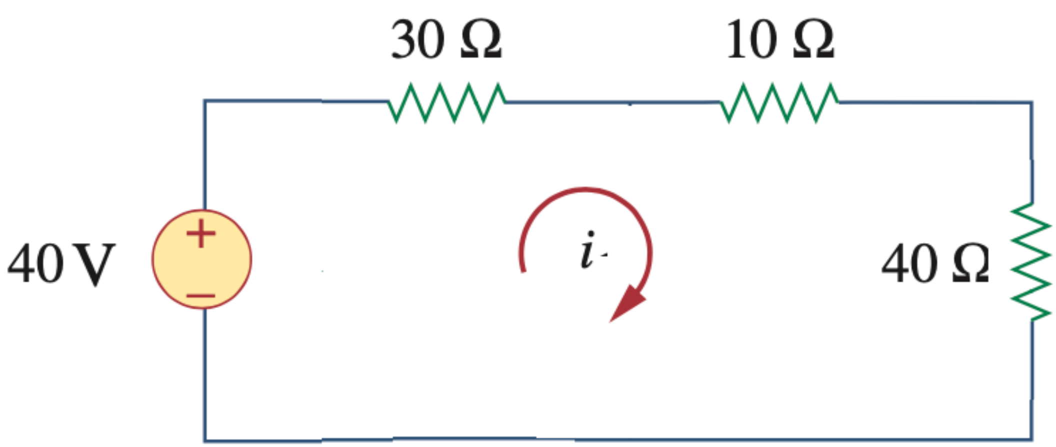
Phasors – 7

AC power

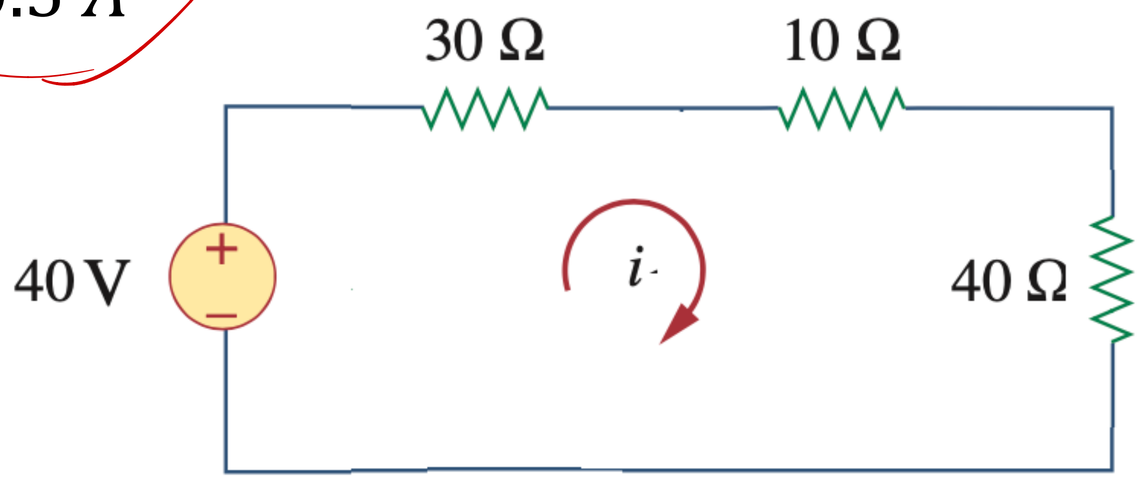
Power – DC Resistive Circuits

- General expression: $P = v\, i$



- Solve for clockwise current: $i = \frac{40\ V}{80\ \Omega} = 0.5\ A$

- With $i = \frac{40\,V}{80\,\Omega} = 0.5\,A$

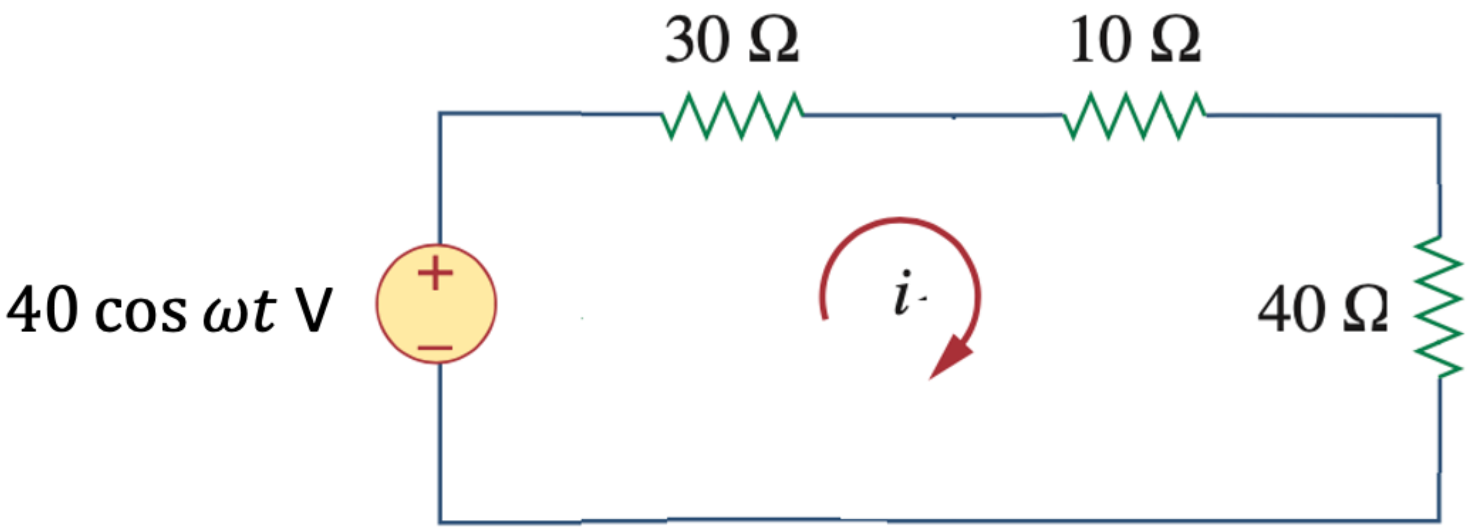


- Using $v = R\,i$ and $P = v\,i$

