Throughput-Optimal Scheduling Design with Regular Service Guarantees in Wireless Networks

Bin Li, Ruogu Li, and Atilla Eryilmaz

***Abstract*—Motivated by the regular service requirements of video applications for improving Quality-of-Experience (QoE) of users, we consider the design of scheduling strategies in multi-hop wireless networks that not only *maximize system throughput* but also *provide regular inter-service times* for all links. Since the service regularity of links is related to the higher-order statistics of the arrival process and the policy operation, it is challenging to characterize and analyze directly. We overcome this obstacle by introducing a new quantity, namely the *time-since-last-service* (TSLS), which tracks the time since the last service. By combining it with the queue-length in the weight, we propose a novel maximum-weight type scheduling policy, called Regular Service Guarantee (RSG) Algorithm. The unique evolution of the TSLS counter poses significant challenges for the analysis of the RSG Algorithm.**

To tackle these challenges, we first propose a novel Lyapunov function to show the throughput optimality of the RSG Algorithm. Then, we prove that the RSG Algorithm can provide service regularity guarantees by using the Lyapunov-drift based analysis of the steady-state behavior of the stochastic processes. In particular, our algorithm can achieve a degree of service regularity within a factor of a fundamental lower bound we derive. This factor is a function of the system statistics and design parameters and can be as low as two in some special networks. Our results, both analytical and numerical, exhibit significant service regularity improvements over the traditional throughput-optimal policies, which reveals the importance of incorporating the metric of time-since-last-service into the scheduling policy for providing regulated service.

***Index Terms*—Wireless scheduling, throughput-optimality, service regularity, Quality-of-Experience (QoE), real-time traffic.**

I. Introduction

During the past years, there has been increasing deployment of a variety of real-time applications over the wireless networks, especially streaming multi-media applications. Unlike its non-real-time counterpart, the real-time traffic often has various quality-of-service (QoS) requirements besides throughput. Such requirements usually include end-to-end delay constraints, packet delivery ratio requirements, and the regularity of the inter-service times. Unlike the traditional long-term mean throughput based requirements, these QoS requirements often have a complex dependence on the higher-order statistics of the arrival process as well as the system operation. Thus, the canonical optimization-based approaches that aim to optimize the throughput performance (e.g., [22], [4], [14], [18], [15]) do not apply.

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Recently, valuable efforts have been exerted in the design of algorithms that improve various aspects of the QoS, especially on the delay performance of the algorithms. For example, some works focus on designing algorithms with low end-to-end delay performance, such as [1], [25], [23]. Constant delay bounds (e.g. [16]) and delivery ratio requirements for deadline-constrained traffic (e.g. [7], [8], [9], [10], [12]) are some of the other QoS metrics considered in the literature.

However, these QoS metrics do not fully characterize the Quality-of-Experience (QoE) of users in video applications in wireless networks. To see it, we can envision the network scenario where each individual user wants to download its video from the base station, as shown in Fig. 1. Each mobile

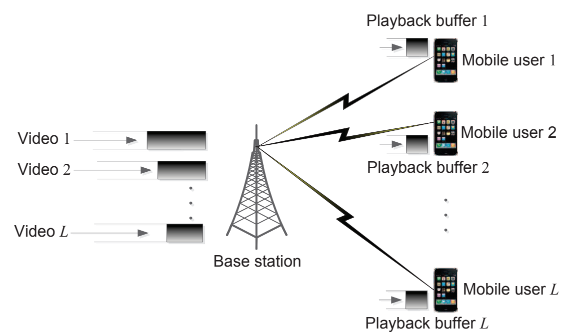


Fig. 1: Cellular network with a single base station and L users

user would like to receive the data from the base station regularly. Indeed, the QoE of users is highly related to the average Perceived Video Quality (PVQ) across the sequence of scenes forming the video, where the PVQ traditionally is a local quality measure associated with a particular scene or a short period of time. In [24], [11], the authors point out that the variance in PVQ leads to the worse QoE than the constant quality video with even smaller average PVQ. Yet, both the time-varying nature of wireless channels and the scheduling policy significantly affect the variance of the received data of each mobile user. Traditional scheduling policies aiming to maximize the system throughput or minimize the delay at the base station side do not take users' experience into account and thus lead to the high variance of the received data of users.

This motivates us to reduce variability of arrivals to the mobile users, which can be achieved by providing regulated inter-service times for the arriving flows at the base station end. However, the inter-service time characteristics are difficult to analyze directly due to: its complex dependence on the high-order statistics of the arrival and service processes, and its non-Markovian evolution. To overcome this, we need to find new approaches to study the inter-service time behavior. To the best of our knowledge, this is the first work that rigorously studies the service regularity of the scheduling policies. Our contributions in this work can be summarized as follows:

• We introduce a new quantity (cf. Section II), namely the *time-since-last-service,* that has a tight relationship with the service regularity performance, and hence enables novel design strategies. Yet, this new parameter has its unique evolution, drastically different from a queue, which poses new challenges for its analysis.

• We develop a novel maximum-weight type scheduling policy that combines the time-since-last-service parameter and the queue-length in its weight measure (cf. Section III). Then, we show that the proposed scheduling policy possesses the desirable throughput optimality property by using a novel Lyapunov function.

• We derive lower and upper bounds on the service regularity performance (cf. Section IV) by utilizing a novel Lyapunov-drift-based argument, inspired by the approach in [3]. We further show that, by properly scaling the design parameter in our policy, we can guarantee a degree of service regularity within a factor of our fundamental lower bound. This factor is a function of the system statistics and design parameters and can be as low as two under symmetric arrival rates in some special networks.

• We support our analytical results with extensive numerical investigations (cf. Section V), which show significant performance gains in the service regularity over the traditional throughput-optimal policies. Furthermore, the numerical investigations indicate that the service regularity performance of our policy actually approaches the lower bounds as the weight of the time-since-last-service increases in some special networks.

This work significantly extends our earlier work [13] in several key aspects: (1) we conduct novel analyses that extend both throughput optimality and service regularity guarantee results to general multi-hop fading networks; (2) we show the existence of all moments of the system state under our proposed algorithm, which establishes the foundation to utilize the Lyapunov-drift based analysis of the steady-state behavior of stochastic processes; (3) we conduct simulations to compare our policy with traditional throughput-optimal scheduling algorithms in more general setups, including switch topologies and fading scenarios.

II. SYSTEM MODEL

We consider a wireless network with L links, where a link represents a pair of a transmitter and a receiver that are within the transmission range of each other. We assume that the system operates in slotted time with normalized slots $t \in \{1, 2, ...\}$. Due to the interference-limited nature of wireless transmissions, the success or failure of a transmission over a link depends on whether an interfering link is also active in the same slot, which is called the *link-based conflict model*. We call a set of links that can be active simultaneously as a *feasible schedule* and denote it as $\mathbf{S}[t] = (S_l[t])_{l=1}^L$, where $S_l[t] = 1$ if the link l is scheduled in slot t and $S_l[t] = 0$, otherwise. We use $\mathcal{S}$ to denote the set of all feasible schedules.

We capture the channel fading over link l via a non-negative-integer-valued random variable $C_l[t]$, with $C_l[t] \le C_{\max}$, $\forall l, t$, for some $C_{\max} < \infty$, which measures the maximum amount of service available in slot t, if the link l is scheduled. We assume that $\mathbf{C}[t] = (C_l[t])_{l=1}^L$, $\forall t \ge 0$, are independently and identically distributed (i.i.d.) over time. We assume that $\bar{c}_{\min} \triangleq \min_l \mathbb{E}[C_l[t]] > 0$. Let $\mathcal{S}^{(\mathbf{c})} \triangleq \{\mathbf{Sc} : \mathbf{S} \in \mathcal{S}\}$ denote the set of feasible rate vectors when the channel is in state $\mathbf{c}$, where $\mathbf{ab} = (a_l b_l)_{l=1}^L$ denotes the component-wise product of two vectors $\mathbf{a}$ and $\mathbf{b}$. Then, the *capacity region* is defined as

$$\mathcal{R} \triangleq \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t] = \mathbf{c}\} \cdot \mathrm{CH}\{\mathcal{S}^{(\mathbf{c})}\}, \tag{1}$$

where $\mathrm{CH}\{\mathcal{A}\}$ denotes a convex hull of the set $\mathcal{A}$, and the summation is a Minkowski addition of sets.

We assume a per-link traffic model[1], where $A_l[t]$ denotes the number of packets arriving at link l in slot t that are independently distributed over links, and i.i.d. over time with finite mean $\lambda_l > 0$, and $A_l[t] \le A_{\max}$, $\forall l, t$, for some $A_{\max} < \infty$. Accordingly, a queue is maintained for each link l with $Q_l[t]$ denoting its queue length at the beginning of time slot t. Then, the evolution of queue l is described as follows:

$$Q_l[t+1] = (Q_l[t] + A_l[t] - C_l[t]S_l[t])^+, \forall l, \tag{2}$$

where $(x)^+ = \max\{x, 0\}$. We say that the queue l is *strongly stable* if it satisfies

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[Q_l[t]] < \infty. \tag{3}$$

We call system *stable* if all queues are strongly stable. In this paper, we consider the policies under which the system evolves as a Markov Chain. We call an algorithm *throughput-optimal* if it makes all queues strongly stable for any arrival rate vector $\boldsymbol{\lambda} = (\lambda_l)_{l=1}^L$ that lies strictly within the capacity region.

In this work, we are interested in providing regular service for each link, which relates to the statistics of the *inter-service time*. We use $I_l[m]$ to denote the time between the $(m-1)^{th}$ and the m^{th} service for link l. If the system is stable, the *steady-state distribution* of the underlying Markov Chain exists (see [17]) and thus we use $\overline{\mathbf{Q}} = (\overline{Q}_l)_{l=1}^L$, $\overline{\mathbf{S}} = (\overline{S}_l)_{l=1}^L$ and $\overline{\mathbf{I}} = (\overline{I}_l)_{l=1}^L$ to denote the random vector with the same steady-state distribution of the queue-length, service processes and inter-service time, respectively. We use the normalized second moment of the inter-service time under the steady-state distribution, i.e., $\mathbb{E}[\overline{I}_l^2]/(\mathbb{E}[\overline{I}_l])^2$, as a measure of the "regularity" of the service that link l receives. Noting that $\mathbb{E}[\overline{I}_l^2]/(\mathbb{E}[\overline{I}_l])^2 = \mathrm{Var}(\overline{I}_l)/(\mathbb{E}[\overline{I}_l])^2 + 1$, the normalized

[1]We note that our algorithm can be extended to serve multi-hop traffic, but the notion of service regularity is clearer in the per-link context.

second moment of the inter-service time reflects its normalized variance. Hence, the smaller the normalized second moment of the inter-service time, the smaller its normalized variance and thus the received service is more regular.

We would like to develop throughput-optimal policies that achieve low values of a linear increasing function of $(\mathbb{E}[\overline{I}_l^2]/(\mathbb{E}[\overline{I}_l])^2)_{l=1}^L$ in steady-state, implying more regular service. However, unlike queue-lengths with Markovian evolution, the dynamics of inter-service times do not lend themselves to commonly used Markovian analysis methods. To overcome this obstacle, we introduce the following related quantity, namely the *time-since-last-service*, which has much more tractable form of evolution, and whose mean has a close relationship to the normalized second moment of the inter-service time (cf. Lemma 1).

For each link l, we introduce a counter T_l, namely Time-Since-Last-Service (TSLS), to keep track of the time since it was lastly *served*, i.e., it was scheduled and the channel was available. Let

$$\tau_l[t] \triangleq \max_{\tau=\{1,\ldots,t-1\}} \left\{ \begin{array}{l} S_l[\tau]C_l[\tau] > 0, S_l[\tau+1]C_l[\tau+1] = \\ \cdots = S_l[t-1]C_l[t-1] = 0 \end{array} \right\},$$

be the last time when link l was served before time slot t, then $T_l[t] = t - \tau_l[t] - 1$. By definition, each counter T_l increases by 1 in each time slot when link l has zero transmission rate, either because it is not scheduled, or because its channel is unavailable, i.e., $C_l[t] = 0$, and drops to 0, otherwise. More precisely, the evolution of the counter T_l can be written as

$$T_l[t+1] = \begin{cases} 0 & \text{if } S_l[t]C_l[t] > 0; \\ T_l[t]+1 & \text{if } S_l[t]C_l[t] = 0. \end{cases} \tag{4}$$

It can be seen from (4) that the evolution of $T_l[t]$ differs significantly from that of a traditional queue (also see Fig. 2). In particular, unlike the slowly evolving nature of queue-lengths, the $T_l[t]$ is incremented until link l receives service at which time it drops to zero. In our design, we will consider policies that not only use queue-lengths to achieve throughput-optimality, but also include TSLS to improve service regularity.

The evolution of T_l is tightly related to the inter-service time I_l, where I_l is the time between two consecutive instances when T_l hits zero, as shown in Fig. 2. In fact, we have the following lemma relating the two in steady-state.

