

OPTIMAL TRANSMIT SIGNAL DESIGN FOR ACTIVE SONAR/RADAR

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ABSTRACT

This work is concerned with the optimization of an active sonar or radar transmit signal, to maximize the probability of detecting a nonmoving point target in the presence of signal-dependent reverberation and colored ambient noise whose power spectral densities are known. An analytical solution first reported by Kooij is corrected and extended. A simple example for the white ambient noise case is included to provide insight into the transmit signal optimization. This example shows that the effect of the solution is to “pre-emphasize” the transmit signal, which results in the whitening of the signal-dependent reverberation power spectral density.

1. INTRODUCTION

When an active sonar or radar system transmits a short pulse, it receives an echo from each of any targets present, and from the many random scatterers present in the vicinity of the source. The summation of the returns from these random scatterers is known as reverberation in the sonar literature, and known as clutter in the radar literature. This signal-dependent reverberation or clutter often renders the detection of any target echoes present much more difficult. This is especially true if a target does not maintain an appreciable radial motion with respect to the active source, because then its target echo will not undergo sufficient Doppler shift that it can be readily detected outside the frequency band of the reverberation. Hence, the detection of echoes from nonmoving targets in the presence of signal-dependent reverberation has always been considered a most difficult detection problem. The presence of ambient noise at the receiver further complicates the detection of target echoes.

An optimal receiver is usually defined as one which maximizes the probability of detecting a target in the presence

of interference (i.e., reverberation plus ambient noise) for a specified probability of false alarm. The solution for the design of such an optimal receiver, given a statistical characterization of the interference, is available in a number of standard texts [1, 2]. It can be shown that the detection performance of the optimal receiver is a monotonic function of the signal to interference ratio (SIR). When the interference is white in frequency, the SIR depends only on the total energy of the transmit signal. However, when the interference is not white, and furthermore, signal-dependent, the design of the transmit signal can have a significant impact upon the detection performance of the optimal receiver.

It was pointed out two decades ago [3] that the performance of the optimal receiver in detecting target echoes from a point target in signal-dependent interference depends only upon the channel power spectral density (PSD), the ambient noise PSD, the Doppler frequency shift induced by the point target’s motion, and the power spectrum of the transmit signal. When each of these factors is specified, the solution to the design of the optimal receiver is determined. Of all these factors, the only one that is under the control of an active sonar/radar system designer is the transmit signal.

For this reason, the design of a transmit signal which maximizes the performance of an optimal receiver in detecting a point target in signal-dependent interference has been an active research topic for many years [4, 5, 6, 7, 8, 9]. In particular, Kooij [7] has reported an analytical solution to the problem of transmit signal optimization in the presence of signal-dependent reverberation plus ambient noise. However, Kooij assumed that the ambient noise spectral level was white (i.e., constant over the transmit frequency band). Also, he employed a calculus of variations approach that optimized the transmit signal power spectrum by solving for a local extremum, but he did not attempt to show that his analytical solution was indeed a global maximum.

Work by the current authors corrects and extends Kooij’s work, and provides an analytical solution to the problem of optimizing the transmit signal power spectrum to maximize the detection performance of an optimal receiver in detecting target echoes from a nonmoving point target in the pres-

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ence of signal-dependent reverberation and colored ambient noise. In addition, this paper interprets the analytical solution through a simple example, and provides the results of numerical calculations which illustrate the performance gain which may be achieved under certain conditions.

2. PROBLEM STATEMENT

We wish to detect a complex signal $As(t)$ embedded in wide-sense stationary colored and complex Gaussian noise. The signal is known, except for its amplitude A , which is modeled as a complex Gaussian random variable with zero mean and variance σ_A^2 . The signal is furthermore assumed to be bandlimited to W Hz.

The interference has the PSD

$$P(f) = P_c(f)P_s(f) + P_n(f),$$

for $|f| \leq W$, where $P_c(f)$ is the PSD of the random channel, $P_s(f)$ is $|S(f)|^2$, where $S(f)$ is the Fourier transform of $s(t)$, and $P_n(f)$ is the PSD of the ambient noise.

The optimal receiver can be shown to compute the test statistic

$$T = \left| \int_{-W}^{+W} \frac{X(f)S^*(f)}{P(f)} df \right|^2,$$

where $X(f)$ is the Fourier transform of $x(t)$, the received waveform. This detector is an asymptotic form, under the assumption of stationary processes and long observation times (SPLOT) [1]. It is valid if the observation time is much greater than the correlation time of the noise. Under the SPLOT assumption, the performance of the optimal detector is

$$P_D = P_F^{1/(1+\Delta)},$$

where P_D is the probability of detection, P_F is the probability of false alarm, and Δ is the detection index defined by