$$v_{30} = 15\,V \qquad P_{30} = 7.5\,W$$
$$v_{10} = 5\,V \qquad \mathbf{P_{10} = 2.5\,W}$$
$$v_{40} = 20\,V \qquad P_{40} = 10\,W$$
$$i_{source} = -0.5\,A \qquad P_{source} = -20\,W$$

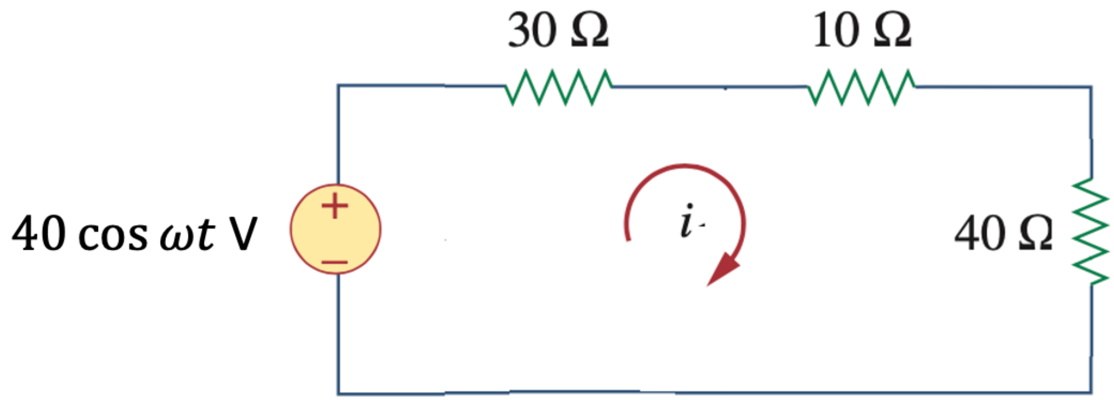
Power sums to zero

Sinusoidal Source



- Using phasors, clockwise current phasor is $\mathbf{I} = \frac{40}{80} = 0.5$ so $i(t) = 0.5\cos\omega t$ A

