, we have $f(\mathbf{D}^*, d) \leq f(\mathbf{B}, d)$.

(2) If $\mathbf{A}^* \bigcap \mathbf{D}^* \neq \emptyset$ and there are some users in $\mathbf{A}^* \bigcap \mathbf{D}^*$ whose queue lengths are less than Q_d, let $\mathbf{T} \triangleq \mathbf{A}^* \bigcap \mathbf{D}^*$, $\mathbf{B'} \triangleq \mathbf{B} \setminus \mathbf{T}$ and $\mathbf{D'} \triangleq \mathbf{D}^* \setminus \mathbf{T}$. Figure 9 characterizes the relationship among all these sets.

We define

$$\begin{aligned} &g(\mathbf{E}, \mathbf{F}, e) \triangleq \\ &-\mathbb{E}\left[\min\left(\max_{l\in\mathbf{E}} \min\{Q_l C_l, Q_e C_e\}, \max_{l\in\mathbf{F}} \min\{Q_l C_l, Q_e C_e\}\right)\right], \end{aligned}$$

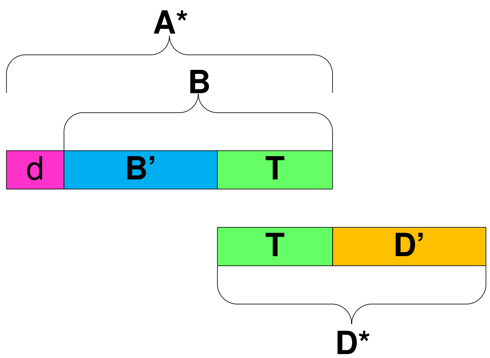


Fig. 9: The relations among all sets

where $\mathbf{E}\bigcap\mathbf{F}=\emptyset$ and $e\notin\mathbf{E}, e\notin\mathbf{F}$. Then, we have

$$\begin{aligned}
f(\mathbf{B},d) &= \mathbb{E}\left[\max_{l\in\mathbf{B}}\min\{Q_lC_l,Q_dC_d\}\right]\\
=&\mathbb{E}\left[\max\left(\max_{l\in\mathbf{B}'}\min\{Q_lC_l,Q_dC_d\},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_dC_d\}\right)\right]\\
=&\mathbb{E}\left[\max_{l\in\mathbf{B}'}\min\{Q_lC_l,Q_dC_d\}\right]+\mathbb{E}\left[\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_dC_d\}\right]\\
-&\mathbb{E}\left[\min\left(\max_{l\in\mathbf{B}'}\min\{Q_lC_l,Q_dC_d\},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_dC_d\}\right)\right]\\
=&f(\mathbf{B}',d)+f(\mathbf{T},d)+g(\mathbf{B}',\mathbf{T},d), \qquad (68)
\end{aligned}$$

where we use the maximum-minimums identity. Similarly, we have

$$f(\mathbf{D}^*,d)=f(\mathbf{D}',d)+f(\mathbf{T},d)+g(\mathbf{D}',\mathbf{T},d). \qquad (69)$$

Thus, to show $f(\mathbf{D}^*,d)\le f(\mathbf{B},d)$, we only need to show

$$f(\mathbf{D}',d)+g(\mathbf{D}',\mathbf{T},d)\le f(\mathbf{B}',d)+g(\mathbf{B}',\mathbf{T},d). \qquad (70)$$

Note that $Q_d\le\min_{i\in\mathbf{B}'}Q_i$. Without loss of generality, we assume that K_2 users in $\mathbf{D}'$ whose queue lengths are less than or equal to Q_d, that is $Q_{j_1}\le Q_{j_2}\le ...\le Q_{j_{K_2}}\le Q_d\le Q_{j_{K_2+1}}\le ...\le Q_{j_{|\mathbf{D}'|}}$. We denote $\mathbf{D'}_1\triangleq\{j_1,j_2,...,j_{K_2}\}$ and $\mathbf{D'}_2\triangleq\{j_{K_2+1},j_{K_2+2},...,j_{|\mathbf{D}'|}\}$. By using similar technique in deriving equation (67), we have

$$f(\mathbf{D}',d)-f(\mathbf{B}',d)=\sum_{k=1}^{K_2}p^2(1-p)^{|\mathbf{D}'|-k}(Q_{j_k}-Q_d). \quad (71)$$

Next, let's focus on the term $g(\mathbf{B}',\mathbf{T},d)$.

$$\begin{aligned}
&g(\mathbf{B}',\mathbf{T},d)\\
=&-\mathbb{E}\left[\min\left(\max_{l\in\mathbf{B}'}\min\{Q_lC_l,Q_dC_d\},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_dC_d\}\right)\right]\\
=&-p\mathbb{E}\left[\min\left(\max_{l\in\mathbf{B}'}\min\{Q_lC_l,Q_d\},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}\right)\right].
\end{aligned}$$

Let $\mathcal{J}$ be the event that at least one user in $\mathbf{B}'$ has the available channel. Then, we have

$$\max_{l\in\mathbf{B}'}\min\{Q_lC_l,Q_d\}=\begin{cases}Q_d & \text{, if event } \mathcal{J} \text{ happens;}\\ 0 & \text{, otherwise.}\end{cases} \qquad (72)$$

Thus, we get

$$\begin{aligned}
&\min\left(\max_{l\in\mathbf{B}'}\min\{Q_lC_l,Q_d\},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}\right)\\
=&\begin{cases}\min\left(Q_d,\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}\right) & \text{, if event } \mathcal{J} \text{ happens;}\\ 0 & \text{, otherwise}\end{cases}\\
=&\begin{cases}\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\} & \text{, if event } \mathcal{J} \text{ happens;}\\ 0 & \text{, otherwise.}\end{cases} \qquad (73)
\end{aligned}$$

where we use Lemma 7. Since $\Pr\{\mathcal{J}\}=(1-(1-p)^{|\mathbf{B}'|})$, we have

$$\begin{aligned}
g(\mathbf{B}',\mathbf{T},d)&=-p\left(1-(1-p)^{|\mathbf{B}'|}\right)\mathbb{E}[\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}]\\
&=\left((1-p)^{|\mathbf{B}'|}-1\right)f(\mathbf{T},d). \qquad (74)
\end{aligned}$$