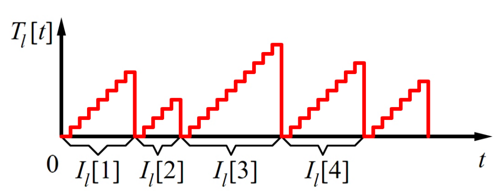


Fig. 2: A sample trajectory of $I_l[m]$ and $T_l[t]$, where the curve shows the evolution of $T_l[t]$.

Lemma 1: For any policy under which the steady-state distribution of the underlying Markov Chain exists, we have

$$\mathbb{E}[\overline{T}_l] = \frac{1}{2}\left(\frac{1}{\mathbb{E}[\overline{I}_l]}\mathbb{E}[\overline{I}_l^2] - 1\right), \tag{5}$$

where $\overline{T}_l$ and $\overline{I}_l$ denote the steady-state TSLS and inter-service time at link l, respectively.

Proof: The detailed proof is provided in Appendix A. ■

Lemma 1 reveals the connection between the second moment of the inter-service time $\overline{I}_l$ and the mean of TSLS $\overline{T}_l$ in steady-state. This can be intuitively seen in Fig. 2, where the area of each "triangle" under the trajectory of $T_l[t]$ is roughly $\frac{1}{2}I_l^2$. In this work, we are interested in designing throughput-optimal algorithms that reduce the total weighted-sum[2] of the normalized second moment of the inter-service time, i.e., $\sum_{l=1}^L \beta_l\rho_l\mathbb{E}[\overline{I}_l^2]/\left(\mathbb{E}[\overline{I}_l]\right)^2$, where $\mu_l \triangleq 1/\mathbb{E}[\overline{I}_l]$, $\rho_l \triangleq \lambda_l/\mu_l$, and $\beta_l \geq 0$ is some parameter related to the link. We can set $\beta_l > 0$ if link l prefers regular service and $\beta_l = 0$ otherwise.

According to Lemma 1, we have

$$\sum_{l=1}^L \beta_l\rho_l\frac{\mathbb{E}[\overline{I}_l^2]}{\left(\mathbb{E}[\overline{I}_l]\right)^2} = 2\sum_{l=1}^L \beta_l\lambda_l\mathbb{E}[\overline{T}_l] + \sum_{l=1}^L \beta_l\lambda_l. \tag{6}$$

Since $\sum_{l=1}^L \beta_l\lambda_l$ only depends on the system parameters, we will use $\sum_{l=1}^L \beta_l\lambda_l\mathbb{E}[\overline{T}_l]$ as our measure for the service regularity. In this work, we aim to design a scheduling policy that is not only throughput-optimal, but also yields provable good characteristics in the service regularity.

We achieve this dual objective by developing a parametric class of throughput-optimal schedulers (cf. Section III-B) that utilize a combination of queue-lengths and TSLS in its decisions. Our policy is shown to guarantee a ratio (as a function of the system statistics) in its service regularity with respect to a fundamental lower bound (cf. Section IV-A).

III. Algorithm design for regular service

In this section, we first discuss the inefficiency of the well-known throughput-optimal Maximum Weight Scheduling (MWS) Algorithm in terms of service regularity. We then propose Regular Service Guarantee policy which can be shown that not only achieves the throughput optimality but also possesses good service regularity performance.

A. Inefficiency of the MWS Algorithm

In this subsection, we describe a well-known scheduling policy, namely the Maximum Weight Scheduling (MWS) Algorithm and discuss its inefficiency in terms of service regularity performance. We first give the definition of the MWS Algorithm for completeness.

Algorithm 1 (Maximum Weight Scheduling (MWS) Algorithm): Under our model, the MWS Algorithm selects a schedule $\mathbf{S}^{(\text{MWS})}[t]$ with the largest total sum of the product of queue-length and the maximum channel available rate within that

[2]The weighting parameter ρ_l is the arrival intensity at link l, and indicates that the link with higher load prefers more regular service. Despite this, ρ_l is included in the objective function primarily for technical reasons. Noting that the link preference parameter β_l can be any non-negative real number, and thus this weighted form is still general enough.

schedule, i.e., it chooses a schedule $\mathbf{S}^{(\text{MWS})}[t]$ such that

$$\mathbf{S}^{(\text{MWS})}[t] \in \arg\max_{\mathbf{S}\in\mathcal{S}} \sum_{l=1}^{L} Q_l[t] C_l[t] S_l[t]. \quad (7)$$

The MWS Algorithm is known to be throughput-optimal (e.g., [22], [15], [19], [2]), i.e., it stabilizes the network for any arrival rate vector $\boldsymbol{\lambda}$ that strictly lies within the capacity region $\mathcal{R}$. In our setup, the MWS Algorithm can be expected to have close-to-lower-bound average delay performance (see [5]). It has also been shown to be heavy-traffic optimal (see [20], [3]), i.e., it minimizes the mean steady-state queue-length under heavy-traffic conditions, where the arrival rate vector approaches the boundary of the capacity region from below.

However, despite its throughput optimality and a number of favorable properties on the delay performance, the MWS Algorithm may result in poor performance in terms of service regularity. This can be observed in Fig. 3 when the MWS Algorithm serves a set of links with heterogeneous arrival statistics in a non-fading fully-connected network with 8 links, where each link can transfer one packet in one time slot if scheduled and at most one link can be scheduled in each slot.

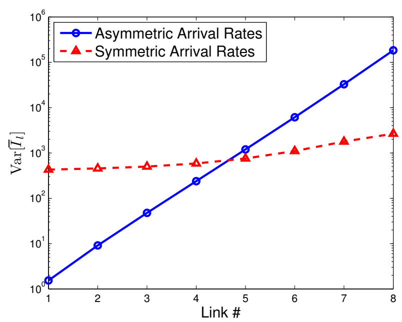


Fig. 3: The variance of the inter-service time under the MWS Algorithm for links with different arrival processes in fully-connected networks with 8 links. The links with smaller rates or more bursty arrivals suffer from high variance of the inter-service time.

In Fig. 3, the blue line shows a scenario where the l^{th} link has a Bernoulli arrival with rate 2^{-l} for $l \in \{1, \ldots, 8\}$. In this case, we observe that the variance of the inter-service time increases exponentially as the arrival rate of the link reduces. The red curve illustrates a different scenario where all 8 links have the same mean arrival rate, but increasing variances (i.e., burstiness) in their arrivals, where we observe that the link with more bursty arrivals suffers from higher variance in its inter-service time.

B. The Regular Service Guarantee Policy

As discussed above, the MWS Algorithm is throughput-optimal but inefficient in providing regular services. Note that the introduced TSLS counter has a direct impact on service regularity: the smaller the mean TSLS value, the more regular the service. This interesting connection motivates the following parametrized policy which is later revealed to possess the characteristics of throughput optimality and service regularity.

Algorithm 2 (Regular Service Guarantee (RSG) Algorithm): In each time slot t, select a schedule $\mathbf{S}^*[t]$ such that

$$\mathbf{S}^*[t] \in \arg\max_{\mathbf{S}\in\mathcal{S}} \sum_{l=1}^{L} (\alpha_l Q_l[t] + \gamma \beta_l T_l[t]) C_l[t] S_l, \quad (8)$$

where $\alpha_l > 0$ and $\gamma \geq 0$ are fixed control parameters.

We note that there are two sets of control parameters in the RSG Algorithm[3] and they affect different behaviors of the algorithm. Yet, it will be revealed later that none of them affects its throughput optimality. The parameters α_l are weighing factors for the queue-lengths, where a larger α_l will result in a smaller average queue-length. The parameter γ is a common weighing factor of TSLS for all links. It will be revealed in Section IV-B that the design parameter γ can improve the service regularity as it increases. Also note that when $\gamma = 0$, our policy coincides with the MWS Algorithm. When $\gamma > 0$, with the addition of $T_l[t]$ terms in the weight of each link, our algorithm operates completely different from the MWS and its approximate algorithms, which, to the best of our knowledge, are the only known policies possessing the throughput-optimality characteristic in general multi-hop network topologies. Despite of this, we can still show that our algorithm is throughput-optimal.

Proposition 1: The RSG Algorithm with any $\alpha_l > 0$, $\beta_l \geq 0$ and $\gamma \geq 0$, is throughput-optimal, i.e., for any arrival rate vector $\boldsymbol{\lambda} \in \text{Int}(\mathcal{R})$, the RSG Algorithm stabilizes the system, with

$$\limsup_{K\to\infty} \frac{1}{K} \sum_{t=0}^{K-1} \sum_{l=1}^{L} \alpha_l \mathbb{E}[Q_l[t]] \leq \frac{B(\boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma)}{2\epsilon}, \quad (9)$$

where Int($\mathcal{A}$) denotes the interior points of the region $\mathcal{A}$, and $B(\boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma) \triangleq 4\gamma C_{\max} \sum_{l=1}^{L} \beta_l + \sum_{l=1}^{L} \alpha_l \mathbb{E}\left[A_l^2[t] + C_l^2[t]\right]$.

Proof: Consider the Lyapunov function

$$W(\mathbf{Q}[t], \mathbf{T}[t]) \triangleq \sum_{l=1}^{L} \alpha_l Q_l^2[t] + 4\gamma C_{\max} \sum_{l=1}^{L} \beta_l T_l[t]. \quad (10)$$

It is shown in Appendix B that there exists a positive constant $\epsilon > 0$ such that

$$\begin{aligned}\Delta W \triangleq& \mathbb{E}\left[W(\mathbf{Q}[t+1], \mathbf{T}[t+1]) - W(\mathbf{Q}[t], \mathbf{T}[t]) | \mathbf{Q}[t], \mathbf{T}[t]\right] \\ \leq& -2\epsilon \sum_{l=1}^{L} \alpha_l Q_l[t] + B(\boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma). \quad (11)\end{aligned}$$

Taking the expectation on the both sides of (11) and summing over $t = 0, 1, ..., K-1$, we have the desired result. ■

[3]The RSG Algorithm inherits the same complexity issue as the well-known MWS Algorithm. The low complexity or the distributed implementations of the RSG Algorithm are always attractive in practical networks and are left for future research.

Proposition 1 establishes the throughput optimality of the RSG Algorithm, thus $Q_l[t]$ and $T_l[t]$ will converge in distribution to $\overline{Q}_l^*$ and $\overline{T}_l^*$, which attain the steady-state distribution under our policy. Proposition 1 also gives an upper bound for the expected total queue-length under the steady-state, which increases linearly with the design parameter γ. It will be revealed later that γ controls the tradeoff between the average total queue-length, and the service regularity performance, especially in the heterogenous networks.

Next, we will show that all moments of steady-state system variables, such as queue-lengths and TSLS, are bounded under the RSG Algorithm, which enables us to analyze the service regularity performance by using the Lyapunov-type approach developed in [3]. In [6], the sufficient condition for all moments of state variables of a Markov Chain to exist in steady state is given as finding a Lyapunov function that satisfies: (1) it has a negative Lyapunov drift when the system variable is large enough; (2) the absolute value of the Lyapunov drift is bounded or has an exponential tail. From equation (11), we know that the first condition holds. Yet, the second condition is hard to hold for the Lyapunov function $W(\mathbf{Q},\mathbf{T})$ or its variant $\sqrt{W(\mathbf{Q},\mathbf{T})}$ due to the unique evolution of TSLS counters (see equation (4)), which have bounded increment but unbounded decrement. We tackle this challenge by properly partitioning the system space.

Proposition 2: For any arrival rate vector $\boldsymbol{\lambda} \in \text{Int}(\Lambda)$, all moments of steady-state queue length and TLSL exist under the RSG Algorithm with any $\alpha_l > 0$, $\beta_l \geq 0$ and $\gamma > 0$.

Proof: We show the boundedness of $\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t]\|_2}\right]$ for some $\eta > 0$ by intelligently partitioning the system space, where $\mathbf{Y}[t] \triangleq \left(\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t], \sqrt{4\gamma C_{\max}\boldsymbol{\beta}\mathbf{T}[t]}\right)$, $\sqrt{\mathbf{x}}$ denotes the component-wise square root of the vector $\mathbf{x}$, and $\mathbf{xy}$ denotes the component-wise product of the vectors $\mathbf{x}$ and $\mathbf{y}$. Please see Appendix C for details. ■

Having established the throughput optimality and the moment existence of the system state variables of the RSG Algorithm, we are ready to analyze the service regularity performance, i.e., $\sum_{l=1}^{L}\beta_l\lambda_l\mathbb{E}[\overline{T}_l]$.

IV. Service Regularity Performance Analysis

In this section, we study the service regularity performance of our proposed RSG Algorithm analytically. We first establish a fundamental lower bound on the service regularity for any feasible scheduling algorithm. Then, we derive an upper bound on the service regularity under the RSG Algorithm. These investigations reveal that the service regularity performance of the RSG Algorithm can be guaranteed to remain within a factor of the lower bound, which is expressed as a function of the system statistics and the design parameters, and can be as low as 2 in some special networks. We assume the parameter $\gamma > 0$ throughout this section.