$$\Delta = \sigma_A^2 \int_{-W}^{+W} \frac{P_s(f)}{P(f)} df.$$

Note that for a fixed P_F , P_D increases as Δ increases. For the purpose of simplicity, we assume that $\sigma_A^2 = 1$, so that the optimal receiver detection performance is solely dependent on the index:

$$\Delta = \int_{-W}^{+W} \frac{P_s(f)}{P_c(f)P_s(f) + P_n(f)} df$$

For best performance, we would like to maximize Δ over all signals. Note that for the SPLOT condition, it is

only the magnitude of the Fourier transform of $s(t)$ that affects Δ . Hence, we seek to maximize Δ over all transmit signal power spectra $P_s(f)$ that are bandlimited to W Hz, and for which the total energy is constrained. We assume that the total transmit signal energy to be constrained to E , or

$$\int_{-W}^{+W} |S(f)|^2 df = \int_{-W}^{+W} P_s(f) df = E,$$

and that $P_s(f) \geq 0$ for $|f| \leq W$.

In the next section, the solution to this optimization problem is given.

3. THEOREM STATEMENT AND INTERPRETATION

Theorem: Given the functional

$$J = \int_{-W}^{+W} \frac{P_s(f)}{P_c(f)P_s(f) + P_n(f)} df,$$

where $P_c(f)$ and $P_n(f)$ are known functions for $|f| \leq W$, and $E > 0$ is known, and where the selection of the function $P_s(f)$ is subject to the constraint

$$\int_{-W}^{+W} P_s(f) df = E,$$

and to the constraint that $P_s(f) \geq 0$ for $|f| \leq W$, then the function $P_s(f)$ which maximizes J is given by the following expression:

$$P_s(f)_{opt} = \max \left(\frac{\sqrt{P_n(f)/\lambda} - P_n(f)}{P_c(f)}, 0 \right),$$

where λ can be found by numerically solving the following expression:

$$\int_{-W}^{+W} \max \left(\frac{\sqrt{P_n(f)/\lambda} - P_n(f)}{P_c(f)}, 0 \right) df = E.$$

We can better understand this theorem by looking at a simple example. Let us assume that $P_n(f) = N_0$. Then the optimal function $P_s(f)$ must maximize

$$J = \int_{-W}^{+W} \frac{P_s(f)}{P_c(f)P_s(f) + N_0} df,$$

subject to the constraints $\int_{-W}^{+W} P_s(f) df = E$ and $P_s(f) \geq 0$ for $|f| \leq W$.

The theorem in this case yields the optimal signal PSD

$$P_s(f)_{opt} = \max \left(\frac{\sqrt{N_0/\lambda} - N_0}{P_c(f)}, 0 \right),$$

where λ is found by solving

$$\int_{-W}^{+W} \max\left(\frac{\sqrt{N_0/\lambda} - N_0}{P_c(f)}, 0\right) df = E.$$

Since $(\sqrt{N_0/\lambda} - N_0)$ must be a positive constant to satisfy the constraint, we have that

$$(\sqrt{N_0/\lambda} - N_0) = \frac{E}{\int_{-W}^{+W} 1/P_c(f) df},$$

and therefore

$$P_s(f)_{opt} = \frac{\sqrt{N_0/\lambda} - N_0}{P_c(f)} = \frac{E/P_c(f)}{\int_{-W}^{+W} 1/P_c(f) df}.$$

For example, if $P_c(f) = 1$, then the optimal $P_s(f) = E/2W$, which agrees with a well known result [4]. Because this expression for $P_s(f)$ results from the assumption that $P_c(f) = 1$, we often denote this expression as $P_s(f)_{flat}$. Similarly, we denote as J_{flat} the value of the functional J that results when the transmit signal power spectrum $P_s(f)$ is selected to be $P_s(f)_{flat}$, even when $P_c(f)$ is not equal to 1.

4. SIMPLE EXAMPLE WITH NUMERICAL RESULTS

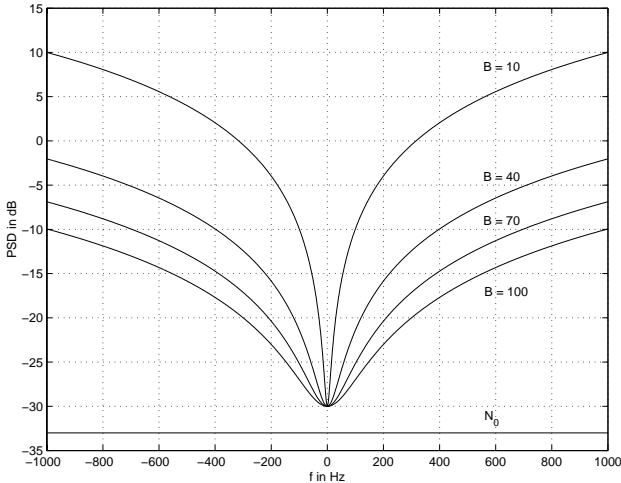


Fig. 1. Random channel PSD $P_c(f)$ and ambient noise PSD N_0 for different values of the 3 dB bandwidth B

To illustrate that the selection of an optimal transmit signal power spectrum can improve the detection performance of an optimal receiver, we used MATLAB to calculate the gain in the functional J for $P_s(f) = P_s(f)_{opt}$ over the choice of $P_s(f) = P_s(f)_{flat}$.