Let's consider the term $g(\mathbf{D}',\mathbf{T},d)$.

$$\begin{aligned}
&g(\mathbf{D}',\mathbf{T},d)\\
=&-\mathbb{E}\left[\min\left(\max_{l\in\mathbf{D}'}\min\{Q_lC_l,Q_dC_d\},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_dC_d\}\right)\right]\\
=&-p\mathbb{E}\left[\min\left(\max_{l\in\mathbf{D}'}\min\{Q_lC_l,Q_d\},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}\right)\right].
\end{aligned}$$

Let $\mathcal{K}$ be the event that at least one user in $\mathbf{D'}_2$ has the available channel. Let $\mathcal{L}_k$ be the event that $C_{j_k}=1, C_{j_i}=0$ for $k<i\le K_2$, $k=1,2,...,K_2$ and $\mathcal{L}_{K_2}$ be the event that $C_{j_{K_2}}=1$. Then, by using Lemma 8, we have

$$\max_{l\in\mathbf{D}'}\min\{Q_lC_l,Q_d\}=\begin{cases}Q_d & \text{, if event } \mathcal{K} \text{ happens;}\\ Q_{j_k} & \text{, if event } \mathcal{K}^c\bigcap\mathcal{L}_k \text{ happens,}\\ & \quad \text{for } k=1,2.,,,.,K_2.\end{cases}$$

Thus, we get

$$\begin{aligned}
&\min\left(\max_{l\in\mathbf{D}'}\min\{Q_lC_l,Q_d\},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}\right)\\
=&\begin{cases}\min\left(Q_d,\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}\right) & \text{, if } \mathcal{K} \text{ happens;}\\ \min\left(Q_{j_k},\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}\right) & \text{, if } \mathcal{K}^c\bigcap\mathcal{L}_k \text{ happens,}\\ & \quad \text{for } k=1,2.,,,.,K_2\end{cases}\\
=&\begin{cases}\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\} & \text{, if } \mathcal{K} \text{ happens;}\\ \max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_{j_k}\} & \text{, if } \mathcal{K}^c\bigcap\mathcal{L}_k \text{ happens,}\\ & \quad \text{for } k=1,2.,,,.,K_2,\end{cases}
\end{aligned}$$

where we use Lemma 7. Hence, we have

$$\begin{aligned}
&g(\mathbf{D}',\mathbf{T},d)\\
=&-p\mathbb{E}\left[\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_d\}\right]\Pr\{\mathcal{K}\}\\
&-\sum_{k=1}^{K_2}p\mathbb{E}\left[\max_{l\in\mathbf{T}}\min\{Q_lC_l,Q_{j_k}\}\right]\Pr\left\{\mathcal{K}^c\bigcap\mathcal{L}_k\right\}\\
=&-\Pr\{\mathcal{K}\}f(\mathbf{T},d)-\sum_{k=1}^{K_2}\Pr\left\{\mathcal{K}^c\bigcap\mathcal{L}_k\right\}f(\mathbf{T},j_k). \qquad (75)
\end{aligned}$$

Note that $\Pr\{\mathcal{K}\} = 1-(1-p)^{|\mathbf{D}'|-K_2}$ and $\Pr\{\mathcal{K}^c \bigcap \mathcal{L}_k\} = p(1-p)^{|\mathbf{D}'|-k}$. Thus, we have

$$\begin{aligned}&g(\mathbf{D}', \mathbf{T}, d)\\=&\sum_{k=1}^{K_2}(-p)(1-p)^{|\mathbf{D}'|-k} f(\mathbf{T}, j_k) + \left((1-p)^{|\mathbf{D}'|-K_2} - 1\right) f(\mathbf{T}, d).\end{aligned}$$

Note that $|\mathbf{B}'| = |\mathbf{D}'|$. Thus, we have

$$\begin{aligned}&g(\mathbf{D}', \mathbf{T}, d) - g(\mathbf{B}', \mathbf{T}, d)\\=&\sum_{k=1}^{K_2}(-p)(1-p)^{|\mathbf{D}'|-k} f(\mathbf{T}, j_k)\\&+\left((1-p)^{|\mathbf{D}'|-K_2} - (1-p)^{|\mathbf{D}'|}\right) f(\mathbf{T}, d).\end{aligned} \tag{76}$$

Note that $(1-p)^{|\mathbf{D}'|-K_2} - (1-p)^{|\mathbf{D}'|} = p\sum_{k=1}^{K_2}(1-p)^{|\mathbf{D}'|-k}$. Thus, (76) becomes

$$\begin{aligned}&g(\mathbf{D}', \mathbf{T}, d) - g(\mathbf{B}', \mathbf{T}, d)\\=&\sum_{k=1}^{K_2}(-p)(1-p)^{|\mathbf{D}'|-k}\left(f(\mathbf{T}, j_k) - f(\mathbf{T}, d)\right).\end{aligned} \tag{77}$$

