hp calculators

HP 50g Solving differential equations

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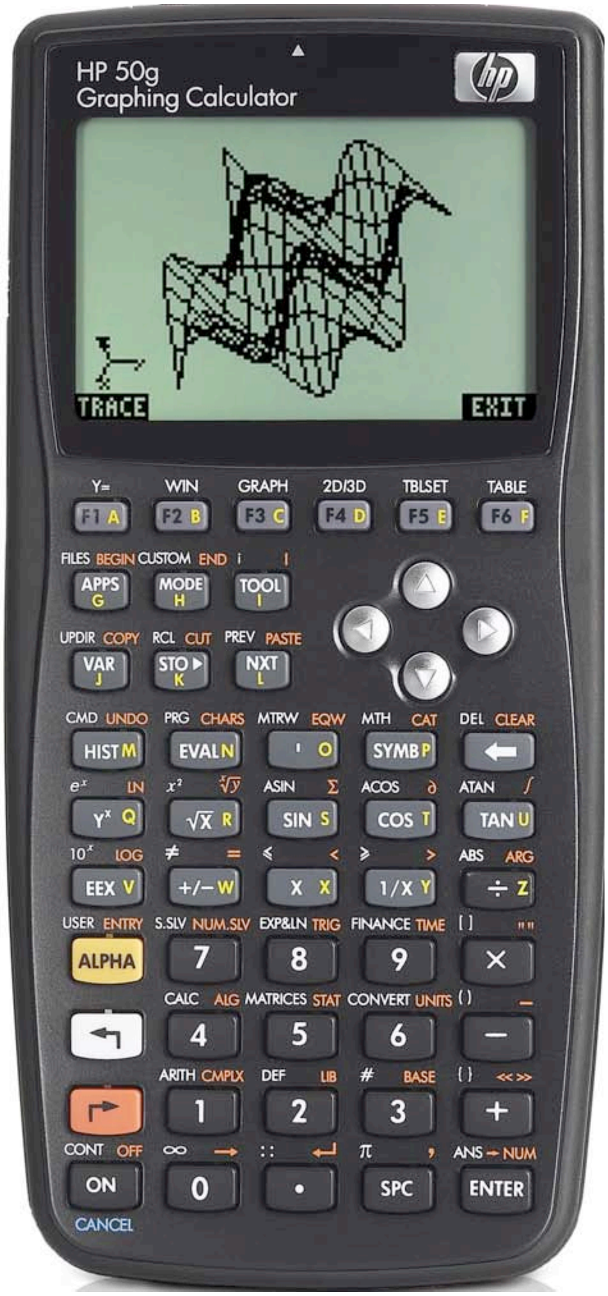

The menu SYMBOLIC SOLVE

The menu SYMBOLIC SOLVE menu is t he WHITE shifted function of the 7 key. This menu contains commands for symbolically solving a great variety of problems. Two of these commands, DESOLVE and LDEC are for symbolically solving differential equations. To access the menu you press ← S.SLV .

Figure 1

How differential equations can be displayed

On the HP50g differential equations can be denoted in two different ways. The first is to use some of the built-in differentiation commands. For example the differential equation:

$$\frac{\partial Y(X)}{\partial X}=Y(X)$$

can be denoted as:

$$\frac{\partial}{\partial X}(Y(X))=Y(X)$$

or

DERIV(Y(X),X)=Y(X)

or (if the current variable VX is X)

DERVX(Y(X))=Y(X)

The second way to denote differential equations is to use the formal derivatives of the HP50g. Formal derivatives are written starting with the letter "d", then a number that indicates the position of the independent variable in the unknown formal function, then the unknown formal function followed by its arguments in parentheses. For example the above differential equation could be also written as:

d1Y(X)=Y(X)

where d1Y(X) means the derivative for the first variable (i.e. X) of Y(X). According to this rule of syntax the differential equation:

$$\frac{\partial^2 Y(X)}{\partial X^2}=Y(X)$$

can be written as:

$$\frac{\partial}{\partial X}\left(\frac{\partial}{\partial X}(Y(X))\right)=Y(X)$$

or

DERIV(DERIV(Y(X),X),X)=Y(X)

or (if the current variable VX is X)

DERVX(DERVX(Y(X)))=Y(X)

or

d1d1Y(X)=Y(X)

All intermixed notions will be also accepted as valid arguments. For example one could also write:

$$\frac{\partial}{\partial X}\left(d1Y(X)\right)=Y(X)$$

Notice that in all notions the unknown function is written as Y(X) and not simply as Y. This allows for the use of any possible name for the unknown function and its independent variable.