A. Lower Bound Analysis

In this subsection, we derive a lower bound based on a Lyapunov drift argument inspired by the technique used in [3]. To study the lower bound of the service regularity by the Lyapunov drift argument, we consider a class of policies, called $\mathcal{P}$, that not only stabilize the system but also yield the bounded second moment of the steady-state TSLS[4]. Note that our proposed algorithm, as well as the MWS algorithm, falls into this class by Propositions 1 and 2.

Let $\overline{T}_l^{(p)}$ and $\overline{S}_l^{(p)}$ be the steady-state TSLS and scheduling variable for link l under policy $p \in \mathcal{P}$, respectively. The following lemma gives key identities for the first and second moment of the steady-state TSLS, which are useful in deriving a lower bound on the service regularity.

Lemma 2: For any policy $p \in \mathcal{P}$, we have

$$\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}^{(p)}}\beta_l\lambda_l\overline{T}_l^{(p)}\right] = \sum_{l=1}^{L}\beta_l\lambda_l - \mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}^{(p)}}\beta_l\lambda_l\right], \quad (12)$$

$$\begin{aligned} 2\sum_{l=1}^{L}\beta_l\lambda_l\mathbb{E}\left[\overline{T}_l^{(p)}\right] &= \sum_{l=1}^{L}\beta_l\lambda_l - \mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}^{(p)}}\beta_l\lambda_l\right] \\ &\quad + \mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}^{(p)}}\beta_l\lambda_l\left(\overline{T}_l^{(p)}\right)^2\right], \quad (13)\end{aligned}$$

where $\overline{\mathbf{H}}^{(p)} \triangleq \{l : \overline{C}_l\overline{S}_l^{(p)} > 0\}$, and $\overline{\mathbf{C}} = (\overline{C}_l)_{l=1}^{L}$ has the same probability distribution as $\mathbf{C}[t] = (C_l[t])_{l=1}^{L}$.

Proof: See Appendix G for the proof. ■

We are ready to give a lower bound on the service regularity for any feasible policy $p \in \mathcal{P}$.

Proposition 3: For any policy $p \in \mathcal{P}$, we have

$$\sum_{l=1}^{L}\beta_l\lambda_l\mathbb{E}\left[\overline{T}_l^{(p)}\right] \geq \frac{1}{2}\left(\frac{\sum_{l=1}^{L}\beta_l\lambda_l}{\max_{\mathbf{S}\in\mathcal{S}}\sum_{l\in\mathbf{S}}\beta_l\lambda_l} - 1\right)\sum_{l=1}^{L}\beta_l\lambda_l.$$

Proof: In the rest of proof, we will omit superscript p for conciseness. For any sample path, by Cauchy-Schwarz inequality, we have

$$\begin{aligned}\left(\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l\right)^2 &= \left(\sum_{l\in\overline{\mathbf{H}}}\sqrt{\beta_l\lambda_l}\cdot\sqrt{\beta_l\lambda_l}\overline{T}_l\right)^2 \\ &\leq \left(\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\right)\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l^2, \quad (14)\end{aligned}$$

where we recall that $\overline{\mathbf{H}} \triangleq \{l : \overline{C}_l\overline{S}_l > 0\}$. This implies

$$\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l^2 \geq \frac{\left(\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l\right)^2}{\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l}. \quad (15)$$

[4]We conjecture that the second moment of the steady-state TSLS is bounded as long as the system is stable.

Hence, we have

$$\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l^2\right] \geq \mathbb{E}\left[\frac{\left(\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l\right)^2}{\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l}\right]$$
$$\stackrel{(a)}{\geq}\frac{\left(\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l\right]\right)^2}{\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\right]}$$
$$\stackrel{(b)}{=}\frac{\left(\sum_{l=1}^{L}\beta_l\lambda_l-\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\right]\right)^2}{\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\right]}, \quad (16)$$

where the step (a) uses the fact that $f(x,y)=\frac{x^2}{y}$ is convex and Jensen's inequality for a multi-variable function; step (b) follows from (12). By substituting (16) into (13), we have

$$\sum_{l=1}^{L}\beta_l\lambda_l\mathbb{E}\left[\overline{T}_l\right]\geq\frac{1}{2}\left(\frac{\sum_{l=1}^{L}\beta_l\lambda_l}{\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\right]}-1\right)\sum_{l=1}^{L}\beta_l\lambda_l. \quad (17)$$

Note that

$$\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\right]=\mathbb{E}\left[\sum_{l=1}^{L}\beta_l\lambda_l\mathbb{1}_{\{\overline{C}_l\overline{S}_l>0\}}\right]$$
$$=\sum_{l=1}^{L}\beta_l\lambda_l\Pr\{\overline{C}_l\overline{S}_l>0\}$$
$$\leq\sum_{l=1}^{L}\beta_l\lambda_l\Pr\{\overline{S}_l=1\}\leq\max_{\mathbf{S}\in\mathcal{S}}\sum_{l\in\mathbf{S}}\beta_l\lambda_l. \quad (18)$$

By substituting (18) into (17), we have the desired result. ■

Consider a fully-connected non-fading network, where only one link is scheduled in each time slot. Let $\beta_l=\beta$ and $\lambda_l=\lambda$ for each link l. Then, the lower bound becomes

$$\sum_{l=1}^{L}\mathbb{E}\left[\overline{T}_l^{(p)}\right]\geq\frac{1}{2}L(L-1). \quad (19)$$

This lower bound can be achieved by the Round-Robin (RR) policy, which serves each link periodically. Indeed, in the steady-state, the TSLS vector under the RR policy is a permutation of $\{0,1,2,...,L-1\}$ and thus $\sum_{l=1}^{L}\mathbb{E}\left[\overline{T}_l^{(\mathrm{RR})}\right]=\frac{1}{2}L(L-1)$.

Yet, we would like to point out that the RR policy is not throughput-optimal. Thus, for an arrival rate vector $\boldsymbol{\lambda}$ that cannot be supported by the RR policy, we do not expect a throughput-optimal policy to approach the above lower bound when serving it. However, for the arrival rate vectors that can be supported by the RR policy, we shall see in our numerical results that the performance of our policy can approach this lower bound when we increase the scaling parameter γ.

B. Upper Bound Analysis

In this subsection, we obtain an upper bound on the service regularity under the RSG Algorithm. Let $\overline{Q}_l^*$, $\overline{S}_l^*$ and $\overline{T}_l^*$ be the steady-state queue-length, scheduling variable and TSLS for link l under the RSG Algorithm, respectively.

Proposition 4: For the RSG Algorithm, we have

$$\sum_{l=1}^{L}\beta_l\lambda_l\mathbb{E}\left[\overline{T}_l^*\right]\leq\frac{C_{\max}}{1+\epsilon}\left(\sum_{l=1}^{L}\beta_l-\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}^*}\beta_l\right]\right)$$
$$+\frac{1}{2\gamma(1+\epsilon)}\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{A}_l^2+\overline{C}_l^2\right], \quad (20)$$

where $\epsilon>0$ satisfies $\boldsymbol{\lambda}(1+\epsilon)\in\mathcal{R}$, $\overline{\mathbf{H}}^*\triangleq\{l:\overline{C}_l\overline{S}_l^*>0\}$, and $\overline{\mathbf{A}}=(\overline{A}_l)_{l=1}^L$ has the same distribution as $\mathbf{A}[t]=(A_l[t])_{l=1}^L$.

Proof: See Appendix H for the details. ■

Note that the second term of the right hand side of (20) captures various random effects in the network: the burstiness of the arrival processes and the channel variations. Under our policy these effects diminish as the scaling factor γ goes to infinity. Hence, together with Proposition 1, Proposition 4 reveals a tradeoff: when increasing γ, the upper bound on the total queue-length increases linearly with γ, but the upper bound for the service regularity decreases.

Consider the fully-connected non-fading network as in Section IV-A. Let $\beta_l=\beta$ and $\lambda_l=\lambda=\frac{1}{L(1+\epsilon)}$ for each link l. Then, as γ goes to infinity, (20) becomes

$$\sum_{l=1}^{L}\mathbb{E}\left[\overline{T}_l^*\right]\leq L(L-1), \quad (21)$$

which is always within twice the value of the lower bound (19). In the more general case, the upper bound converges to a constant that is determined by the system statistics and design parameters as γ goes to infinity. Moreover, we shall see in the numerical results presented in Section V-B that as γ increases, the service regularity performance under the RSG Algorithm actually converges to the lower bound (19) in the fully-connected non-fading network with the symmetric parameters.

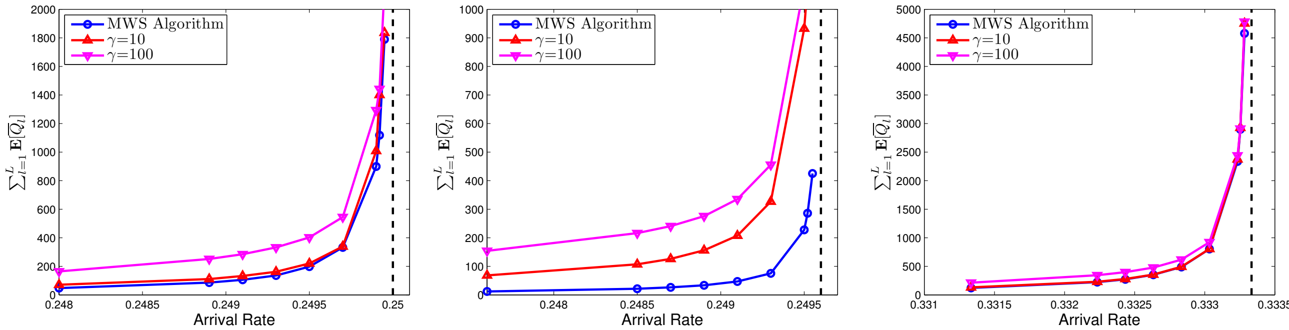
V. Numerical Results

In this section, we provide simulation results for our proposed RSG Algorithm and compare its performance to the MWS Algorithm and the Oldest Cell First (OCF) Algorithm[5] (see [**?**]). In addition to investigating the throughput (cf. Section V-A) and service regularity (cf. Section V-B) performances of our policy in both fully-connected network with $L=4$ links and 3×3 switch, we also look at the behavior of the RSG Algorithm as well as the potential benefit of the service regularity (cf. Section V-C). In the first two simulations, we assume Bernoulli arrivals to each link and $\alpha_l=\beta_l=1$ for each link l.

A. Throughput Performance

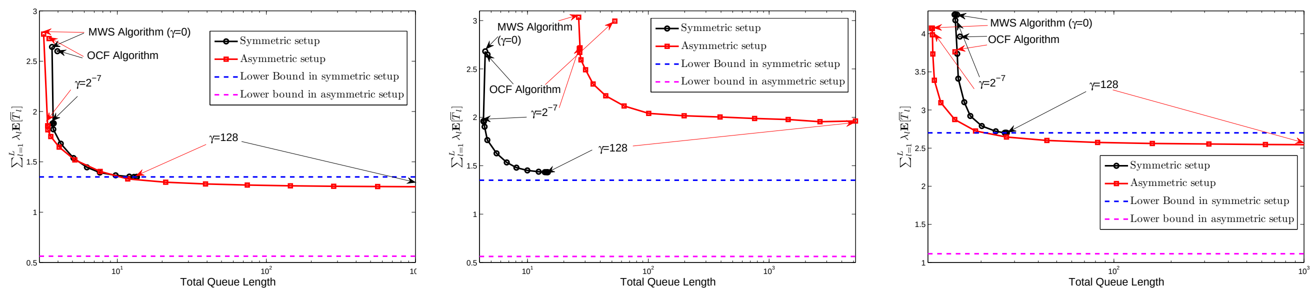
In this subsection, we illustrate the throughput performance of the RSG Algorithm in three different network setups with symmetric arrivals: (i) fully-connected non-fading network,

[5]The OCF Algorithm prioritizes activation of links with the greatest Head-of-Line (HOL) delay.



(a) Fully-connected non-fading network (b) Fully-connected fading network (c) 3×3 switch

Fig. 4: The throughput performance of the RSG Algorithm



(a) Fully-connected non-fading network (b) Fully-connected fading network (c) 3×3 switch

Fig. 5: Trade-off between mean queue length and the service regularity

(ii) fully-connected network with symmetric ON-OFF fading channels with probability $q = 0.8$ that the channel is available, and (iii) 3×3 switch. The achievable rate regions for these three networks, respectively, are

$$\Lambda_1 \triangleq \left\{ \boldsymbol{\lambda} = (\lambda_l)_{l=1}^4 : \lambda_1 = \lambda_2 = ... = \lambda_4 < \frac{1}{4} \right\},$$
$$\Lambda_2 \triangleq \left\{ \boldsymbol{\lambda} = (\lambda_l)_{l=1}^4 : \lambda_1 = \lambda_2 = ... = \lambda_4 < \frac{1-(1-q)^4}{4} \right\},$$
$$\Lambda_3 \triangleq \left\{ \boldsymbol{\lambda} = (\lambda_l)_{l=1}^9 : \lambda_1 = \lambda_2 = ... = \lambda_9 < \frac{1}{3} \right\}.$$

In Fig. 4, we compare the total mean queue-length under the MWS Algorithm, as well as the RSG Algorithm with different γ values. It can be observed in Fig. 4 that the RSG Algorithm can stabilize the system in the above network setups. It also can be seen that the total mean queue-length of the RSG Algorithm increases with the parameter γ. This is expected since as γ increases, it becomes more likely for the RSG Algorithm to choose a queue with less packet to serve, potentially wasting some service while improving the service regularity, as we shall see next.