Consider the term $f(\mathbf{T}, j_k) - f(\mathbf{T}, d)$. Without loss of generality, we assume n_k users in $\mathbf{T}$ whose queue lengths are less than or equal to Q_{j_k} and n_d users whose queue lengths are less than or equal to Q_d, that is, $Q_{i_1} \le Q_{i_2} \le ... \le Q_{i_{n_k}} \le Q_{j_k} \le Q_{i_{n_k+1}} \le ... \le Q_{i_{n_d}} \le Q_d \le Q_{i_{n_d+1}} \le Q_{i_{|\mathbf{T}|}}$. Note that $n_k \le n_d$. Thus, by using Corollary 4, we have

$$\begin{aligned}&f(\mathbf{T}, j_k) - f(\mathbf{T}, d)\\=&\sum_{l=1}^{n_k} p^2(1-p)^{|\mathbf{T}|-l} Q_{i_l} + p\left(1-(1-p)^{|\mathbf{T}|-n_k}\right) Q_{j_k}\\&-\sum_{l=1}^{n_d} p^2(1-p)^{|\mathbf{T}|-l} Q_{i_l} - p\left(1-(1-p)^{|\mathbf{T}|-n_d}\right) Q_d\\=&p\left(1-(1-p)^{|\mathbf{T}|-n_k}\right) Q_{j_k} - p\left(1-(1-p)^{|\mathbf{T}|-n_d}\right) Q_d\\&-\sum_{l=n_k+1}^{n_d} p^2(1-p)^{|\mathbf{T}|-l} Q_{i_l}\\\ge&p\left(1-(1-p)^{|\mathbf{T}|-n_k}\right) Q_{j_k} - p\left(1-(1-p)^{|\mathbf{T}|-n_d}\right) Q_d\\&-Q_d\sum_{l=n_k+1}^{n_d} p^2(1-p)^{|\mathbf{T}|-l}\\=&p\left(1-(1-p)^{|\mathbf{T}|-n_k}\right) Q_{j_k} - p\left(1-(1-p)^{|\mathbf{T}|-n_d}\right) Q_d\\&-p\left((1-p)^{|\mathbf{T}|-n_d} - (1-p)^{|\mathbf{T}|-n_k}\right) Q_d\\=&p\left(1-(1-p)^{|\mathbf{T}|-n_k}\right)\left(Q_{j_k} - Q_d\right).\end{aligned} \tag{78}$$

Thus, we have

$$\begin{aligned}&g(\mathbf{D}', \mathbf{T}, d) - g(\mathbf{B}', \mathbf{T}, d)\\=&\sum_{k=1}^{K_2}(-p)(1-p)^{|\mathbf{D}'|-k}\left(f(\mathbf{T}, j_k) - f(\mathbf{T}, d)\right)\\\le&\sum_{k=1}^{K_2} p^2(1-p)^{|\mathbf{D}'|-k}(Q_{j_k} - Q_d)\left((1-p)^{|\mathbf{T}|-n_k} - 1\right)\\=&\sum_{k=1}^{K_2}(Q_{j_k} - Q_d)p^2\left((1-p)^{|\mathbf{D}'|+|\mathbf{T}|-n_k-k} - (1-p)^{|\mathbf{D}'|-k}\right).\end{aligned}$$

Hence, we have

$$\begin{aligned}&f(\mathbf{D}^*, d) - f(\mathbf{B}, d)\\\le&\sum_{k=1}^{K_2}(Q_{j_k} - Q_d)p^2(1-p)^{|\mathbf{D}'|-k}\\&+\sum_{k=1}^{K_2}(Q_{j_k} - Q_d)p^2\left((1-p)^{|\mathbf{D}'|+|\mathbf{T}|-n_k-k} - (1-p)^{|\mathbf{D}'|-k}\right)\\=&\sum_{k=1}^{K_2}(Q_{j_k} - Q_d)p^2(1-p)^{|\mathbf{D}'|+|\mathbf{T}|-n_k-k} \le 0.\end{aligned} \tag{79}$$

Thus, we have the desired result. ■

Appendix H
Proof for Lemma 4

Proof: We only prove the first part in (25). The second part follows the similar argument. It is enough to show

$$\max_i a_i C_i^{\min}[t] \le_{st} \max_i a_i C_i[t], \forall t, \tag{80}$$

holds for any constants $a_i \ge 0, \forall i$, where $W_1 \le_{st} W_2$ means that the random variable W_1 is stochastically smaller than the random variable W_2 [19]. At any time slot t, according to the definition of stochastically smaller, we need to show

$$\Pr\{\max_i a_i C_i[t] \le b\} \le \Pr\{\max_i a_i C_i^{\min}[t] \le b\}, \forall b, \tag{81}$$

which is equivalent to showing

$$\Pr\{a_i C_i[t] \le b, \forall i \in \mathbf{N}\} \le \Pr\{a_i C_i^{\min}[t] \le b, \forall i \in \mathbf{N}\}, \forall b.$$

If $b < 0$, we have

$$\Pr\{a_i C_i[t] \le b, \forall i \in \mathbf{N}\} = \Pr\{a_i C_i^{\min}[t] \le b, \forall i \in \mathbf{N}\} = 0,$$

since $a_i \ge 0$ and $C_i[t] \ge 0, \forall i$. Thus, we assume $b \ge 0$ in the rest of the proof. Let $\mathbf{G} \triangleq \{i \in \mathbf{N} : a_i > 0\}$, we have

$$\begin{aligned}\Pr\{a_i C_i[t] \le b, \forall i \in \mathbf{N}\} &= \Pr\{a_i C_i[t] \le b, \forall i \in \mathbf{G}\}\\\Pr\{a_i C_i^{\min}[t] \le b, \forall i \in \mathbf{N}\} &= \Pr\{a_i C_i^{\min}[t] \le b, \forall i \in \mathbf{G}\}\end{aligned}$$

Thus, we only need to show

$$\Pr\{a_i C_i[t] \le b, \forall i \in \mathbf{G}\} \le \Pr\{a_i C_i^{\min}[t] \le b, \forall i \in \mathbf{G}\},$$

which is equivalent to proving

$$\Pr\left\{C_i[t] \leq \frac{b}{a_i}, \forall i \in \mathbf{G}\right\} \leq \Pr\left\{C_i^{\min}[t] \leq \frac{b}{a_i}, \forall i \in \mathbf{G}\right\}. \tag{82}$$