The basic commands for symbolic solutions

The command DESOLVE takes two arguments: An ordinary differential equation of first or second order and a formal function for which the differential equation must be solved. The second argument of DESOLVE is the unknown function. The command returns the symbolic solutions for the unknown function. These solutions contain also the constants of integration, which are denoted as cC0, cC1, cC2, and so on. The command DESOLVE accepts also a vector as its first argument, that contains the differential equation and the initial conditions of the problem. The solutions do not contain any constants of integration if a vector is used as input.

The command LDEC finds solutions of linear ordinary differential equations of any order with constant coefficients. It takes two arguments: the right hand side of the differential equation and the characteristic polynomial in the current variable VX (usually X) of the differential equation. It returns the symbolic solutions for the unknown function with constants of integration, denoted as cC0, cC1, cC2, and so on. The command can also solve systems of linear differential equations. In this case it needs a vector containing the right hand side of the differential equations, and a matrix containing the constant coefficients of the characteristic polynomials of the differential equations.

The built-in form for numeric solutions

Though the HP 50g can solve many differential equations symbolically there will be many cases in which it can't deliver such a solution, or which don't have an closed analytic solution at all. Even in such cases the HP 50g provides a great variety of ways to solve such equations numerically. The most convenient way to numerically solve a differential equation is the built-in numeric differential equation solver and its input form. This built-in application is accessed in several ways. For example you can press ↱ NUM.SLV to get the CHOOSE box with all numeric solvers available in the system:

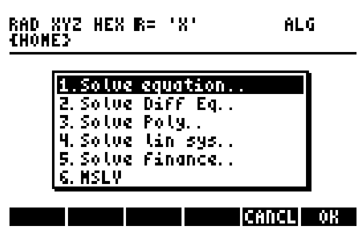

Figure 2

The first menu item is for solving equations numerically. The second is for solving differential equations numerically. The third provides numerical solutions for polynomials of any degree. The fourth solves systems of linear equations numerically. The fifth is for financial problems. The sixth is for numerical solutions of systems of arbitrary equations.

Press 2 ENTER to start the built-in numeric differential equation solver.

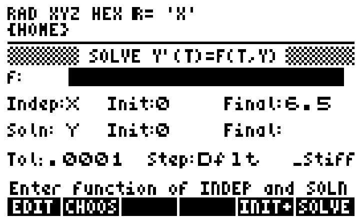

Figure 3

The input field f: is where we enter the right hand side of the differential equation of the form Y'(t)=F(T,Y). The input field Indep: is for specifying the independent variable of the differential equation. In the same row, Init: takes the initial value and Final: the final value of the independent variable of differential equation. The field Soln: is for entering the dependent variable. Init: takes the initial value of the dependent variable. Final: is the field where the numeric solution is put when the HP50g solves the problem. The field Tol: is for specifying the maximum tolerance for the answer, Step: is for the maximum step size of the calculation. (The solution is calculated stepwise according to the Runge-Kutta-Fehlberg method.) The option _Stiff is for specifying that the problem is "stiff", which causes the HP50g to use a special method suitable for such problems. The menu key EDIT is for editing the data of an input field. CHOOS allows selecting between several alternatives. INIT+ replaces the current initial values by the current final values. And finally SOLVE returns the solution for the differential equation.

The next few examples barely begin to explore differential equations on the 50g.

Practice solving differential equations

Example 1: The differential equation of the concentration c(t) of a chemical substance that undergoes decay in a first order reaction is given by:

$$\frac{\partial}{\partial t}(c(t))=-k\cdot c(t)$$

Find c(t) if the initial concentration is c(0).

Solution: Assume algebraic mode and CHOOSE boxes. Press [←] S.SLV

Figure 4

Choose DESOLVE and enter the differential equation.