B. Service Regularity Performance

In this subsection, we investigate the service regularity performance of our RSG Algorithm, as well as illustrate the tradeoff between the total mean queue-length and the service regularity. We present our results in three different networks: fully-connected non-fading network, fully-connected fading network and 3×3 switch. In both fully-connected nonfading and fading networks, we consider the symmetric setup with the arrival rate vector $\boldsymbol{\lambda} \triangleq [0.225, 0.225, 0.225, 0.225]$, and the asymmetric setup with the arrival rate vector $\boldsymbol{\lambda} \triangleq [0.4, 0.3, 0.15, 0.05]$. For a fully-connected ON-OFF fading network, the probability vectors that the channels are available are $\mathbf{q} = [0.8, 0.8, 0.8, 0.8]$ in symmetric setup and $\mathbf{q} = [0.6, 0.5, 0.4, 0.3]$ in asymmetric setup. For a 3×3 switch, we consider the symmetric setup with the arrival rate vector $\boldsymbol{\lambda} \triangleq [0.3, 0.3, 0.3; 0.3, 0.3, 0.3; 0.3, 0.3, 0.3]$ and the asymmetric setup with the arrival rate vector $\boldsymbol{\lambda} \triangleq [0.5, 0.3, 0.1; 0.2, 0.4, 0.3; 0.1, 0.2, 0.5]$. In all simulations, we choose the scaling parameter γ to be the powers of 2, ranging from 2^{-7} to 2^7.

Fig. 5 shows the relationship between the total mean queue-length and the service regularity in different network setups. The tradeoff between the service regularity and the total mean queue-length can be clearly seen: as γ increases, the service regularity improves while the total mean queue-length also increases. Also, we can observe that the OCF Algorithm does not improve the service regularity performance significantly compared to the MWS Algorithm. The reason lies in that both MWS and OCF Algorithms perform similarly among the links that have continuous arrivals, which can be seen from the Little's Law. In addition, it has been shown that the

MWS Algorithm is delay-optimal in fully-connected networks with symmetric Bernoulli arrivals and ON-OFF fading channels (see [**?**]). From Fig. 5(b), we can see that the delay-optimal algorithm may fail to meet the certain service regularity requirement. Furthermore, it can be observed that the simulated values converge to the fundamental lower bound in non-fading networks with symmetric setup (Figs 5(a) and (c)), while they stay away from the lower bound in asymmetric setups. This motivates us to refine the lower bound analysis in asymmetric setups, which is left for future investigation. Here, it is worth mentioning that even with very small γ values (e.g., 2^{-5}), our RSG Algorithm significantly improves the service regularity, while introducing negligible increase in the total mean queue-length.

C. Benefit of the Service Regularity

In this subsection, we study various performance metrics (such as mean unused service, service regularity, mean queue-length and variance of queue-length) among links to illustrate the behavior of the RSG Algorithm as well as the benefit of the service regularity. To that end, we consider a fully-connected non-fading network with two links. Each link can serve 4 packets in each time slot if scheduled. There is always 1 packet arriving at the first link in each time slot, while the number of packets arriving at the second link is either $2K$ with probability $1/K$ or 0, where K is a natural number. We compare the performance between the MWS Algorithm, the RSG Algorithm and the variant of the RSG Algorithm whose TSLS counter increases only when the link does not receive service and its queue-length is non-zero. In both RSG Algorithm and its variant, we set $\beta_1 = 1, \beta_2 = 0, \gamma = 10$, i.e., we assume that the first link prefers the regular service while the second link does not have such a requirement. In the following simulations, we set $K = 5$.

From Fig. 6a, we observe that compared to the MWS Algorithm, under the RSG Algorithm, the mean unused service in the first link slightly increases, while in the second it slightly decreases. This is expected since the TSLS counter increases even when the queue-length is non-zero. Yet, the total amount of mean unused service under the RSG Algorithm remains the same as that under the MWS Algorithm. For the variant of the RSG Algorithm, the mean unused service for each individual link almost does not change.

From Fig. 6b, we can see that both the RSG Algorithm and its variant improve the service regularity compared to the MWS Algorithm. Also, the RSG Algorithm yields the better service regularity performance than its variant. This is because the TSLS counter under the variant of the RSG Algorithm is not as aggressive as that under the original RSG Algorithm. As can be seen in Fig. 6c and Fig. 6d, providing more regular service is extremely beneficial for the link with constant arrivals since it leads to the smaller mean and variance of delay that each packet experiences in that link.

Next, we would like to reveal the relationship between the service regularity of the first link with constant arrivals and the burstiness of arrivals at the second link that is reflected by the parameter K. The larger the K, the more bursty the arrivals at the second link. Fig. 7 shows the impact of the bursty arrivals on the service regularity of the link with the constant arrival under both MWS and RSG Algorithms. We can observe from Fig. 7 that the service regularity of the first link under the MWS Algorithm degrades much faster than that under the RSG Algorithm as the the burstiness of the second link increases. Also, as γ increases, under the RSG Algorithm, the service regularity of the first link improves significantly, and it is almost independent of the burstiness of the second link when $\gamma = 100$.

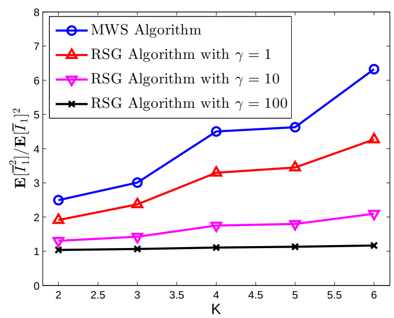


Fig. 7: The impact of the bursty arrivals on the service regularity of the constant flow

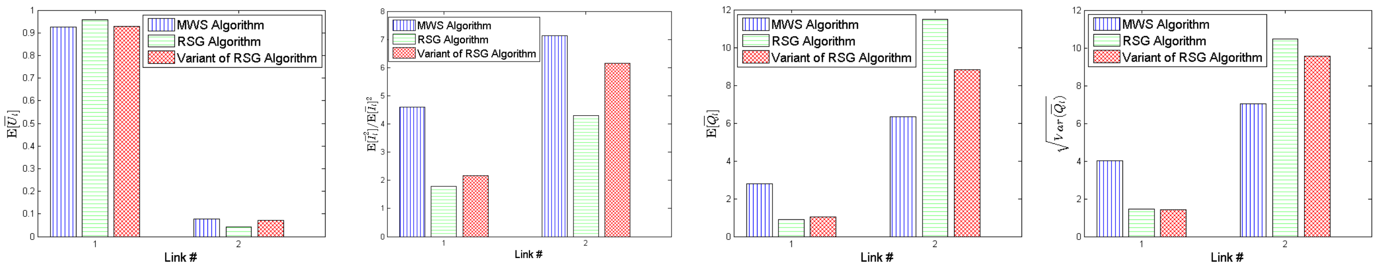
VI. Conclusion

In this work, we investigated the problem of designing a scheduling policy that is both throughput-optimal and possesses favorable service regularity characteristics. We introduced a new parameter of time-since-last-service, and proposed a novel scheduling policy that combines this parameter with the queue-lengths in its weight. After establishing the throughput optimality of our policy, we showed that it also has provable service regularity performance. In particular, the service regularity of our policy can be guaranteed to remain within a factor distance of a fundamental lower bound for any feasible scheduling policy. We explicitly expressed this factor as a function of the system statistics and the design parameters. We performed extensive numerical studies to illustrate the significant gains achieved by our policy over the traditional queue-length or head-of-line-delay based policies. Our results show the significance of utilizing the time-since-last-service in improving the service regularity performance of throughput-optimal policies.

Appendix A
Proof of Lemma 1

Without loss of generality, assume that link l is served at time 0. For any positive integer M, there exists an m such that

$$\begin{aligned} I_l[1] + \cdots + I_l[m] &\leq M, \\ I_l[1] + \cdots + I_l[m] + I_l[m+1] &\geq M. \end{aligned}$$



(a) Mean unused service (b) Service regularity performance (c) Mean queue length (d) Standard deviation of queue length

Fig. 6: Performance comparison between the RSG Algorithm and the MWS Algorithm

We can write

$$\frac{1}{M}\sum_{t=1}^{M}T_l[t] = \frac{1}{M}\Bigg(\sum_{t=1}^{I_l[1]}T_l[t] + \sum_{t=I_l[1]+1}^{I_l[1]+I_l[2]}T_l[t] + \dots + \sum_{t=I_l[1]+\dots+I_l[m]+1}^{M}T_l[t]\Bigg). \quad (22)$$

We observe the following fact: assume link l receives its $(m-1)^{th}$ and m^{th} service at time slot t_1 and t_2, respectively, where $t_2 > t_1$. Then, by definition, $I_l[m] = t_2 - t_1$, $T_l[t_1+1] = 0$ and $T_l[t_2] = t_2 - t_1 - 1 = I_l[m] - 1$. Using this fact, we know the k^{th} summation on the right hand side of (22) gives $\frac{1}{2}(I_l[k](I_l[k]-1))$, except for the last one. Thus we have:

$$\frac{1}{M}\sum_{t=1}^{M}T_l[t] \geq \frac{1}{M}\sum_{k=1}^{m}\frac{I_l[k](I_l[k]-1)}{2}, \quad (23)$$

$$\frac{1}{M}\sum_{t=1}^{M}T_l[t] \leq \frac{1}{M}\sum_{k=1}^{m+1}\frac{I_l[k](I_l[k]-1)}{2}. \quad (24)$$

By the definition of m and the fact that $\mathbb{E}[\overline{I}_l] < \infty$ when each link is served with strictly positive probability, we know that $m \to \infty$ when $M \to \infty$. Since the policy is assumed to be ergodic, we have

$$\lim_{M\to\infty}\frac{1}{M}\sum_{t=1}^{M}T_l[t] = \mathbb{E}[\overline{T}_l],$$

$$\lim_{m\to\infty}\frac{1}{m}\sum_{k=1}^{m}I_l[k] = \mathbb{E}[\overline{I}_l],$$

$$\lim_{m\to\infty}\frac{1}{m}\sum_{k=1}^{m}I_l^2[k] = \mathbb{E}[\overline{I}_l^2].$$

Note that

$$\frac{1}{m}\sum_{k=1}^{m}I_l[k] \leq \frac{M}{m} \leq \frac{m+1}{m}\frac{1}{m+1}\sum_{k=1}^{m+1}I_l[k].$$

By taking limit as M goes to infinity, we get

$$\lim_{M\to\infty}\frac{M}{m} = \mathbb{E}[\overline{I}_l].$$

By taking limit on (23) and (24) as $M \to \infty$, we have

$$\begin{aligned}&\lim_{M\to\infty}\frac{m}{M}\frac{1}{m}\sum_{k=1}^{m}\frac{I_l[k](I_l[k]-1)}{2}\\ \leq\ &\lim_{M\to\infty}\frac{1}{M}\sum_{t=1}^{M}T_l[t]\\ \leq\ &\lim_{M\to\infty}\frac{m+1}{M}\frac{1}{m+1}\sum_{k=1}^{m+1}\frac{I_l[k](I_l[k]-1)}{2},\end{aligned}$$

and thus we have the desired result.

Appendix B
Proof of inequality (11)

$$\begin{aligned}\Delta W \triangleq&\mathbb{E}\left[W(\mathbf{Q}[t+1],\mathbf{T}[t+1]) - W(\mathbf{Q}[t],\mathbf{T}[t])|\mathbf{Q}[t],\mathbf{T}[t]\right]\\ =&\mathbb{E}\Bigg[\sum_{l=1}^{L}\alpha_l Q_l^2[t+1] + 4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t+1]\\ &-\sum_{l=1}^{L}\alpha_l Q_l^2[t] - 4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t]\Bigg|\mathbf{Q}[t],\mathbf{T}[t]\Bigg]\\ \leq&\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[(Q_l[t]+A_l[t]-C_l[t]S_l^*[t])^2 - Q_l^2[t]\big|\mathbf{Q}[t],\mathbf{T}[t]\right]\\ &+4\gamma C_{\max}\mathbb{E}\left[\sum_{l=1}^{L}\beta_l T_l[t+1] - \sum_{l=1}^{L}\beta_l T_l[t]\Bigg|\mathbf{Q}[t],\mathbf{T}[t]\right],\end{aligned} \quad (25)$$

where the last step follows from the evolution of each queue, and $(\max\{x,0\})^2 \leq x^2$.