Next, we will show that (82) is true. Let $\mathbf{H} \triangleq \{i \in \mathbf{G} : \frac{b}{a_i} \geq c_1\}$. We have

$$\begin{aligned}
&\Pr\{C_i[t] \leq \frac{b}{a_i}, \forall i \in \mathbf{G}\} \\
&\leq \Pr\{C_i[t] \leq \frac{b}{a_i}, \forall i \in \mathbf{G} \setminus \mathbf{H}\} \\
&= \Pr\{C_i[t] = 0, \forall i \in \mathbf{G} \setminus \mathbf{H}\}.
\end{aligned} \tag{83}$$

From the construction of independent ON-OFF fading channel $\mathbf{C}^{\min}$, we have

$$\Pr\{C_i[t] = 0\} \leq \Pr\{C_i^{\min}[t] = 0\}, \forall i. \tag{84}$$

Since condition (24) holds, we have

$$\Pr\{C_i[t] = 0, \forall i \in \mathbf{G} \setminus \mathbf{H}\} \leq \Pr\{C_i^{\min}[t] = 0, \forall i \in \mathbf{G} \setminus \mathbf{H}\}.$$

Thus, we get

$$\begin{aligned}
&\Pr\left\{C_i[t] \leq \frac{b}{a_i}, \forall i \in \mathbf{G}\right\} \\
&\leq \Pr\{C_i^{\min}[t] = 0, \forall i \in \mathbf{G} \setminus \mathbf{H}\} \\
&= \Pr\left\{C_i^{\min}[t] \leq \frac{b}{a_i}, \forall i \in \mathbf{G} \setminus \mathbf{H}\right\} (\text{since } C_i^{\min}[t] = 0 \text{ or } c_1) \\
&= \Pr\left\{C_i^{\min}[t] \leq \frac{b}{a_i}, \forall i \in \mathbf{G}\right\}.
\end{aligned}$$

■

Appendix I
Proof for Lemma 5

Consider a system over fading channel $\mathbf{C}^{II}$. We show that the JPT algorithm where we use channel statistics and rates of channel $\mathbf{C}^{I}$ in the probing component can support any arrival rate vector $\lambda \in \Lambda(\mathbf{m}, \mathbf{C}^I)$. By choosing the same Lyapunov function and following the same steps as in the proof for Proposition 2, we have

$$\begin{aligned}
\Delta L \leq& \sum_{i=1}^{N} \lambda_i Q_i - \sum_{i=1}^{N} m_i U_i + B_{2,\max} \\
&- \mathbb{E}\left[\max_i Q_i[t] X_i^*[t] C_i^{II}[t] - \sum_{i=1}^{N} U_i[t] X_i^*[t] | \mathbf{Q}[t], \mathbf{U}[t]\right].
\end{aligned}$$

Given any value of $\mathbf{Q}[t]$ and $\mathbf{U}[t]$ at slot t, $Q_i[t]X_i^*[t], \forall i$, are just non-negative constant numbers. Thus, by the condition (26), we have

$$\begin{aligned}
&\mathbb{E}[\max_i Q_i[t] X_i^*[t] C_i^{I}[t] | \mathbf{Q}[t], \mathbf{U}[t]] \\
\leq& \mathbb{E}[\max_i Q_i[t] X_i^*[t] C_i^{II}[t] | \mathbf{Q}[t], \mathbf{U}[t]].
\end{aligned} \tag{85}$$

Hence, we get

$$\begin{aligned}
\Delta L \leq& \sum_{i=1}^{N} \lambda_i Q_i - \sum_{i=1}^{N} m_i U_i + B_{2,\max} \\
&- \mathbb{E}\left[\max_i Q_i[t] X_i^*[t] C_i^{I}[t] - \sum_{i=1}^{N} U_i[t] X_i^*[t] | \mathbf{Q}[t], \mathbf{U}[t]\right]
\end{aligned}$$

For any $\boldsymbol{\lambda} \in \Lambda(\mathbf{m}, \mathbf{C}^I)$, by Proposition 2, the considered JPT algorithm can support this arrival rate vector and thus we have the desired result.

Appendix J
Proof for Lemma 6

Proof: By maximum-minimums identity, we have

$$\begin{aligned}
h(p) =& \frac{1}{p}\Bigg(\sum_{l \in \mathbf{A}} \mathbb{E}[X_l Z_l] - \sum_{\substack{l_1, l_2 \in \mathbf{A} \\ l_1 < l_2}} \mathbb{E}[\min\{X_{l_1} Z_{l_1}, X_{l_2} Z_{l_2}\}] \\
&+ ... + (-1)^{|\mathbf{A}|-1} \mathbb{E}[\min\{X_1 Z_1, ..., X_{|\mathbf{A}|} Z_{|\mathbf{A}|}\}]\Bigg) \\
=& \sum_{l \in \mathbf{A}} \mathbb{E}[X_l] - p \sum_{\substack{l_1, l_2 \in \mathbf{A} \\ l_1 < l_2}} \mathbb{E}[\min\{X_{l_1}, X_{l_2}\}] \\
&+ ... + (-1)^{|\mathbf{A}|-1} p^{|\mathbf{A}|-1} \mathbb{E}[\min\{X_1, ..., X_{|\mathbf{A}|}\}].
\end{aligned} \tag{86}$$

Note that $\min_{l \in \mathbf{L}} X_l = \prod_{l \in \mathbf{L}} X_l, \forall \mathbf{L} \subseteq \mathbf{A}$. Thus, we have

$$\begin{aligned}
h(p) =& \sum_{l \in \mathbf{A}} \mathbb{E}[X_l] - p \sum_{\substack{l_1, l_2 \in \mathbf{A} \\ l_1 < l_2}} \mathbb{E}[X_{l_1} X_{l_2}] \\
&+ ... + (-1)^{|\mathbf{A}|-1} p^{|\mathbf{A}|-1} \mathbb{E}[\prod_{l \in \mathbf{A}} X_l].
\end{aligned} \tag{87}$$