OK [→] ∂ [ALPHA] [ALPHA] [←] [T] [←] () [C] [←] () [←] [T] [▶] [▶] [→] = [−] [←] [K] [×] [C] [←] () [←] [T] [▶] [→] , [C] [←] () [←] [T] [ENTER]

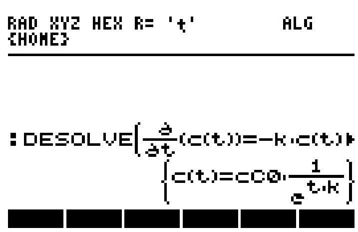

Figure 5

Notice that the variable VX was changed to t (upper edge of the screen). Extract the solution from the list.

[←] PRG [6] [ENTER] [ENTER] [7] [ENTER] [←] ANS [ENTER]

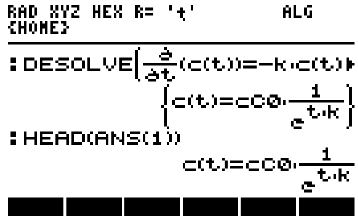

Figure 6

Substitute t=0 in the above solution.

↰ ANS ↱ | ↰ () ALPHA ↰ T ↱ = 0 ENTER

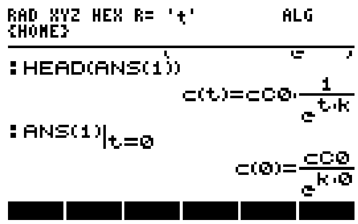

Figure 7

Solve for the integration constant cC0.

↰ S.SLV 6 ENTER ↰ ANS ↱ , ALPHA ALPHA ↰ C C 0 ENTER

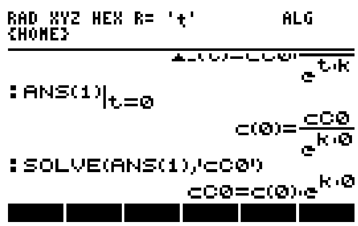

Figure 8

↱ ALG 2 ENTER ↰ ANS ENTER (expand)

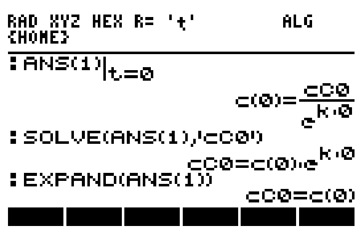

Figure 9

Substitute the constant cC0 in the general solution. The general solution of the differential equation is the fourth most recent answer of the HP50g. Use ANS(4) to refer to it.

↱ ALG 8 ENTER ↰ ANS ◀ ⇐ 4 ▶ ↱ , ↰ ANS ENTER

Figure 10

If you don't like the exponential in the denominator linearize the expression.

[→] ALG [5] [ENTER] [←] ANS [ENTER]

Figure 11

Get information about the kind of the differential equation. (The calculator creates a variable in the current directory where it stores the type of the differential equation.)

ODETY [ENTER]

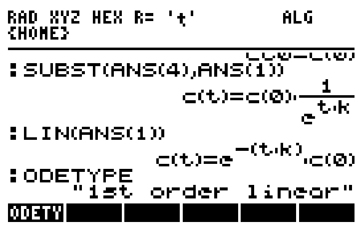

Figure 12

Answer: $c(t)=c(0)\cdot e^{-k\cdot t}$

Example 2: A physical body moves under the influence of a constant force F in a viscous liquid. The differential equation of its motion is:

$$\frac{\partial v(t)}{\partial t}=\frac{F}{m}-\frac{k}{m}\cdot v(t)$$

Find the velocity of the body at t=1s, t=2s, t=5s, t=∞ if the initial velocity was 0.

with m=0.8Kg, F=3.5N, k=2.3Kg/s

Solution: Start the numeric differential equation solver.

[→] NUM.SLV [2] [ENTER]

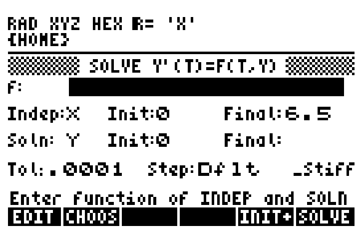

Figure 13

Enter the right hand side of the differential equation.