Let $\mathbf{H}^* \triangleq \{l : S_l^*[t]C_l[t] > 0\}$. According to the definition of the TSLS counter, we have

$$\begin{aligned}&\sum_{l=1}^{L}\beta_l T_l[t+1]\\&=\sum_{l\notin\mathbf{H}^*}\beta_l\left(T_l[t]+1\right)\\&=\sum_{l=1}^{L}\beta_l T_l[t]-\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]+\sum_{l=1}^{L}\beta_l-\sum_{l\in\mathbf{H}^*}\beta_l \qquad (26)\\&\le\sum_{l=1}^{L}\beta_l T_l[t]-\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]+\sum_{l=1}^{L}\beta_l. \qquad (27)\end{aligned}$$

By substituting inequality (27) into (25), we have

$$\begin{aligned}&\Delta W\le\\&\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[(Q_l[t]+A_l[t]-C_l[t]S_l^*[t])^2-Q_l^2[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&+4\gamma C_{\max}\mathbb{E}\left[\sum_{l=1}^{L}\beta_l-\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&\le\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[2Q_l[t](A_l[t]-C_l[t]S_l^*[t])|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&+\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[(A_l[t]-C_l[t]S_l^*[t])^2\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l-4\gamma C_{\max}\mathbb{E}\left[\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&\le 2\sum_{l=1}^{L}\alpha_l\lambda_l Q_l[t]-2\mathbb{E}\left[\sum_{l=1}^{L}\alpha_l Q_l[t]C_l[t]S_l^*[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&-4\gamma C_{\max}\mathbb{E}\left[\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]+B(\boldsymbol{\alpha},\boldsymbol{\beta},\gamma), \qquad (28)\end{aligned}$$

where $B(\boldsymbol{\alpha},\boldsymbol{\beta},\gamma)$ is defined in Proposition 1.

Let $\mathbf{S}^{(\mathrm{MWS})}[t]\in\arg\max_{\mathbf{S}\in\mathcal{S}}\sum_{l=1}^{L}\alpha_l Q_l[t]C_l[t]S_l$. Then, by the definition of the RSG Algorithm, we have

$$\begin{aligned}&\sum_{l=1}^{L}(\alpha_l Q_l[t]+\gamma\beta_l T_l[t])C_l[t]S_l^*[t]\\&\ge\sum_{l=1}^{L}(\alpha_l Q_l[t]+\gamma\beta_l T_l[t])C_l[t]S_l^{(\mathrm{MWS})}[t]\\&\ge\sum_{l=1}^{L}\alpha_l Q_l[t]C_l[t]S_l^{(\mathrm{MWS})}[t],\end{aligned}$$

which implies

$$\begin{aligned}&\sum_{l=1}^{L}\alpha_l Q_l[t]C_l[t]S_l^*[t]\\&\ge\sum_{l=1}^{L}\alpha_l Q_l[t]C_l[t]S_l^{(\mathrm{MWS})}[t]-\gamma\sum_{l=1}^{L}\beta_l T_l[t]C_l[t]S_l^*[t]. \qquad (29)\end{aligned}$$

By substituting (29) into (28), we have

$$\begin{aligned}&\Delta W\\&\le 2\sum_{l=1}^{L}\alpha_l\lambda_l Q_l[t]-2\mathbb{E}\left[\sum_{l=1}^{L}\alpha_l Q_l[t]C_l[t]S_l^{(\mathrm{MWS})}[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&+2\gamma\mathbb{E}\left[\sum_{l=1}^{L}\beta_l T_l[t]C_l[t]S_l^*[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&-4\gamma C_{\max}\mathbb{E}\left[\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]+B(\boldsymbol{\alpha},\boldsymbol{\beta},\gamma). \qquad (30)\end{aligned}$$

Given $\mathbf{Q}[t]$ and $\mathbf{T}[t]$, we have

$$\begin{aligned}&C_{\max}\mathbb{E}\left[\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&\ge\mathbb{E}\left[\sum_{l=1}^{L}\beta_l T_l[t]C_l[t]S_l^*[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right], \qquad (31)\end{aligned}$$

where we recall that $\mathbf{H}^*=\{l:S_l^*[t]C_l[t]>0\}$. By substituting (31) into (30), we have

$$\begin{aligned}&\Delta W\\&\le 2\sum_{l=1}^{L}\alpha_l\lambda_l Q_l[t]-2\mathbb{E}\left[\sum_{l=1}^{L}\alpha_l Q_l[t]C_l[t]S_l^{(\mathrm{MWS})}[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\\&-2\gamma\mathbb{E}\left[\sum_{l=1}^{L}\beta_l T_l[t]C_l[t]S_l^*[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]+B(\boldsymbol{\alpha},\boldsymbol{\beta},\gamma). \qquad (32)\end{aligned}$$

Note that the capacity region $\mathcal{R}$ (see [21]) is also equivalent to a set of arrival rate vectors $\boldsymbol{\lambda}$ such that there exist non-negative numbers $\theta(\mathbf{c};\mathbf{s})$ satisfying

$$\sum_{\mathbf{s}\in\mathcal{S}}\theta(\mathbf{c};\mathbf{s})=1,\forall\mathbf{c}, \qquad (33)$$

$$\lambda_l\le\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\sum_{\mathbf{s}\in\mathcal{S}}\theta(\mathbf{c};\mathbf{s})c_l s_l,\forall l, \qquad (34)$$

where $\mathbf{s}=(s_l)_{l=1}^{L}$. For any $\boldsymbol{\lambda}\in\mathrm{Int}(\mathcal{R})$, there exists an $\epsilon>0$ such that

$$\lambda_l\le\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\sum_{\mathbf{s}\in\mathcal{S}}\theta(\mathbf{c};\mathbf{s})c_l s_l-\epsilon,\forall l. \qquad (35)$$

Hence, we have

$$\sum_{l=1}^{L}\alpha_l\lambda_l Q_l[t]+\epsilon\sum_{l=1}^{L}\alpha_l Q_l[t]$$
$$\le\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\sum_{\mathbf{s}\in\mathcal{S}}\theta(\mathbf{c};\mathbf{s})\sum_{l=1}^{L}\alpha_l Q_l[t]c_l s_l$$
$$\overset{(a)}{\le}\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\sum_{\mathbf{s}\in\mathcal{S}}\theta(\mathbf{c};\mathbf{s})\sum_{l=1}^{L}\alpha_l Q_l[t]c_l S_l^{(\mathrm{MWS})}[t]$$
$$=\mathbb{E}\left[\sum_{l=1}^{L}\alpha_l Q_l[t]C_l[t]S_l^{(\mathrm{MWS})}[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]. \tag{36}$$

where the step (a) follows from the definition of $\mathbf{S}^{(\mathrm{MWS})}$. By substituting (36) into (32), we have

$$\Delta W\le-2\epsilon\sum_{l=1}^{L}\alpha_l Q_l[t]+B(\boldsymbol{\alpha},\boldsymbol{\beta},\gamma)$$
$$-2\gamma\mathbb{E}\left[\sum_{l=1}^{L}\beta_l T_l[t]C_l[t]S_l^*[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right] \tag{37}$$
$$\le-2\epsilon\sum_{l=1}^{L}\alpha_l Q_l[t]+B(\boldsymbol{\alpha},\boldsymbol{\beta},\gamma). \tag{38}$$

Appendix C
Proof of Proposition 2

Consider the Lyapunov function

$$V(\mathbf{Y}[t])\triangleq\|\mathbf{Y}[t]\|_2=\sqrt{\sum_{l=1}^{L}\alpha_l Q_l^2[t]+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t]},$$

where $\mathbf{Y}[t]\triangleq(\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t],\sqrt{4\gamma C_{\max}\boldsymbol{\beta}\mathbf{T}[t]})$, $\sqrt{\mathbf{x}}$ denotes the component-wise square root of the vector $\mathbf{x}$, and $\mathbf{xy}$ denotes the component-wise product of the vectors $\mathbf{x}$ and $\mathbf{y}$.

It is shown in Appendix D that there exist positive constants b and δ such that if $\|\mathbf{Y}[t]\|_2>b$, then,

$$\Delta V\triangleq\mathbb{E}\left[\|\mathbf{Y}[t+1]\|_2-\|\mathbf{Y}[t]\|_2|\mathbf{Q}[t],\mathbf{T}[t]\right]<-\delta. \tag{39}$$

We first consider the term $\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2}\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]$. Let $l^*[t]\in\arg\max_l(\alpha_l Q_l[t]+\gamma\beta_l T_l[t])$ and $\lambda_{\min}\triangleq\frac{1}{2}\min_l\lambda_l$. Then, we partition the space $(\mathbf{Q}[t],\mathbf{T}[t])$ into sets $\mathcal{F}_1$, $\mathcal{F}_2$ and $\mathcal{F}_3$, where

$$\mathcal{F}_1\triangleq\{\|\mathbf{Y}[t]\|_2\le b\};$$
$$\mathcal{F}_2\triangleq\left\{\|\mathbf{Y}[t]\|_2>b,Q_{l^*[t]}[t]>\lambda_{\min}T_{l^*[t]}[t]\right\};$$
$$\mathcal{F}_3\triangleq\left\{\|\mathbf{Y}[t]\|_2>b,Q_{l^*[t]}[t]\le\lambda_{\min}T_{l^*[t]}[t]\right\}.$$

Then, we have

$$\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2}\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]=\sum_{i=1}^{3}\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2};\mathcal{F}_i\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]. \tag{40}$$

Next, we consider each term in (40) individually.

(i) On event $\mathcal{F}_1$, we have

$$\|\mathbf{Y}[t]\|_2=\sqrt{\sum_{l=1}^{L}\alpha_l Q_l^2[t]+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t]}\le b,$$

which implies $\sum_{l=1}^{L}\alpha_l Q_l^2[t]\le b^2$ and $\sum_{l=1}^{L}\alpha_l Q_l^2[t]+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t]\le b^2$. For $\|\mathbf{Y}[t+1]\|_2$, we have

$$\|\mathbf{Y}[t+1]\|_2^2$$
$$=\sum_{l=1}^{L}\alpha_l Q_l^2[t+1]+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t+1]$$
$$\le\sum_{l=1}^{L}\alpha_l(Q_l[t]+A_{\max})^2+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l(T_l[t]+1)$$
$$=\sum_{l=1}^{L}\alpha_l Q_l^2[t]+2A_{\max}\sum_{l=1}^{L}\alpha_l Q_l[t]+A_{\max}^2\sum_{l=1}^{L}\alpha_l$$
$$+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t]+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l$$
$$\overset{(a)}{\le}\left(\sum_{l=1}^{L}\alpha_l Q_l^2[t]+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t]\right)$$
$$+2A_{\max}\sqrt{\sum_{l=1}^{L}\alpha_l\sum_{l=1}^{L}\alpha_l Q_l^2[t]}$$
$$+A_{\max}^2\sum_{l=1}^{L}\alpha_l+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l$$
$$\le b^2+2A_{\max}b\sqrt{\sum_{l=1}^{L}\alpha_l}+A_{\max}^2\sum_{l=1}^{L}\alpha_l+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l$$
$$\triangleq D_1^2, \tag{41}$$

where step (a) follows from Cauchy-Schwarz's inequality. Hence, we have $\|\mathbf{Y}[t+1]\|_2\le D_1$, which implies

$$\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2};\mathcal{F}_1\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]\le e^{\eta D_1} \tag{42}$$

To consider other two terms in (40), we need the following lemma, which is shown in Appendix E.

Lemma 3: If $\|\mathbf{Y}[t]\|_2>b$, then

$$|\|\mathbf{Y}[t+1]\|_2-\|\mathbf{Y}[t]\|_2|$$
$$\le 2L\max\{A_{\max},C_{\max}\}\sqrt{\alpha_{\max}}+\frac{4\gamma C_{\max}\sum_{l=1}^{L}\beta_l}{b}$$
$$+4\gamma C_{\max}\frac{\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]}{\sqrt{\sum_{l=1}^{L}\alpha_l Q_l^2[t]+4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t]}}, \tag{43}$$

where $\alpha_{\max}\triangleq\max_l\alpha_l$, and $\mathbf{H}^*\triangleq\{l:S_l^*[t]C_l[t]>0\}$ is defined in Appendix B.