Hence, we have

$$\begin{aligned}
h'(p) =& - \sum_{\substack{l_1, l_2 \in \mathbf{A} \\ l_1 < l_2}} \mathbb{E}[X_{l_1} X_{l_2}] + 2p \sum_{\substack{l_1, l_2, l_3 \in \mathbf{A} \\ l_1 < l_2 < l_3}} \mathbb{E}[X_{l_1} X_{l_2} X_{l_3}] \\
&+ ... + (-1)^{|\mathbf{A}|-1} (|\mathbf{A}| - 1) p^{|\mathbf{A}|-2} \mathbb{E}[\prod_{l \in \mathbf{A}} X_l].
\end{aligned} \tag{88}$$

Let γ_i ($i = 0, 1, 2, ..., |\mathbf{A}|$) be the probability that i users in set $\mathbf{A}$ probe the channel. Let's consider the term $\sum_{\substack{l_1, l_2, ..., l_k \in \mathbf{A} \\ l_1 < l_2 < ... < l_k}} \mathbb{E}[\prod_{i=1}^{k} X_{l_i}]$ ($k = 2, 3, ..., |\mathbf{A}|$). Let $\mathcal{U}$ be the event that users $l_1, l_2, ..., l_k$ probe the channel and $\mathcal{V}_j$ ($j = 0, 1, 2, ..., |\mathbf{A}|$) be the event that j users probe the channel. By the law of total probability, we have

$$\mathbb{E}[\prod_{i=1}^{k} X_{l_i}] = \Pr\{\mathcal{U}\} = \sum_{j=k}^{|\mathbf{A}|} \gamma_j \Pr\{\mathcal{U} | \mathcal{V}_j\}. \tag{89}$$

Thus, we have

$$\sum_{\substack{l_1,l_2,...,l_k\in\mathbf{A}\\ l_1<l_2<...<l_k}} \mathbb{E}[\prod_{i=1}^{k} X_{l_i}] = \sum_{\substack{l_1,l_2,...,l_k\in\mathbf{A}\\ l_1<l_2<...<l_k}} \sum_{j=k}^{|\mathbf{A}|} \gamma_j \Pr\{\mathcal{U}|\mathcal{V}_j\}$$
$$= \sum_{j=k}^{|\mathbf{A}|} \gamma_j \sum_{\substack{l_1,l_2,...,l_k\in\mathbf{A}\\ l_1<l_2<...<l_k}} \Pr\{\mathcal{U}|\mathcal{V}_j\}. \tag{90}$$

Note that if $\{l_1, l_2, ..., l_k\}$ is a subset of the set of selected j users, then $\Pr\{\mathcal{U}|\mathcal{V}_j\} = 1$; otherwise, $\Pr\{\mathcal{U}|\mathcal{V}_j\} = 0$. Thus, we have

$$\sum_{\substack{l_1,l_2,...,l_k\in\mathbf{A}\\ l_1<l_2<...<l_k}} \mathbb{E}[\prod_{i=1}^{k} X_{l_i}] = \sum_{j=k}^{|\mathbf{A}|} \gamma_j \binom{j}{k}. \tag{91}$$

By substituting (91) into (88), we have

$$h'(p) = -\left(\binom{2}{2}\gamma_2 + \binom{3}{2}\gamma_3 + \binom{4}{2}\gamma_4 + ... + \binom{|\mathbf{A}|}{2}\gamma_{|\mathbf{A}|}\right)$$
$$+ 2p\left(\binom{3}{3}\gamma_3 + \binom{4}{3}\gamma_4 + ... + \binom{|\mathbf{A}|}{3}\gamma_{|\mathbf{A}|}\right) + ...$$
$$+ (-1)^{|\mathbf{A}|-1}(|\mathbf{A}|-1)p^{|\mathbf{A}|-2}\binom{|\mathbf{A}|}{|\mathbf{A}|}\gamma_{|\mathbf{A}|}$$
$$= -\sum_{k=2}^{|\mathbf{A}|} \gamma_k \sum_{i=2}^{k} (-1)^i \binom{k}{i}(i-1)p^{i-2}. \tag{92}$$

Using mathematical induction, it is not hard to show

$$\sum_{i=2}^{n} (-1)^i \binom{n}{i}(i-1)p^{i-2} = \frac{1 - np(1-p)^{n-1} - (1-p)^n}{p^2},$$

for any $n \geq 2$. Noting that $1 - np(1-p)^{n-1} - (1-p)^n = \sum_{k=2}^{n} \binom{n}{k} p^k (1-p)^{n-k} \geq 0$ for any $n \geq 2$, we have $\sum_{i=2}^{n} (-1)^i \binom{n}{i}(i-1)p^{i-2} \geq 0$ for any $n \geq 2$. Hence, we have $h'(p) \leq 0$, and thus $h(p)$ is a decreasing function. ∎

APPENDIX K
PROOF FOR PROPOSITION 4

Proof: Under Assumption 1, by Lemma 4 and Lemma 5, we have

$$\Lambda(\mathbf{m}, \mathbf{C}^{\min}) \subseteq \Lambda(\mathbf{m}, \mathbf{C}) \subseteq \Lambda(\mathbf{m}, \mathbf{C}^{\max}).$$