[→] EQW [3] [.] [5] [÷] [.] [8] [▶] [−] [2] [.] [3] [÷] [.] [8] [▶] [×] [ALPHA] [←] [V] [ENTER]

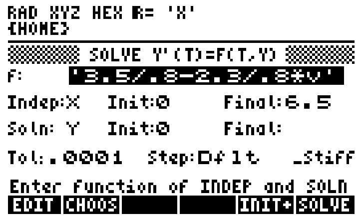

Figure 14

Enter the independent variable.

▶ ' ALPHA ◀ T ENTER

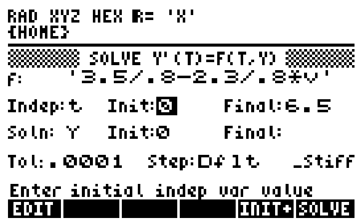

Figure 15

Enter the final value of the independent variable.

▶ 1 ENTER

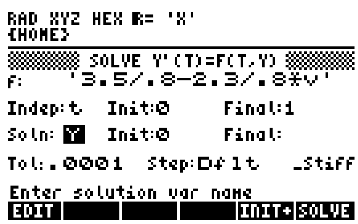

Figure 16

Enter the dependent variable.

' ALPHA ◀ V ENTER

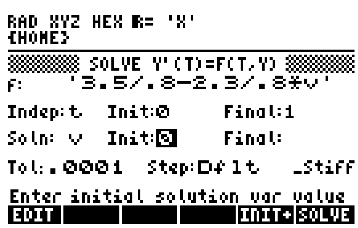

Figure 17

Solve for the final value of the dependent variable.

▶ SOLVE

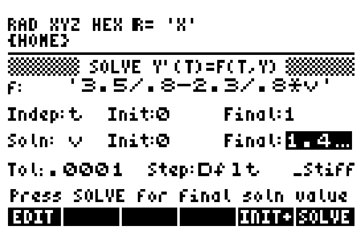

Figure 18

View the solution

EDIT

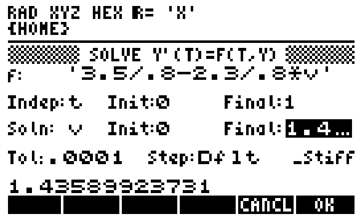

Figure 19

CANCL to finish viewing. Use the final values as new initial values.

INIT+

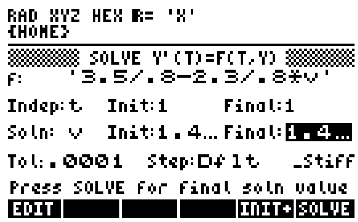

Figure 20

Enter the new final value for t and solve again.

▲ 2 ENTER ▶ ▶ SOLVE

Figure 21

Repeat the process for t=3s.

INIT+ ▲ 3 ENTER ▶ ▶ SOLVE:

Figure 22

Repeat the process for t=5s.

INIT+ ▲ 5 ENTER ▶ ▶ SOLVE

Figure 23

Obviously as the time passes by the velocity seems to approach a limit. Press INIT+ ▲ 1 0 0 ENTER ► ► SOLVE to calculate velocity after "infinite" time. The calculation takes a couple of seconds.

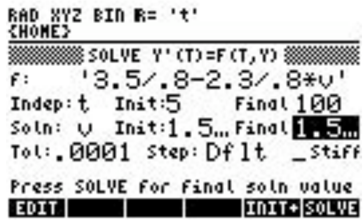

Figure 24

Press NXT OK to quit and return to the stack. The HP 50g put copies of all results in the stack as pairs of tolerance and solution values. If you press VAR you see that it also stored the equation in variable EQ, the last value of t in variable t, and the last calculated value for v in variable v:

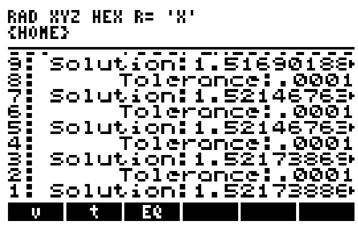

Figure 25

It seems that v really approaches a limit. If this differential equation were to be solved symbolically, we would see that this limit is given by F/k. If we calculate this using the values that we used here, we find 3.5/2.3=1.52173913043 which agrees very well with our numeric calculation of v for growing values of t.

Answer: 1.43589923731m/s, 1.51690188958m/s, 1.52146763983m/s, 1.52173869336m/s, 1.52173886357m/s