(ii) On event $\mathcal{F}_2$, we have

$$\begin{aligned}&\alpha_l Q_l[t] + \gamma\beta_l T_l[t]\\ \leq&\alpha_{l^*[t]} Q_{l^*[t]}[t] + \gamma\beta_{l^*[t]} T_{l^*[t]}[t]\\ \leq&\left(\alpha_{\max} + \frac{\gamma\beta_{\max}}{\lambda_{\min}}\right) Q_{l^*[t]}[t], \forall l,\end{aligned} \tag{44}$$

where $\beta_{\max} \triangleq \max_l \beta_l$. This implies

$$\gamma\beta_l T_l[t] \leq \left(\alpha_{\max} + \frac{\gamma\beta_{\max}}{\lambda_{\min}}\right) Q_{l^*[t]}[t], \forall l. \tag{45}$$

Thus, we have

$$\begin{aligned}&\frac{\gamma\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]}{\sqrt{\sum_{l=1}^L \alpha_l Q_l^2[t] + 4\gamma C_{\max}\sum_{l=1}^L \beta_l T_l[t]}}\\ \leq&\frac{L\left(\alpha_{\max} + \frac{\gamma\beta_{\max}}{\lambda_{\min}}\right) Q_{l^*[t]}[t]}{\sqrt{\alpha_{\min}} Q_{l^*[t]}[t]}\\ =&\frac{L}{\sqrt{\alpha_{\min}}}\left(\alpha_{\max} + \frac{\gamma\beta_{\max}}{\lambda_{\min}}\right),\end{aligned} \tag{46}$$

where $\alpha_{\min} \triangleq \min_l \alpha_l > 0$. By substituting (46) into (43), we get

$$|\|\mathbf{Y}[t+1]\|_2 - \|\mathbf{Y}[t]\|_2| \leq D_2, \tag{47}$$

where $D_2 \triangleq 2L\max\{A_{\max}, C_{\max}\}\sqrt{\alpha_{\max}} + \frac{4\gamma C_{\max}\sum_{l=1}^L \beta_l}{b} + \frac{4C_{\max}L}{\sqrt{\alpha_{\min}}}\left(\alpha_{\max} + \frac{\gamma\beta_{\max}}{\lambda_{\min}}\right)$. Noting that (39) and (47) satisfy conditions of Lemma 2.2 in [6], there exists $\eta_1 > 0$ and $\rho \in (0, 1)$ such that

$$\mathbb{E}\left[e^{\eta(\|\mathbf{Y}[t+1]\|_2 - \|\mathbf{Y}[t]\|_2)}; \mathcal{F}_2 \middle| \mathbf{Q}[t], \mathbf{T}[t]\right] \leq \rho, \forall 0 < \eta < \eta_1.$$

Thus, for any $0 < \eta < \eta_1$, we have

$$\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2}; \mathcal{F}_2 \middle| \mathbf{Q}[t], \mathbf{T}[t]\right] \leq \rho e^{\eta\|\mathbf{Y}[t]\|_2}. \tag{48}$$

(iii) On event $\mathcal{F}_3$, we have

$$\begin{aligned}&\alpha_l Q_l[t] + \gamma\beta_l T_l[t]\\ \leq&\alpha_{l^*[t]} Q_{l^*[t]}[t] + \gamma\beta_{l^*[t]} T_{l^*[t]}[t]\\ \leq&\left(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max}\right) T_{l^*[t]}[t], \forall l,\end{aligned} \tag{49}$$

which implies

$$\gamma\beta_l T_l[t] \leq \left(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max}\right) T_{l^*[t]}[t], \forall l, \tag{50}$$

$$\alpha_l Q_l[t] \leq \left(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max}\right) T_{l^*[t]}[t], \forall l. \tag{51}$$

Thus, we have

$$\begin{aligned}&\frac{\gamma\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]}{\sqrt{\sum_{l=1}^L \alpha_l Q_l^2[t] + 4\gamma C_{\max}\sum_{l=1}^L \beta_l T_l[t]}}\\ \leq&\frac{L(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max})}{b} T_{l^*[t]}[t].\end{aligned} \tag{52}$$

By substituting (52) into (43), we get

$$\begin{aligned}&|\|\mathbf{Y}[t+1]\|_2 - \|\mathbf{Y}[t]\|_2|\\ \leq&2L\max\{A_{\max}, C_{\max}\}\sqrt{\alpha_{\max}} + \frac{4\gamma C_{\max}\sum_{l=1}^L \beta_l}{b}\\ &+ \frac{4C_{\max}L(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max})}{b} T_{l^*[t]}[t].\end{aligned} \tag{53}$$

In addition, on event $\mathcal{F}_3$, we have

$$\begin{aligned}\|\mathbf{Y}[t]\|_2^2 =& \sum_{l=1}^L \alpha_l Q_l^2[t] + 4\gamma C_{\max}\sum_{l=1}^L \beta_l T_l[t]\\ \leq&(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max})^2 T_{l^*[t]}^2[t]\sum_{l=1}^L \frac{1}{\alpha_l}\\ &+ 4C_{\max}L(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max}) T_{l^*[t]}[t]\\ =&F_3^2 T_{l^*[t]}^2[t],\end{aligned} \tag{54}$$

where $F_3^2 \triangleq (\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max})^2\sum_{l=1}^L \frac{1}{\alpha_l} + 4C_{\max}L(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max})$. Hence, we have

$$\begin{aligned}\|\mathbf{Y}[t+1]\|_2 \leq&\|\mathbf{Y}[t]\|_2 + |\|\mathbf{Y}[t+1]\|_2 - \|\mathbf{Y}[t]\|_2|\\ \leq&F_1 T_{l^*[t]}[t] + F_2,\end{aligned} \tag{55}$$

where $F_1 \triangleq \frac{4C_{\max}L(\lambda_{\min}\alpha_{\max} + \gamma\beta_{\max})}{b} + F_3$ and $F_2 \triangleq 2L\max\{A_{\max}, C_{\max}\}\sqrt{\alpha_{\max}} + \frac{4\gamma C_{\max}\sum_{l=1}^L \beta_l}{b}$. Thus, we have

$$\begin{aligned}&\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2}; \mathcal{F}_3 \middle| \mathbf{Q}[t], \mathbf{T}[t]\right]\\ \leq&e^{\eta F_2} e^{\eta F_1 T_{l^*[t]}[t]} \mathbb{1}_{\{Q_{l^*[t]}[t] < \lambda_{\min} T_{l^*[t]}[t]\}}.\end{aligned} \tag{56}$$

By substituting (42), (48) and (56) into (40), we have

$$\begin{aligned}&\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2} | \mathbf{Q}[t], \mathbf{T}[t]\right]\\ \leq&e^{\eta D_1} + \rho e^{\eta\|\mathbf{Y}[t]\|_2} + e^{\eta F_2} e^{\eta F_1 T_{l^*[t]}[t]} \mathbb{1}_{\{Q_{l^*[t]}[t] < \lambda_{\min} T_{l^*[t]}[t]\}}.\end{aligned}$$

By taking expectation on both sides, we have

$$\begin{aligned}&\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2}\right]\\ \leq&e^{\eta D_1} + \rho\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t]\|_2}\right]\\ &+ e^{\eta F_2}\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]} \mathbb{1}_{\{Q_{l^*[t]}[t] < \lambda_{\min} T_{l^*[t]}[t]\}}\right].\end{aligned} \tag{57}$$

It is shown in Appendix F that there exists positive constants η_2 and D_3 such that

$$\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]} \mathbb{1}_{\{Q_{l^*[t]}[t] < \lambda_{\min} T_{l^*[t]}[t]\}}\right] \leq D_3, \tag{58}$$

holds for any $0 < \eta < \eta_2$. By substituting (58) into (57), then,

$$\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t+1]\|_2}\right] \leq \rho\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t]\|_2}\right] + D, \tag{59}$$

holds for any $0 < \eta < \min\{\eta_1, \eta_2\}$, where $D \triangleq e^{\eta D_1} + D_3 e^{\eta F_2}$. By using inequality (59) and iterating over t, we have

$$\mathbb{E}\left[e^{\eta\|\mathbf{Y}[t]\|_2}\right] \leq \rho^t e^{\eta\|\mathbf{Y}[0]\|} + \frac{1-\rho^t}{1-\rho} D \leq e^{\eta\|\mathbf{Y}[0]\|} + \frac{D}{1-\rho},$$

which implies the existence of all moments of steady-state queue length and TSLS counter.

Appendix D
Proof of Inequality (39)

For notational convenience, we replace $B(\boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma)$ with B in the rest of the proof.

$$\begin{aligned}\Delta V &\triangleq \mathbb{E}\left[V(\mathbf{Y}[t+1]) - V(\mathbf{Y}[t]) | \mathbf{Q}[t], \mathbf{T}[t]\right] \\ &= \mathbb{E}\left[\|\mathbf{Y}[t+1]\|_2 - \|\mathbf{Y}[t]\|_2 | \mathbf{Q}[t], \mathbf{T}[t]\right] \\ &= \mathbb{E}\left[\sqrt{\|\mathbf{Y}[t+1]\|_2^2} - \sqrt{\|\mathbf{Y}[t]\|_2^2} \middle| \mathbf{Q}[t], \mathbf{T}[t]\right] \\ &\leq \frac{1}{2\|\mathbf{Y}[t]\|_2} \mathbb{E}\left[\|\mathbf{Y}[t+1]\|_2^2 - \|\mathbf{Y}[t]\|_2^2 \middle| \mathbf{Q}[t], \mathbf{T}[t]\right], \quad (60)\end{aligned}$$

where the last step follows from the fact that $f(x) = \sqrt{x}$ is concave for $x \geq 0$ so that $f(x_1) - f(x_2) \leq f'(x_2)(x_1 - x_2) = \frac{x_1 - x_2}{2\sqrt{x_2}}$ with $x_1 \triangleq \|\mathbf{Y}[t+1]\|_2^2$ and $x_2 \triangleq \|\mathbf{Y}[t]\|_2^2$.

Note that the difference in (60) is exactly ΔW in (25). Next, we consider the term ΔW. Since

$$\begin{aligned}\sum_{l=1}^{L} \alpha_l Q_l[t] \geq& \frac{1}{C_{\max}} \mathbb{E}\left[\sum_{l=1}^{L} \alpha_l Q_l[t] C_l[t] \middle| \mathbf{Q}[t], \mathbf{T}[t]\right] \\ \geq& \frac{1}{C_{\max}} \mathbb{E}\left[\sum_{l=1}^{L} \alpha_l Q_l[t] C_l[t] S_l^*[t] \middle| \mathbf{Q}[t], \mathbf{T}[t]\right].\end{aligned}$$

By substituting above inequality into (37), we have

$$\begin{aligned}\Delta W \leq& -\frac{2\epsilon}{C_{\max}} \mathbb{E}\left[\sum_{l=1}^{L} \alpha_l Q_l[t] C_l[t] S_l^*[t] \middle| \mathbf{Q}[t], \mathbf{T}[t]\right] \\ &- 2\gamma \mathbb{E}\left[\sum_{l=1}^{L} \beta_l T_l[t] C_l[t] S_l^*[t] \middle| \mathbf{Q}[t], \mathbf{T}[t]\right] + B \\ \leq& -2h_1(\epsilon) \mathbb{E}\left[\sum_{l=1}^{L} (\alpha_l Q_l[t] + \gamma \beta_l T_l[t]) C_l[t] S_l^*[t] \middle| \mathbf{Q}[t], \mathbf{T}[t]\right] + B \\ \overset{(a)}{\leq}& -2h_1(\epsilon) \frac{1}{L} \mathbb{E}\left[\sum_{l=1}^{L} (\alpha_l Q_l[t] + \gamma \beta_l T_l[t]) C_l[t] \middle| \mathbf{Q}[t], \mathbf{T}[t]\right] + B \\ \leq& -\frac{2\overline{c}_{\min}}{L} h_1(\epsilon) \sum_{l=1}^{L} (\alpha_l Q_l[t] + \gamma \beta_l T_l[t]) + B, \quad (61)\end{aligned}$$

where $h_1(\epsilon) \triangleq \min\left\{\frac{\epsilon}{C_{\max}}, 1\right\}$, the step (a) follows from the fact that $\sum_{k=1}^{L} (\alpha_k Q_k[t] + \gamma \beta_k T_k[t]) C_k[t] S_k^*[t] \geq (\alpha_l Q_l[t] + \gamma \beta_l T_l[t]) C_l[t], \forall l = 1, 2, ..., L$, and $\overline{c}_{\min} \triangleq \min_l \mathbb{E}[C_l[t]]$. By substituting (61) into (60), we have

$$\begin{aligned}\Delta V \leq& \frac{1}{2\|\mathbf{Y}[t]\|_2}\left(-\frac{2\overline{c}_{\min}}{L} h_1(\epsilon) \sum_{l=1}^{L} (\alpha_l Q_l[t] + \gamma \beta_l T_l[t]) + B\right) \\ \leq& \frac{1}{2\|\mathbf{Y}[t]\|_2}\left(-h_2(\epsilon) \sum_{l=1}^{L} (\alpha_l Q_l[t] + 4\gamma C_{\max} \beta_l T_l[t]) + B\right),\end{aligned}$$

where $h_2(\epsilon) \triangleq h_1(\epsilon) \frac{2\overline{c}_{\min}}{L}$. Note that

$$\begin{aligned}&4\gamma C_{\max} \beta_l T_l[t] \geq 4\gamma C_{\max} \beta_l T_l[t] \mathbb{1}_{\{4\gamma C_{\max} \beta_l T_l[t] \geq 1\}} \\ \geq& \sqrt{4\gamma C_{\max} \beta_l T_l[t]} \mathbb{1}_{\{4\gamma C_{\max} \beta_l T_l[t] \geq 1\}} \\ =& \sqrt{4\gamma C_{\max} \beta_l T_l[t]} - \sqrt{4\gamma C_{\max} \beta_l T_l[t]} \mathbb{1}_{\{4\gamma C_{\max} \beta_l T_l[t] < 1\}} \\ \geq& \sqrt{4\gamma C_{\max} \beta_l T_l[t]} - 1, \quad (62)\end{aligned}$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function.