By Lemma 2, we have

$$\Lambda(\mathbf{m}, \mathbf{C}^{\max})$$
$$=\{\lambda : \exists \text{ a probability distribution of probing schedule } \mathbf{X}^{\max}$$
$$\text{such that } \sum_{i\in\mathbf{A}} \lambda_i \leq \mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\max} C_i^{\max}[t]], \forall \mathbf{A} \subseteq \mathbf{N} \text{ and}$$
$$\mathbb{E}[X_i^{\max}] \leq m_i, \forall i\}, \tag{93}$$

and

$$\Lambda(\mathbf{m}, \mathbf{C}^{\min})$$
$$=\{\lambda : \exists \text{ a probability distribution of probing schedule } \mathbf{X}^{\min}$$
$$\text{such that } \sum_{i\in\mathbf{A}} \lambda_i \leq \mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\min} C_i^{\min}[t]], \forall \mathbf{A} \subseteq \mathbf{N} \text{ and}$$
$$\mathbb{E}[X_i^{\min}] \leq m_i, \forall i\}. \tag{94}$$

Let

$$\rho_{|\mathbf{A}|} \triangleq \frac{\mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\max} C_i^{\min}[t]]}{\mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\max} C_i^{\max}[t]]}$$
$$= \rho \frac{\frac{1}{p_{\min} c_1}\mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\max} C_i^{\min}[t]]}{\frac{1}{p_{\max} c_M}\mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\max} C_i^{\max}[t]]}. \tag{95}$$

By Lemma 6, we have $\rho_{|\mathbf{A}|} \geq \rho$. Hence, for any $\lambda \in \rho\Lambda(\mathbf{m}, \mathbf{C}^{\max})$, we have $\lambda \in \Lambda(\mathbf{m}, \mathbf{C}^{\min})$. Indeed, for any $\lambda \in \rho\Lambda(\mathbf{m}, \mathbf{C}^{\max})$, we have $\frac{\lambda}{\rho} \in \Lambda(\mathbf{m}, \mathbf{C}^{\max})$, that is, $\sum_{i\in\mathbf{A}} \frac{\lambda_i}{\rho} \leq \mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\max} C_i^{\max}[t]], \forall \mathbf{A} \subseteq \mathbf{N}$, and $\mathbb{E}[X_i^{\max}] \leq m_i, \forall i$. Hence, for any $\mathbf{A} \subseteq \mathbf{N}$, we have

$$\sum_{i\in\mathbf{A}} \lambda_i \leq \rho\mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\max} C_i^{\max}[t]]$$
$$\leq \rho_{|\mathbf{A}|}\mathbb{E}[\max_{l\in\mathbf{A}} X_i^{\max} C_i^{\max}[t]] = \mathbb{E}[\max_{i\in\mathbf{A}} X_i^{\max} C_i^{\min}[t]].$$

By taking the probability distribution of $\mathbf{X}^{\min}$ the same as $\mathbf{X}^{\max}$, we have $\lambda \in \Lambda(\mathbf{m}, \mathbf{C}^{\min})$. Thus, for any $\lambda \in \rho\Lambda(\mathbf{m}, \mathbf{C})$, we have $\lambda \in \Lambda(\mathbf{m}, \mathbf{C}^{\min})$. Next, we will show that the MSGP algorithm, combined with the MWS algorithm in the transmission stage, can support any arrival rate vector $\lambda \in \Lambda(\mathbf{m}, \mathbf{C}^{\min})$, implying that it can at least achieve a fraction $\rho = \frac{p_{\min}}{p_{\max}} \frac{c_1}{c_M}$ of the capacity region.

By choosing the same Lyapunov function and following the same argument as in the proof for Proposition 2, we have

$$\Delta L \triangleq \mathbb{E}[L[t+1] - L[t]|\mathbf{Q}[t] = \mathbf{Q}, \mathbf{U}[t] = \mathbf{U}]$$
$$\leq \sum_{i=1}^{N} \lambda_i Q_i - \sum_{i=1}^{N} m_i U_i + B_{2,\max}$$
$$-\mathbb{E}\left[\max_i Q_i[t] X_i^M[t] C_i[t] - \sum_{i=1}^{N} U_i[t] X_i^M[t] | \mathbf{Q}[t], \mathbf{U}[t]\right],$$

where $\mathbf{X}^M[t] = (X_i^M[t])_{i=1}^N$ is a probing schedule chosen by MSGP algorithm. Given any value of $\mathbf{Q}[t]$ and $\mathbf{U}[t]$ at slot t, $Q_i[t] X_i^M[t], \forall i$ are just non-negative constant numbers. Thus, by Lemma 4, we have

$$\mathbb{E}\left[\max_i Q_i[t] X_i^M[t] C_i[t] | \mathbf{Q}[t], \mathbf{U}[t]\right]$$
$$\geq \mathbb{E}\left[\max_i Q_i[t] X_i^M[t] C_i^{\min}[t] | \mathbf{Q}[t], \mathbf{U}[t]\right].$$

Thus, ΔL becomes

$$\Delta L \leq \sum_{i=1}^{N} \lambda_i Q_i - \sum_{i=1}^{N} m_i U_i + B_{\max}$$
$$-\mathbb{E}\bigg[\max_i Q_i[t] X_i^M[t] C_i^{\min}[t] - \sum_{i=1}^{N} U_i[t] X_i^M[t] |\mathbf{Q}[t], \mathbf{U}[t]\bigg].$$

Then, by using the fact that the MSGP algorithm can find the optimal probing schedule in the symmetric and independent ON-OFF fading channel $\mathbf{C}^{\min}$ and following the same argument as in Proposition 2, we have the desired result. ∎

Appendix L
Proof for Proposition 5

To prove this proposition, we need the following claim:

Claim 1: All edge weights with strictly positive value are lower bounded by a strictly positive constant value.