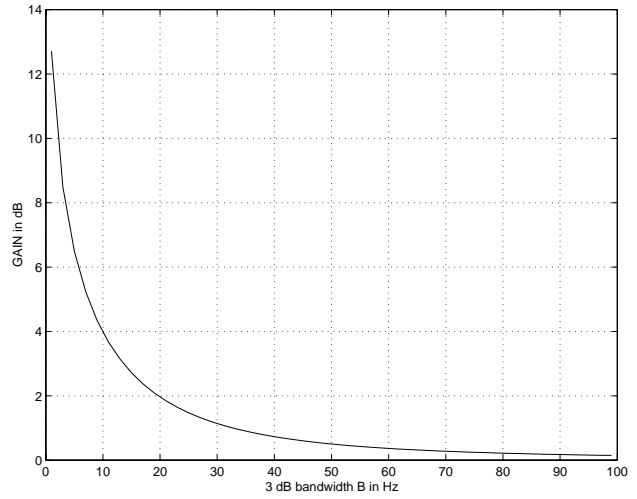


Fig. 2. Performance gain $= 10\log_{10}(J_{max}/J_{flat})$

We assumed that the PSD of the random channel was $P_c(f)$, where

$$P_c(f) = 0.001(1 + ((2\pi f)/a)^2).$$

Note that the 3 dB bandwidth B of this channel is then $a/(2\pi)$. Figure 1 is a graph of this PSD $P_c(f)$ for several values of the 3 dB bandwidth B .

For simplicity, we also assumed that the energy $E = 1$, the signal bandwidth is $W = 1000$ Hz, and the white ambient noise PSD $P_n(f) = N_0 = 0.0005$. MATLAB was used to numerically integrate $1/P_c(f)$ over all frequencies between $-W$ and $+W$, so that $P_s(f)_{opt}$ could be calculated. Next, we calculated J_{max} , the value of the functional J when $P_s(f)_{opt}$ is substituted for $P_s(f)$. Finally, the detection performance gain was calculated as

$$GAIN = 10\log_{10}(J_{max}/J_{flat}).$$

Figure 2 is a graph of this detection performance gain in dB that results from selecting the optimal transmit signal power spectrum, for several different values of the 3 dB channel bandwidth B . It is clear that significant gains may be possible under certain channel conditions.

Figure 3 is a graph of the reverberation PSD ($P_c(f)P_s(f)$) when $P_s(f) = P_s(f)_{flat}$ and when $P_s(f) = P_s(f)_{opt}$. This figure shows that the effect of using $P_s(f)_{opt}$ is to whiten the reverberation PSD.

5. CONCLUSIONS

In this paper, we have presented a new result for transmit signal optimization for active sonar/radar. The result presented corrects and extends an analytical solution originally reported by Kooij. This result is only valid for detecting target echoes from nonmoving point targets embedded in

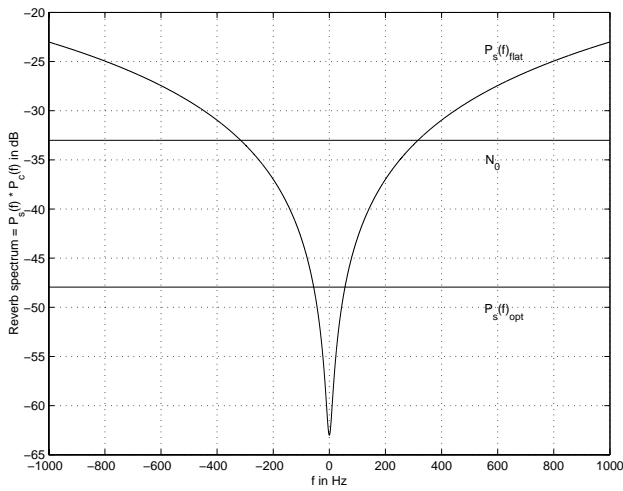


Fig. 3. Ambient noise PSD N_0 , and reverb spectrum $P_c(f)P_s(f)$ when $P_s(f) = P_s(f)_{flat}$ and when $P_s(f) = P_s(f)_{opt}$, for $B = 10$

signal-dependent reverberation and colored ambient noise. A simple example was included, to demonstrate that the effect of the transmit signal optimization can lead to significant detection gains. Also, it was noted that for white ambient noise, the optimal transmit signal is chosen so that the resulting signal-dependent reverberation will have a flat PSD over the transmit frequency band.

6. REFERENCES

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