Also, for any $l = 1, 2, ..., L$, if $\alpha_l \geq 1$, then $\alpha_l \geq \sqrt{\alpha_l}$; if $0 < \alpha_l < 1$, then $\alpha_l \sum_{k=1}^{L} \frac{1}{\alpha_k} \geq 1 \geq \sqrt{\alpha_l}$. Hence,

$$\max\left\{\sum_{k=1}^{L} \frac{1}{\alpha_k}, 1\right\} \alpha_l \geq \sqrt{\alpha_l}, \forall l. \quad (63)$$

Thus, by using (62) and (63), we have

$$\begin{aligned}\Delta V \leq& \frac{1}{2\|\mathbf{Y}[t]\|_2}\Bigg(-h_2(\epsilon)\Bigg(\frac{\sum_{l=1}^{L} \sqrt{\alpha_l} Q_l[t]}{\max\left\{\sum_{l=1}^{L} \frac{1}{\alpha_l}, 1\right\}} \\ &+ \sum_{l=1}^{L} \sqrt{4\gamma C_{\max} \beta_l T_l[t]}\Bigg) + B_1\Bigg) \\ \leq& \frac{1}{2\|\mathbf{Y}[t]\|_2} \left(-h_3(\epsilon) \|\mathbf{Y}[t]\|_1 + B_1\right) \\ \leq& -\frac{h_3(\epsilon)}{2} + \frac{B_1}{2\|\mathbf{Y}[t]\|_2}, \quad (64)\end{aligned}$$

where $h_3(\epsilon) \triangleq h_2(\epsilon) \min\left\{\frac{1}{\max\left\{\sum_{l=1}^{L} \frac{1}{\alpha_l}, 1\right\}}, 1\right\}$, and $B_1 \triangleq B + L h_2(\epsilon)$.

Hence, for any $0 < \delta < \frac{h_3(\epsilon)}{2}$, if

$$\|\mathbf{Y}[t]\|_2 > \frac{B_1}{h_3(\epsilon) - 2\delta} \triangleq b, \quad (65)$$

then, we have

$$\Delta V = \mathbb{E}\left[\|\mathbf{Y}[t+1]\|_2 - \|\mathbf{Y}[t]\|_2 | \mathbf{Y}[t]\right] < -\delta. \quad (66)$$

APPENDIX E
PROOF OF LEMMA 3

If $\|\mathbf{Y}[t]\|_2 > b$, then

$$\begin{aligned}
&\big|\|\mathbf{Y}[t+1]\|_2 - \|\mathbf{Y}[t]\|_2\big| \\
=&\frac{\|\mathbf{Y}[t+1]\|_2^2 - \|\mathbf{Y}[t]\|_2^2}{\|\mathbf{Y}[t+1]\|_2 + \|\mathbf{Y}[t]\|_2} \\
\leq&\frac{\left|\sum_{l=1}^{L}\alpha_l Q_l^2[t+1] - \sum_{l=1}^{L}\alpha_l Q_l^2[t]\right|}{\|\mathbf{Y}[t+1]\|_2 + \|\mathbf{Y}[t]\|_2} \\
&+ \frac{4\gamma C_{\max}\left|\sum_{l=1}^{L}\beta_l T_l[t+1] - \sum_{l=1}^{L}\beta_l T_l[t]\right|}{\|\mathbf{Y}[t+1]\|_2 + \|\mathbf{Y}[t]\|_2} \\
\overset{(a)}{\leq}&\frac{\left|\sum_{l=1}^{L}\alpha_l Q_l^2[t+1] - \sum_{l=1}^{L}\alpha_l Q_l^2[t]\right|}{\sqrt{\sum_{l=1}^{L}\alpha_l Q_l^2[t+1]} + \sqrt{\sum_{l=1}^{L}\alpha_l Q_l^2[t]}} \\
&+ \frac{4\gamma C_{\max}\left|\sum_{l=1}^{L}\beta_l - \sum_{l\in\mathbf{H}^*}\beta_l - \sum_{l\in\mathbf{H}^*}\beta_l T_l[t]\right|}{\|\mathbf{Y}[t+1]\|_2 + \|\mathbf{Y}[t]\|_2} \\
\leq&\big|\|\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t+1]\|_2 - \|\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t]\|_2\big| \\
&+ \frac{4\gamma C_{\max}\sum_{l=1}^{L}\beta_l + 4\gamma C_{\max}\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]}{\|\mathbf{Y}[t+1]\|_2 + \|\mathbf{Y}[t]\|_2} \\
\leq&\big|\|\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t+1]\|_2 - \|\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t]\|_2\big| + \frac{4\gamma C_{\max}\sum_{l=1}^{L}\beta_l}{b} \\
&+ \frac{4\gamma C_{\max}\sum_{l\in\mathbf{H}^*}\beta_l T_l[t]}{\sqrt{\sum_{l=1}^{L}\alpha_l Q_l^2[t] + 4\gamma C_{\max}\sum_{l=1}^{L}\beta_l T_l[t]}},
\end{aligned} \tag{67}$$

where the step (a) follows the fact that $\|\mathbf{Y}[t]\|_2^2 \geq \sum_{l=1}^{L}\alpha_l Q_l^2[t]$, $\forall t$, and the equation (26). Note that

$$\begin{aligned}
&\big|\|\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t+1]\|_2 - \|\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t]\|_2\big| \\
\leq&\|\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t+1] - \sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t]\|_2 \\
\leq&\|\sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t+1] - \sqrt{\boldsymbol{\alpha}}\mathbf{Q}[t]\|_1 \\
\leq&L\max_l\sqrt{\alpha_l}|Q_l[t+1] - Q_l[t]| \\
\leq&2L\max\{A_{\max}, C_{\max}\}\sqrt{\alpha_{\max}},
\end{aligned} \tag{68}$$

where $\alpha_{\max} \triangleq \max_l \alpha_l$. By substituting above inequality into (67), we have the desired result.

APPENDIX F
PROOF OF INEQUALITY (58)

By using the law of total probability, we have

$$\begin{aligned}
&\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]}\mathbb{1}_{\{Q_{l^*[t]}[t]<\lambda_{\min}T_{l^*[t]}[t]\}}\right] \\
=&\sum_{m=0}^{\infty}\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]}\mathbb{1}_{\{Q_{l^*[t]}[t]<\lambda_{\min}T_{l^*[t]}[t]\}}\middle|T_{l^*[t]}[t]=m\right] \\
&\times\Pr\{T_{l^*[t]}[t]=m\}.
\end{aligned} \tag{69}$$

Consider $\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]}\mathbb{1}_{\{Q_{l^*[t]}[t]<\lambda_{\min}T_{l^*[t]}[t]\}}\middle|T_{l^*[t]}[t]=m\right]$.

$$\begin{aligned}
&\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]}\mathbb{1}_{\{Q_{l^*[t]}[t]<\lambda_{\min}T_{l^*[t]}[t]\}}\middle|T_{l^*[t]}[t]=m\right] \\
=&e^{\eta F_1 m}\Pr\left\{Q_{l^*[t]}[t]<\lambda_{\min}T_{l^*[t]}[t]\middle|T_{l^*[t]}[t]=m\right\} \\
\leq&e^{\eta F_1 m}\Pr\left\{\sum_{j=t-m+1}^{t}A_{l^*[t]}[j]<\lambda_{\min}m\middle|T_{l^*[t]}[t]=m\right\},
\end{aligned} \tag{70}$$

where the last step follows from the fact that given $T_{l^*[t]}[t]=m$ (i.e., link $l^*[t]$ is not scheduled in the past m slots), $Q_{l^*[t]}[t] \geq \sum_{j=t-m+1}^{t}A_{l^*[t]}[j]$. By substituting (70) into (69), we have

$$\begin{aligned}
&\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]}\mathbb{1}_{\{Q_{l^*[t]}[t]<\lambda_{\min}T_{l^*[t]}[t]\}}\right] \\
\leq&\sum_{m=0}^{\infty}e^{\eta F_1 m}\Pr\left\{\sum_{j=t-m+1}^{t}A_{l^*[t]}[j]<\lambda_{\min}m, T_{l^*[t]}[t]=m\right\} \\
\leq&\sum_{m=0}^{\infty}e^{\eta F_1 m}\Pr\left\{\sum_{j=t-m+1}^{t}A_{l^*[t]}[j]<\lambda_{\min}m\right\} \\
\leq&\sum_{m=0}^{\infty}e^{\eta F_1 m}\Pr\left\{\bigcup_{l=1}^{L}\left\{\sum_{j=t-m+1}^{t}A_l[j]<\lambda_{\min}m\right\}\right\} \\
\leq&\sum_{m=0}^{\infty}e^{\eta F_1 m}\sum_{l=1}^{L}\Pr\left\{\sum_{j=t-m+1}^{t}A_l[j]<\lambda_{\min}m\right\}.
\end{aligned} \tag{71}$$

For each link l, $A_l[j]$ are i.i.d. over time and its mean is greater than $\lambda_{\min} = \frac{1}{2}\min_l \lambda_l$, by Cramer's theorem, we have

$$\Pr\left\{\sum_{j=t-m+1}^{t}A_l[j]<\lambda_{\min}m\right\} < e^{-mI_l(\lambda_l-\lambda_{\min})}, \tag{72}$$

where $I_l(\cdot)$ is the rate function corresponding to the arrivals for link l. Note that $I_l(\lambda_l-\lambda_{\min}) > 0, \forall l$. Thus, we have

$$\begin{aligned}
&\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]}\mathbb{1}_{\{Q_{l^*[t]}[t]<\lambda_{\min}T_{l^*[t]}[t]\}}\right] \\
\leq&L\sum_{m=0}^{\infty}e^{\eta F_1 m}e^{-m\min_l I_l(\lambda_l-\lambda_{\min})}.
\end{aligned} \tag{73}$$

Hence, for $0 < \eta < \frac{\min_l I_l(\lambda_l-\lambda_{\min})}{F_1} \triangleq \eta_2$, we have

$$\begin{aligned}
&\mathbb{E}\left[e^{\eta F_1 T_{l^*[t]}[t]}\mathbb{1}_{\{Q_{l^*[t]}[t]<\lambda_{\min}T_{l^*[t]}[t]\}}\right] \\
\leq&L\sum_{m=0}^{\infty}e^{-(\min_l I_l(\lambda_l-\lambda_{\min})-F_1\eta)m} \\
=&\frac{L}{1-e^{-(\min_l I_l(\lambda_l-\lambda_{\min})-F_1\eta)}} \triangleq D_3.
\end{aligned} \tag{74}$$

APPENDIX G
PROOF OF LEMMA 2

In the rest of proof, we will omit the superscript p for brevity.

Proof of identity (12):

$$\sum_{l=1}^{L}\beta_l\lambda_l T_l[t+1]=\sum_{l\notin\mathbf{H}}\beta_l\lambda_l\left(T_l[t]+1\right)$$
$$=\sum_{l=1}^{L}\beta_l\lambda_l T_l[t]-\sum_{l\in\mathbf{H}}\beta_l\lambda_l T_l[t]+\sum_{l=1}^{L}\beta_l\lambda_l-\sum_{l\in\mathbf{H}}\beta_l\lambda_l, \quad (75)$$

where $\mathbf{H}\triangleq\{l:S_l[t]C_l[t]>0\}$. Taking expectation on both sides with respect to the steady state distribution of $(\mathbf{Q},\mathbf{T})$ and rearranging terms, we have the desired result.

Proof of identity (13):

$$\sum_{l=1}^{L}\beta_l\lambda_l T_l^2[t+1]=\sum_{l\notin\mathbf{H}}\beta_l\lambda_l\left(T_l[t]+1\right)^2$$
$$=\sum_{l\notin\mathbf{H}}\beta_l\lambda_l T_l^2[t]+2\sum_{l\notin\mathbf{H}}\beta_l\lambda_l T_l[t]+\sum_{l\notin\mathbf{H}}\beta_l\lambda_l$$
$$=\sum_{l=1}^{L}\beta_l\lambda_l T_l^2[t]-\sum_{l\in\mathbf{H}}\beta_l\lambda_l T_l^2[t]+2\sum_{l=1}^{L}\beta_l\lambda_l T_l[t]$$
$$-2\sum_{l\in\mathbf{H}}\beta_l\lambda_l T_l[t]+\sum_{l=1}^{L}\beta_l\lambda_l-\sum_{l\in\mathbf{H}}\beta_l\lambda_l. \quad (76)$$

Taking expectation on both sides with respect to the steady state distribution of $(\mathbf{Q},\mathbf{T})$ and rearranging terms, we have

$$2\sum_{l=1}^{L}\beta_l\lambda_l\mathbb{E}\left[\overline{T}_l\right]=2\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l\right]+\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\overline{T}_l^2\right]$$
$$-\left(\sum_{l=1}^{L}\beta_l\lambda_l-\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}}\beta_l\lambda_l\right]\right). \quad (77)$$

Using Identity (12), we have the desired result.