Proof: Recall that $c_0 = 0$. Since $p_{ij}, \forall i = 1, ..., N, j = 0, 1, ..., M$, and $c_j, j = 1, ..., M$, are rational numbers, let $c_j = \frac{a_j}{b_j}$ and $p_{ij} = \frac{q_{ij}}{d_{ij}}$, where a_j and b_j are co-prime for any $j = 1, ..., M$, and q_{ij} and d_{ij} are co-prime for any $i = 1, ..., N, j = 0, 1, ..., M$. First, we will show that if $\phi_i > 0$, we have

$$\phi_i \triangleq Q_i \mathbb{E}[C_i] - U_i \geq \frac{1}{\prod_{j=1}^{M} b_j d_{ij}}, \forall i. \tag{96}$$

Indeed, we have

$$\begin{aligned}
&Q_i \mathbb{E}[C_i] - U_i \\
&= Q_i \sum_{j=1}^{M} \frac{a_j}{b_j} \frac{q_{ij}}{d_{ij}} - U_i \\
&= \frac{1}{\prod_{j=1}^{M} b_j d_{ij}} \left(Q_i \sum_{j=1}^{M} a_j q_{ij} \prod_{l \neq j} b_l d_{il} - U_i \prod_{j=1}^{M} b_j d_{ij} \right)
\end{aligned}$$

Note that $Q_i \sum_{j=1}^{M} a_j q_{ij} \prod_{l\neq j} b_l d_{il} - U_i \prod_{j=1}^{M} b_j d_{ij}$ is an integer. If $\phi_i > 0$, then $\phi_i \geq \frac{1}{\prod_{j=1}^{M} b_j d_{ij}}$. Next, we will show that if the weight $\phi_i - f(\{i_1, ..., i_{k-1}\}, i) > 0$, then

$$\begin{aligned}
&\phi_i - f(\{i_1, ..., i_{k-1}\}, i) \\
&\geq \frac{1}{\prod_{j=1}^{M} b_j \prod_{j_1=0}^{M} d_{i_1 j_1} \cdots \prod_{j_{k-1}=0}^{M} d_{i_{k-1} j_{k-1}} \prod_{j=0}^{M} d_{ij}}.
\end{aligned} \tag{97}$$

Indeed, we have

$$\begin{aligned}
&\phi_i - f(\{i_1, ..., i_{k-1}\}, i) \\
&= Q_i \mathbb{E}[C_i] - U_i - \mathbb{E}\left[\max_{l \in \{i_1, ..., i_{k-1}\}} \min\{Q_l C_l, Q_i C_i\}\right] \\
&= Q_i \sum_{j=1}^{M} \frac{a_j}{b_j} \frac{q_{ij}}{d_{ij}} - U_i - \sum_{j_1=0}^{M} \Pr\left\{C_{i_1} = \frac{a_{j_1}}{b_{j_1}}\right\} \\
&\cdots \sum_{j_{k-1}=0}^{M} \Pr\left\{C_{i_{k-1}} = \frac{a_{j_{k-1}}}{b_{j_{k-1}}}\right\} \sum_{j=0}^{M} \Pr\left\{C_i = \frac{a_j}{b_j}\right\} \\
&\max\left\{\min\left\{\frac{Q_{i_1} a_{j_1}}{b_{j_1}}, \frac{Q_i a_j}{b_j}\right\}, ..., \min\left\{\frac{Q_{i_{k_1}} a_{j_{k-1}}}{b_{j_{k-1}}}, \frac{Q_i a_j}{b_j}\right\}\right\} \\
&= Q_i \sum_{j=1}^{M} \frac{a_j}{b_j} \frac{q_{ij}}{d_{ij}} - U_i \\
&- \sum_{j_1=0}^{M} \frac{q_{i_1 j_1}}{d_{i_1 j_1}} \cdots \sum_{j_{k-1}=0}^{M} \frac{q_{i_{k-1} j_{k-1}}}{d_{i_{k-1} j_{k-1}}} \sum_{j=0}^{M} \frac{q_{ij}}{d_{ij}} \\
&\max\left\{\min\left\{\frac{Q_{i_1} a_{j_1}}{b_{j_1}}, \frac{Q_i a_j}{b_j}\right\}, ..., \min\left\{\frac{Q_{i_{k_1}} a_{j_{k-1}}}{b_{j_{k-1}}}, \frac{Q_i a_j}{b_j}\right\}\right\} \\
&\overset{(I)}{\geq} \frac{1}{\prod_{j=1}^{M} b_j \prod_{j_1=0}^{M} d_{i_1 j_1} \cdots \prod_{j_{k-1}=0}^{M} d_{i_{k-1} j_{k-1}} \prod_{j_1=0}^{M} d_{ij}},
\end{aligned} \tag{98}$$

where (I) follows the same argument as in (96).

Thus, by combining (96) and (97), it is easy to see that all edge weights with strictly positive value should be lower bounded by $\prod_{j=1}^{M} \frac{1}{b_j} \prod_{i=1}^{N} \prod_{j=0}^{M} \frac{1}{d_{ij}} > 0$. ∎

[**Proof of Proposition 5**:]

Proof: Assume that the node with the optimal value is in level K. Given any $\tau > 0$ and $\delta > 0$. Let W_k^{DMSGP} and W_k^{MSGP} be the weight of an edge selected by DMSGP algorithm and MSGP algorithm from level $k-1$ to level k respectively. Note that W_k^{DMSGP} and W_k^{MSGP} are strictly positive. By claim 1, W_k^{DMSGP} and W_k^{MSGP} are lower bounded by a strictly positive constant value. Thus, by using similar argument in [8], we can show that given any $\tau' > 0$, $\exists G_k > 0$ such that for any $G > G_k$, we have

$$\Pr\{W_k^{DMSGP} > W_k^{MSGP}(1-\delta)\} > 1 - \tau'. \tag{99}$$

Let $W^{DMSGP} = \sum_{k=1}^{K} W_k^{DMSGP}$ and $W^{MSGP} = \sum_{k=1}^{K} W_k^{MSGP}$. Thus, for any $G \geq \max\{G_1, G_2, ..., G_K\}$, we have

$$\begin{aligned}
&\Pr\{W^{DMSGP} > W^{MSGP}(1-\delta)\} \\
\geq &\Pr\{W_k^{DMSGP} > W_k^{MSGP}(1-\delta), \forall k = 1, ..., K\} \\
> &1 - K\tau',
\end{aligned} \tag{100}$$