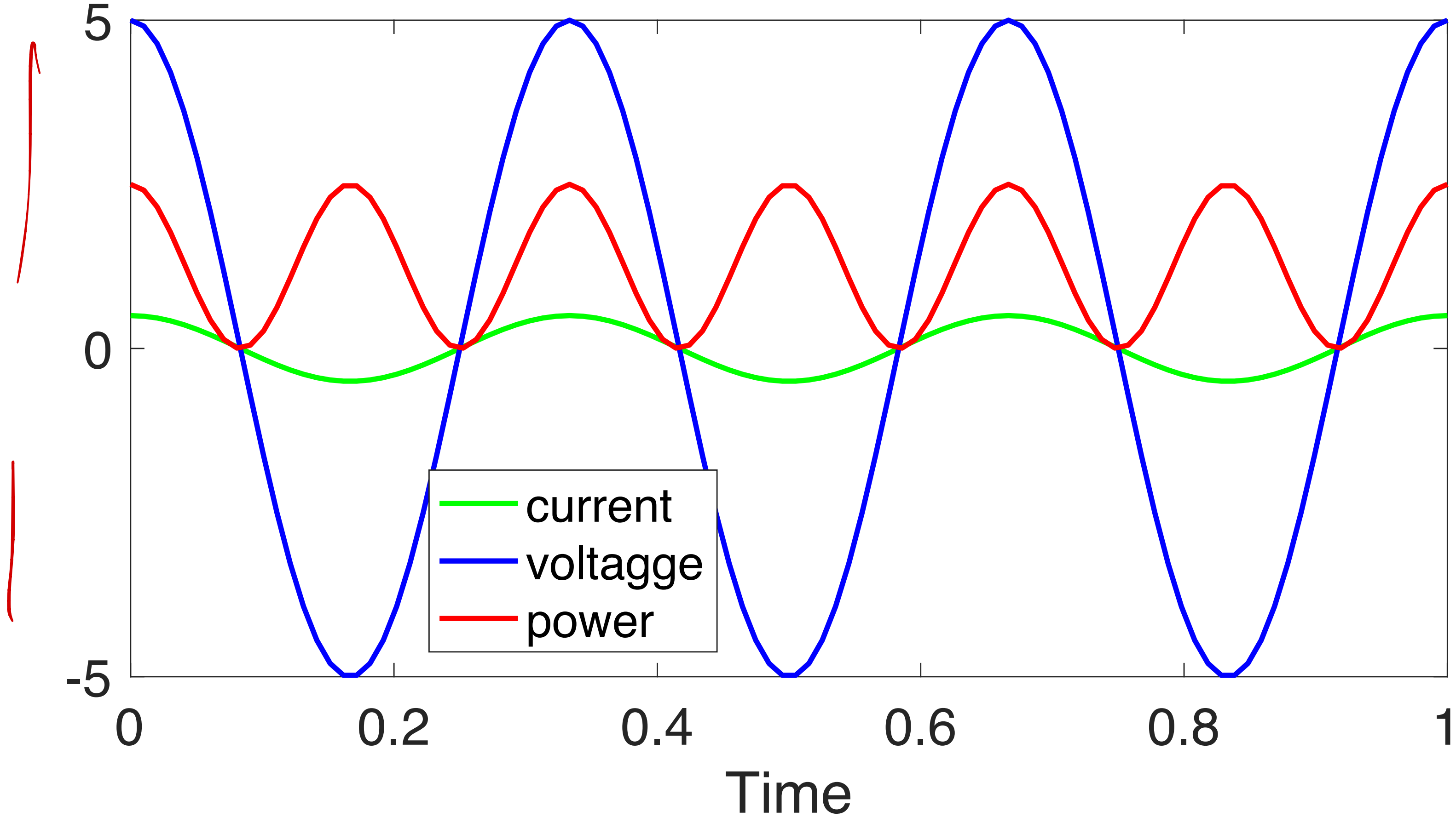
- With $i(t) = 0.5\cos\omega t$



- Consider the 10 Ω resistor:

$$i_{10}(t) = 0.5\cos\omega t \text{ A}$$
$$v_{10}(t) = 10\, i_{40}(t) = 5\cos\omega t \text{ V}$$
$$p_{10}(t) = v_{10}(t)\, i_{10}(t) = 2.5\cos^2\omega t \text{ W}$$
$$= 1.25 + 1.25\cos 2\omega t \text{ W}$$

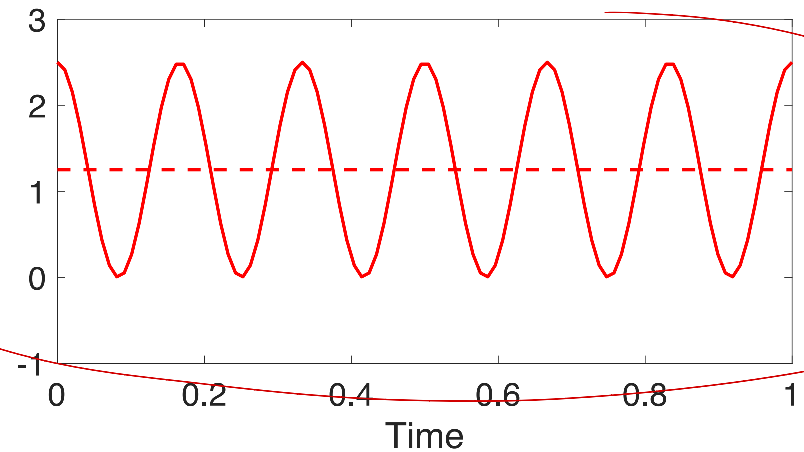
- Graphically:



- To simplify, keep just the average value
  - Since periodic, average over 1 period

$$P = \frac{1}{T}\int_{t}^{t+T} v(t)i(t)dt$$

  - For our example



$$P_{4\Omega} = \frac{1}{T}\int_{0}^{T} (1.25 + 1.25\cos 2\omega t)dt = \mathbf{1.25\,W}$$

5 volts, 0.5 amps, but <u>not 2.5</u> watts

- In general, for AC and resistors V = I R

$$v(\mathrm{t}) = \mathrm{V}\cos\omega t \qquad i(\mathrm{t}) = \mathrm{I}\cos\omega t$$

$$P(t) = v(t)\, i(t) = \mathrm{V\,I}\cos^2\omega t$$

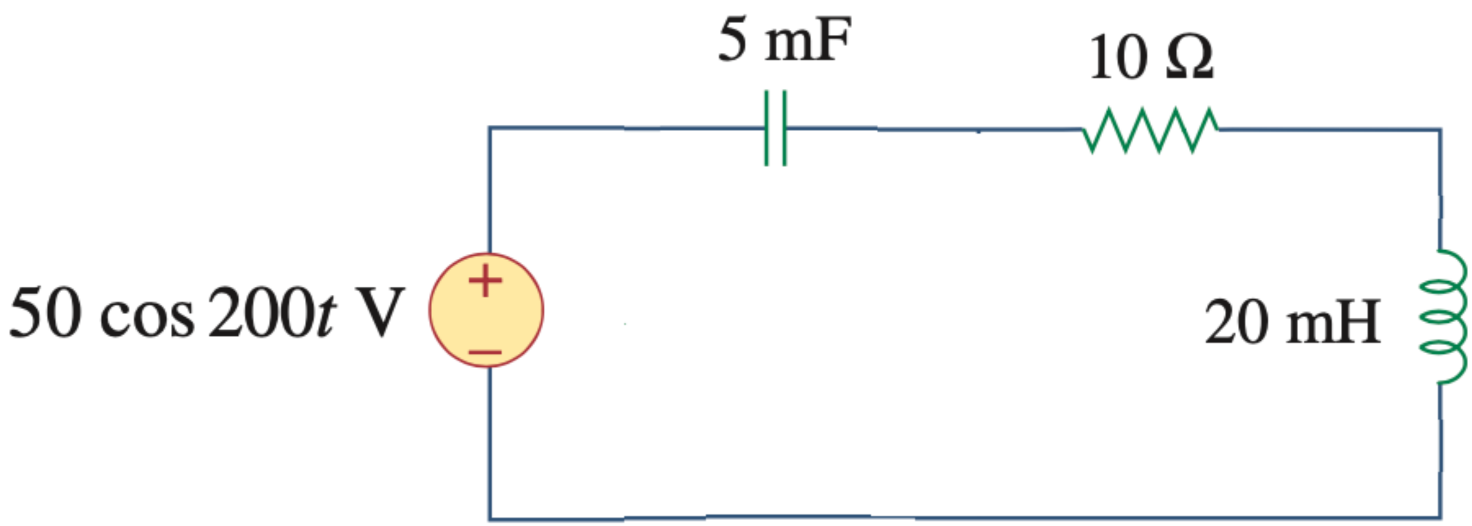
$$= \frac{\mathrm{V\,I}}{2} + \frac{\mathrm{V\,I}}{2}\cos\,2\omega t = \boxed{\frac{\mathrm{V}}{\sqrt{2}}\frac{\mathrm{I}}{\sqrt{2}}} + \frac{\mathrm{V}}{\sqrt{2}}\frac{\mathrm{I}}{\sqrt{2}}\cos\,2\omega t$$

$$P_{average} = \frac{\mathrm{V}}{\sqrt{2}}\frac{\mathrm{I}}{\sqrt{2}} = V_{RMS}\, I_{RMS}$$

What the DMM reads!