Appendix H
Proof of Proposition 4

Consider the quadratic Lyapunov function $W_{\mathbf{Q}}(\mathbf{Q},\mathbf{T})\triangleq\frac{1}{2}\sum_{l=1}^{L}\alpha_l Q_l^2$. We have

$$\Delta W_{\mathbf{Q}}(\mathbf{Q},\mathbf{T})$$
$$=\mathbb{E}\left[W_{\mathbf{Q}}(\mathbf{Q}[t+1],\mathbf{T}[t+1])-W_{\mathbf{Q}}(\mathbf{Q}[t],\mathbf{T}[t])|\mathbf{Q}[t],\mathbf{T}[t]\right]$$
$$=\mathbb{E}\left[\frac{1}{2}\sum_{l=1}^{L}\alpha_l Q_l^2[t+1]-\frac{1}{2}\sum_{l=1}^{L}\alpha_l Q_l^2[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]$$
$$\leq\frac{1}{2}\sum_{l=1}^{L}\mathbb{E}\left[\alpha_l\left(Q_l[t]+A_l[t]-C_l[t]S_l^*[t]\right)^2-\alpha_l Q_l^2[t]\middle|\mathbf{Q}[t],\mathbf{T}[t]\right]$$
$$\leq\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[Q_l[t]\left(A_l[t]-C_l[t]S_l^*[t]\right)|\mathbf{Q}[t],\mathbf{T}[t]\right]$$
$$+\frac{1}{2}\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[A_l^2[t]+C_l^2[t]\right]. \quad (78)$$

Taking expectation on both sides with respect to the steady state distribution of $(\mathbf{Q},\mathbf{T})$, and using the fact that $\mathbb{E}[\Delta W_{\mathbf{Q}}(\overline{\mathbf{Q}},\overline{\mathbf{T}})]=0$ followed from $\mathbb{E}[\overline{Q}_l^2]<\infty$ for all $l\in\mathcal{L}$, we have

$$0\leq\sum_{l=1}^{L}\alpha_l\lambda_l\mathbb{E}\left[\overline{Q}_l^*\right]-\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{Q}_l^*\overline{C}_l\overline{S}_l^*\right]$$
$$+\frac{1}{2}\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{A}_l^2+\overline{C}_l^2\right], \quad (79)$$

which implies

$$\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{Q}_l^*\overline{C}_l\overline{S}_l^*\right]\leq\sum_{l=1}^{L}\alpha_l\lambda_l\mathbb{E}\left[\overline{Q}_l^*\right]+\frac{1}{2}\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{A}_l^2+\overline{C}_l^2\right].$$

Hence, we have

$$\sum_{l=1}^{L}\mathbb{E}\left[\left(\alpha_l\overline{Q}_l^*+\gamma\beta_l\overline{T}_l^*\right)\overline{C}_l\overline{S}_l^*\right]$$
$$\leq\sum_{l=1}^{L}\alpha_l\lambda_l\mathbb{E}\left[\overline{Q}_l^*\right]+\gamma\sum_{l=1}^{L}\beta_l\mathbb{E}\left[\overline{T}_l^*\overline{S}_l^*\overline{C}_l\right]+\frac{1}{2}\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{A}_l^2+\overline{C}_l^2\right]. \quad (80)$$

Recall that given $\mathbf{Q}[t]=\mathbf{Q}$, $\mathbf{T}[t]=\mathbf{T}$ and the channel state $\mathbf{C}[t]$, we have

$$\sum_{l=1}^{L}\left(\alpha_l Q_l[t]+\gamma\beta_l T_l[t]\right)C_l[t]S_l^*[t]$$
$$=\max_{\mathbf{S}\in\mathcal{S}}\sum_{l=1}^{L}\left(\alpha_l Q_l[t]+\gamma\beta_l T_l[t]\right)C_l[t]S_l. \quad (81)$$

According to the definition of the capacity region $\mathcal{R}$, we can show

$$\sum_{l=1}^{L}\mathbb{E}\left[\left(\alpha_l Q_l[t]+\gamma\beta_l T_l[t]\right)C_l[t]S_l^*[t]|\mathbf{Q}[t]=\mathbf{Q},\mathbf{T}[t]=\mathbf{T}\right]$$
$$=\max_{\mathbf{r}\in\mathcal{R}}\sum_{l=1}^{L}\left(\alpha_l Q_l[t]+\gamma\beta_l T_l[t]\right)r_l, \quad (82)$$

The proof is available in Appendix I

Since $\boldsymbol{\lambda}\in\text{Int}(\mathcal{R})$, there exists an $\epsilon>0$ such that $\boldsymbol{\lambda}(1+\epsilon)\in\mathcal{R}$. Hence, we have

$$\sum_{l=1}^{L}\mathbb{E}\left[\left(\alpha_l Q_l[t]+\gamma\beta_l T_l[t]\right)C_l[t]S_l^*[t]|\mathbf{Q}[t]=\mathbf{Q},\mathbf{T}[t]=\mathbf{T}\right]$$
$$\geq\sum_{l=1}^{L}\lambda_l(1+\epsilon)\mathbb{E}\left[\alpha_l Q_l[t]+\gamma\beta_l T_l[t]|\mathbf{Q}[t]=\mathbf{Q},\mathbf{T}[t]=\mathbf{T}\right].$$

Taking expectation on both sides with respect to the steady state distribution of $(\mathbf{Q},\mathbf{T})$, we have

$$\sum_{l=1}^{L}\mathbb{E}\left[\left(\alpha_l\overline{Q}_l^*+\gamma\beta_l\overline{T_l}^*\right)\overline{C}_l\overline{S}_l^*\right]$$
$$\geq\sum_{l=1}^{L}\lambda_l(1+\epsilon)\mathbb{E}\left[\alpha_l\overline{Q}_l^*+\gamma\beta_l\overline{T}_l^*\right].$$

By substituting above inequality into (80) and canceling the common term in both sides, we have

$$\begin{aligned}
&\sum_{l=1}^{L}\beta_l\lambda_l\mathbb{E}\left[\overline{T}_l^*\right]\\
\le&\frac{1}{1+\epsilon}\sum_{l=1}^{L}\beta_l\mathbb{E}\left[\overline{T}_l^*\overline{S}_l^*\overline{C}_l\right]+\frac{1}{2\gamma(1+\epsilon)}\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{A}_l^2+\overline{C}_l^2\right]\\
\le&\frac{C_{\max}}{1+\epsilon}\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}^*}\beta_l\overline{T}_l^*\right]+\frac{1}{2\gamma(1+\epsilon)}\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{A}_l^2+\overline{C}_l^2\right]\\
=&\frac{C_{\max}}{1+\epsilon}\left(\sum_{l=1}^{L}\beta_l-\mathbb{E}\left[\sum_{l\in\overline{\mathbf{H}}^*}\beta_l\right]\right)\\
&+\frac{1}{2\gamma(1+\epsilon)}\sum_{l=1}^{L}\alpha_l\mathbb{E}\left[\overline{A}_l^2+\overline{C}_l^2\right],
\end{aligned}$$

where the last step uses identity (12).

APPENDIX I
PROOF OF EQUATION (82)

We will use the following fact in linear programming.

$$\max_{\mathbf{x}\in\mathcal{A}}\sum_{l=1}^{L}a_lx_l=\max_{\mathbf{x}\in\mathrm{CH}\{\mathcal{A}\}}\sum_{l=1}^{L}a_lx_l,\tag{83}$$

where $\mathbf{x}=(x_l)_{l=1}^L$ is a $L-$dimensional vector, $\mathcal{A}$ is a set of $L-$ dimensional vectors, CH$\{\mathcal{A}\}$ is a convex hull of the set $\mathcal{A}$ and $a_l,\forall l=1,2,...,L$, are real numbers.

Given $\mathbf{Q}[t]$, $\mathbf{T}[t]$ and $\mathbf{C}[t]$, we have

$$\begin{aligned}
&\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])C_l[t]S_l^*[t]\\
=&\max_{\mathbf{S}\in\mathcal{S}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])C_l[t]S_l\\
=&\max_{\mathbf{v}=(v_l)_{l=1}^L\in\mathcal{S}^{(\mathbf{C}[t])}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l,
\end{aligned}\tag{84}$$

where we recall that $\mathcal{S}^{(\mathbf{c})}\triangleq\{\mathbf{Sc}:\mathbf{S}\in\mathcal{S}\}$, and $\mathbf{ab}\triangleq(a_lb_l)_{l=1}^L$ denotes the component-wise product of two vectors $\mathbf{a}$ and $\mathbf{b}$.

Next, we will show that

$$\begin{aligned}
&\sum_{l=1}^{L}\mathbb{E}[(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])C_l[t]S_l^*[t]|\mathbf{Q}[t]=\mathbf{Q},\mathbf{T}[t]=\mathbf{T}]\\
=&\max_{\mathbf{r}=(r_l)_{l=1}^L\in\mathcal{R}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])r_l.
\end{aligned}\tag{85}$$

On one hand,

$$\begin{aligned}
&\sum_{l=1}^{L}\mathbb{E}[(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])C_l[t]S_l^*[t]|\mathbf{Q}[t]=\mathbf{Q},\mathbf{T}[t]=\mathbf{T}]\\
\overset{(a)}{=}&\mathbb{E}\left[\max_{\mathbf{v}\in\mathcal{S}^{(\mathbf{C}[t])}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l\middle|\mathbf{Q}[t]=\mathbf{Q},\mathbf{T}[t]=\mathbf{T}\right]\\
=&\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\max_{\mathbf{v}\in\mathcal{S}^{(\mathbf{c})}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l\\
\overset{(b)}{=}&\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\max_{\mathbf{v}\in\mathrm{CH}\{\mathcal{S}^{(\mathbf{c})}\}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l\\
\overset{(c)}{=}&\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l^{*(\mathbf{c})}\\
=&\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}v_l^{*(\mathbf{c})}\\
\overset{(d)}{\le}&\max_{\mathbf{r}=(r_l)_{l=1}^L\in\mathcal{R}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])r_l,
\end{aligned}\tag{86}$$

where the steps (a) and (b) follow from equation (84) and equation (83), respectively; step (c) is true for

$$\mathbf{v}^{*(\mathbf{c})}=(v_l^{*(\mathbf{c})})_{l=1}^L\in\arg\max_{\mathbf{v}\in\mathrm{CH}\{\mathcal{S}^{(\mathbf{c})}\}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l;$$

and step (d) follows from the fact that $\mathbf{v}^{*(\mathbf{c})}\in\mathrm{CH}\{\mathcal{S}^{(\mathbf{c})}\}$ and $\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\mathbf{v}^{*(\mathbf{c})}\in\mathcal{R}$.

On the other hand,

$$\begin{aligned}
&\max_{\mathbf{r}=(r_l)_{l=1}^L\in\mathcal{R}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])r_l\\
\overset{(a)}{=}&\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])r_l^*\\
\overset{(b)}{=}&\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}v_l^{(\mathbf{c})}\\
=&\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l^{(\mathbf{c})}\\
\le&\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\max_{\mathbf{v}\in\mathrm{CH}\{\mathcal{S}^{(\mathbf{c})}\}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l\\
\overset{(c)}{=}&\sum_{\mathbf{c}}\Pr\{\mathbf{C}[t]=\mathbf{c}\}\max_{\mathbf{v}\in\mathcal{S}^{(\mathbf{c})}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l\\
=&\mathbb{E}\left[\max_{\mathbf{v}\in\mathcal{S}^{(\mathbf{C}[t])}}\sum_{l=1}^{L}(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])v_l\middle|\mathbf{Q}[t]=\mathbf{Q},\mathbf{T}[t]=\mathbf{T}\right]\\
\overset{(d)}{=}&\sum_{l=1}^{L}\mathbb{E}[(\alpha_lQ_l[t]+\gamma\beta_lT_l[t])C_l[t]S_l^*[t]|\mathbf{Q}[t]=\mathbf{Q},\mathbf{T}[t]=\mathbf{T}],
\end{aligned}\tag{87}$$

where the step (a) is true for $\mathbf{r}^* = (r_l^*)_{l=1}^L \in \arg\max_{\mathbf{r}\in\mathcal{R}} \sum_{l=1}^L (\alpha_l Q_l[t] + \gamma\beta_l T_l[t]) r_l$; step (b) follows from the fact that $\mathbf{r}^* \in \mathcal{R}$ and thus $\mathbf{r}^*$ can be written as $\mathbf{r}^* = \sum_{\mathbf{c}} \Pr\{\mathbf{C}[t] = \mathbf{c}\}\mathbf{v}^{(\mathbf{c})}$, where $\mathbf{v}^{(\mathbf{c})} \in \mathrm{CH}\{\mathcal{S}^{(\mathbf{c})}\}$ for each channel state $\mathbf{c}$; and step (c) and (d) follow from equation (83) and equation (84), respectively.

By combing (86) and (87), we have the desired result.

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