where we use the fact [4] that given any two events $\mathcal{E}$ and $\mathcal{F}$ such that $\Pr\{\mathcal{E}\} > 1 - \epsilon_1$ and $\Pr\{\mathcal{F}\} > 1 - \epsilon_2$, we have $\Pr\{\mathcal{E} \bigcap \mathcal{F}\} > 1 - \epsilon_1 - \epsilon_2$. We can pick τ' small enough such that $1 - K\tau' > 1 - \tau$. Hence, we have

$$\Pr\{W^{DMSGP} > W^{MSGP}(1-\delta)\} > 1 - \tau. \tag{101}$$

Then, we have

$$\mathbb{E}[W^{DMSGP}|\mathbf{Q}[t],\mathbf{U}[t]] \geq (1-\delta)(1-\tau)\mathbb{E}[W^{MSGP}|\mathbf{Q}[t],\mathbf{U}[t]].$$

By choosing the same Lyapunov function as in the proof for Proposition 2, the remaining argument follows the similar reasoning as in the proof for Theorem 1 in [8]. ■

REFERENCES

[1] S. Boyd and L. Vandenberghe. *Convex optimization*. Cambridge University Press, 2004.

[2] P. Chaporkar, A. Proutiere, H. Asnani, and A. Karandikar. Scheduling with limited information in wireless systems. In *Proc. ACM international symposium on Mobile ad hoc network and computing (MobiHoc)*, New Orleans, Louisiana, May 2009.

[3] T. Cormen, C. Leiserson, R. Rivest, and C. Stein. *Introduction to algorithms*. The MIT Press, 2002.

[4] T. Cover and J. Thomas. *Elements of information theory*. John Wiley & Sons, Inc., 2005.

[5] J. Ghaderi and R. Srikant. On the design of efficient CSMA algorithms for wireless networks. In *Proc. IEEE International Conference on Decision and Control. (CDC)*, Atlanta, GA, December 2010.

[6] A. Gopalan, C. Caramanis, and S. Shakkottai. On wireless scheduling with partial channel-state information. In *Proc. Allerton Conference on Communication, Control, and Computing*, Monticello, Illinois, September 2007.

[7] L. Jiang and J. Walrand. A CSMA distributed algorithm for throughput and utility maximization in wireless networks. In *Proc. Allerton Conference on Communication, Control, and Computing (Allerton)*, Monticello, Illinois, September 2008.

[8] B. Li and A. Eryilmaz. Optimal distributed scheduling under time-varying conditions: A fast-CSMA algorithm with applications. To appear in *IEEE Transactions on Wireless Communications*, 2013.

[9] B. Li and A. Eryilmaz. Efficient distributed channel probing for opportunistic scheduling over wireless fading channels. Technical Report, 2013. Available online at http://www2.ece.ohio-state.edu/~eryilmaz/LiEryilmaz_TMC13_Report.pdf.

[10] C. Li and M. Neely. Energy-optimal scheduling with dynamic channel acquisition in wireless downlinks. *IEEE Transactions on Mobile Computing*, 9:527–539, April 2010.

[11] X. Lin, N. Shroff, and R. Srikant. A tutorial on cross-layer optimization in wireless networks. *IEEE Journal on Selected Areas in Communications*, 24:1452–1463, August 2006.

[12] X. Liu, E. Chong, and N. Shroff. Opportunistic transmission scheduling with resource-sharing constraints in wireless networks. *IEEE Journal on Selected Areas in Communications*, 19(10):2053–2064, October 2001.

[13] M. Neely. *Stochastic network optimization with application to communication and queueing systems*. Morgan & Claypool, 2010.

[14] M. Neely, E. Modiano, and C. Rohrs. Dynamic power allocation and routing for time varying wireless networks. In *Proc. IEEE International Conference on Computer Communications. (INFOCOM'2003)*, San Francisco, California, April 2003.

[15] M. Neely, E. Modiano, and C. Rohrs. Power allocation and routing in multibeam satellites with time-varying channels. *IEEE/ACM Transactions on Networking*, 11(1):138–152, 2003.

[16] J. Ni, B. Tan, , and R. Srikant. Q-CSMA: Queue length-based CSMA/CA algorithms for achieving maximum throughput and low delay in wireless networks. In *Proc. IEEE International Conference on Computer Communications. (INFOCOM)*, mini-Conference, San Diego, CA, March 2010.

[17] M. Ouyang and L. Ying. On optimal feedback allocation in multichannel wireless downlinks. In *Proc. ACM international symposium on Mobile ad hoc network and computing (MobiHoc)*, Chicago, Illinois, September 2010.

[18] S. Rajagopalan, D. Shah, and J. Shin. Network adiabatic theorem: an efficient randomized protocol for contention resolution. In *Proc. IEEE International Joint Conference on Measurement and Modeling of Computer Systems. (SIGMETRICS)*, Seattle, WA, June 2009.

[19] S. Ross. *Stochastic processes*. John Wiley & Sons, 1995.

[20] S. Ross. *First course in probability*. Prentice Hall, 2005.

[21] L. Tassiulas. Scheduling and performance limits of networks with constantly varying topology. *IEEE Transactions on Information Theory*, 43:1067–1073, May 1997.

[22] L. Tassiulas and A. Ephremides. Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks. *IEEE Transactions on Automatic Control*, 36(12):1936–1948, 1992.

[23] L. Tassiulas and A. Ephremides. Dynamic server allocation to parallel queues with randomly varying connectivity. *IEEE Transactions on Information Theory*, 39(2):466–478, 1993.

[24] L. Ying, R. Srikant, A. Eryilmaz, and G. E. Dullerud. Distributed fair resource allocation in cellular networks in the presence of heterogeneous delays. *IEEE Transactions on Automatic Control*, 52:129–134, 2007.