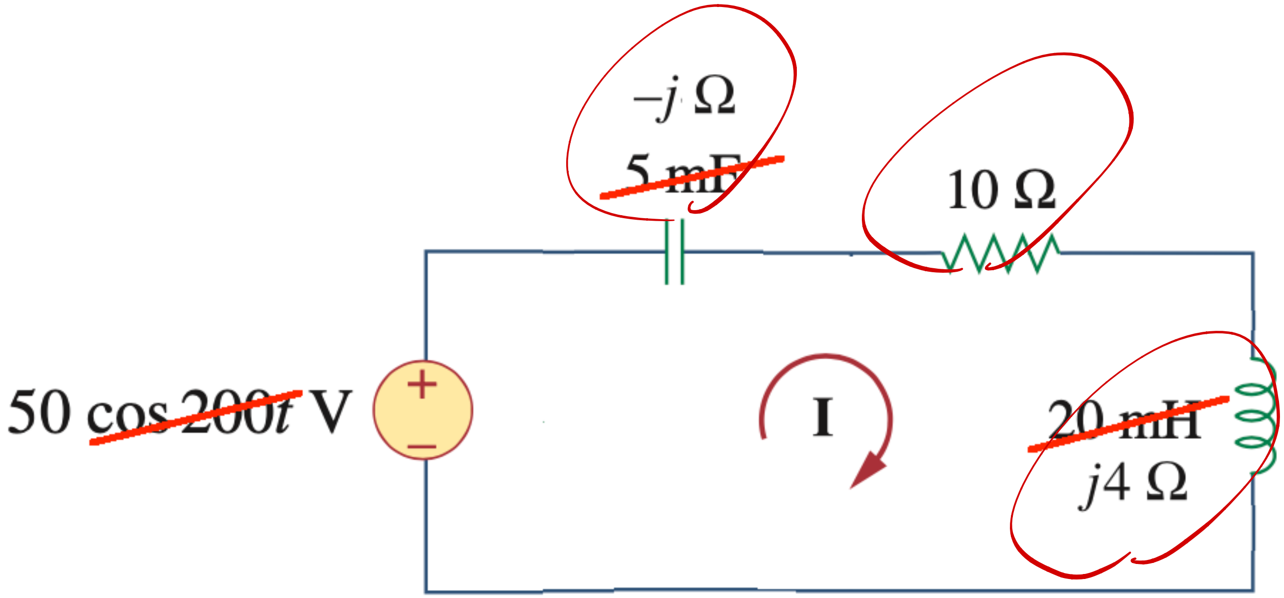
What about an RLC Load?

- Consider



- Solve for the current and voltages using phasors

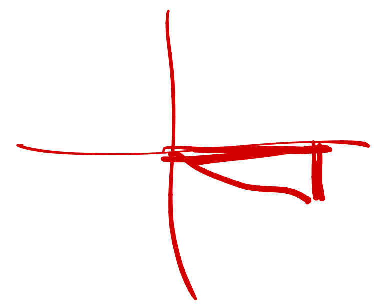
- Convert to phasors and impedances



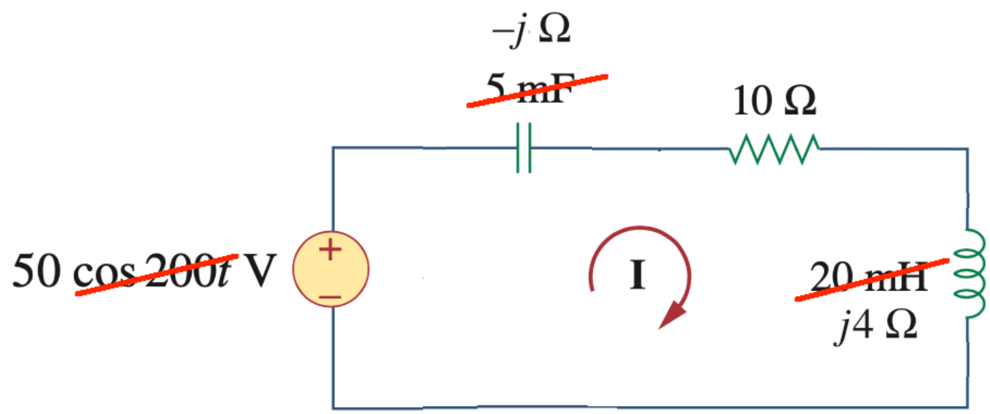
- The clockwise current is

$$\mathbf{I} = \frac{50}{-j1+10+j4} = \frac{50}{10+j3} = 4.59 - j1.38$$

$$i(t) = 4.79\cos(200t - 16.7°)\ \text{A}$$



- Resistor:



$$\mathbf{I} = 4.59 - j1.38$$

$$i(t) = 4.79\cos(200t - 16.7°)\ A$$

$$\mathbf{V} = 10\ \mathbf{I} = 45.9 - j13.8$$

$$v_R(t) = 47.9\cos(200t - 16.7°)\ \text{V}$$

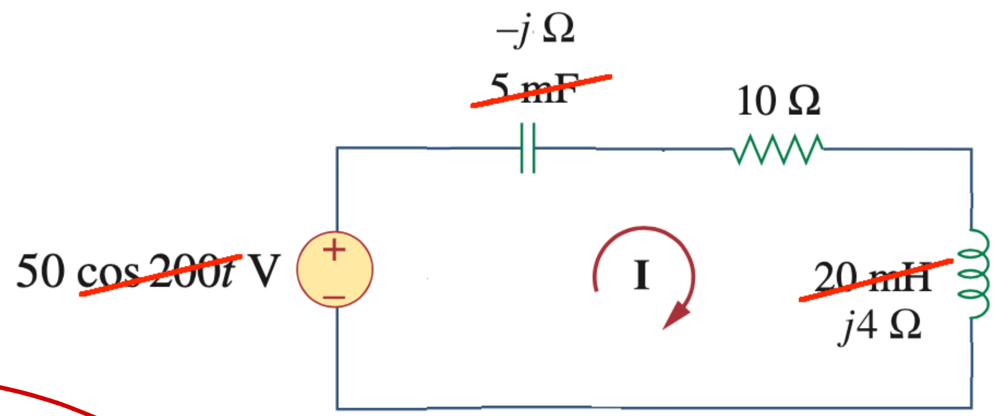
$$p(t) = v(t)\, i(t)$$

$$= 229\cos^2(200t - 16.7°)\ \text{W}$$

$$= 115 + 115\ \cos(400t - 33.4°)\ \text{W}$$

So $P_{average} = 115\ W$

- Inductor:



$$\mathbf{I} = 4.59 - j1.38$$

$$i(t) = 4.79\cos(200t - 16.7°)\ A$$

$$\mathbf{V} = j4\ \mathbf{I} = 5.50 + j18.3$$

$$v_L(t) = 19.2\cos(200t + 73.3°)\ \text{V}$$

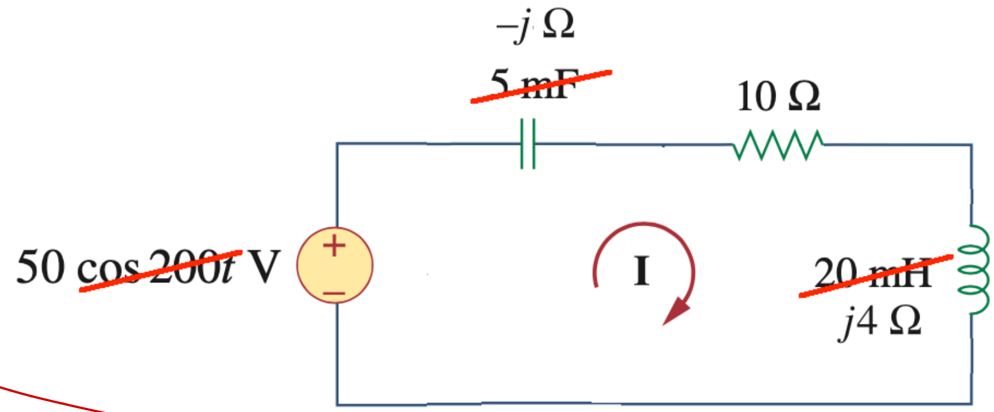
$$p(t) = v(t)\, i(t)$$

$$= 92.0\cos(200t - 16.7°)\cos(200t + 73.3°)\ \text{W}$$

$$= 46.0\ \cos(400t + 56.6°)\ \text{W}$$

So $P_{average} = 0$

- Capacitor:



$$\mathbf{I} = 4.59 - j1.38$$

$$i(t) = 4.79\cos(200t - 16.7°)\ A$$

$$\mathbf{V} = -j1\,\mathbf{I} = -1.38 - j4.59$$

$$v_C(t) = 4.79\cos(200t - 107°)\ \text{V}$$

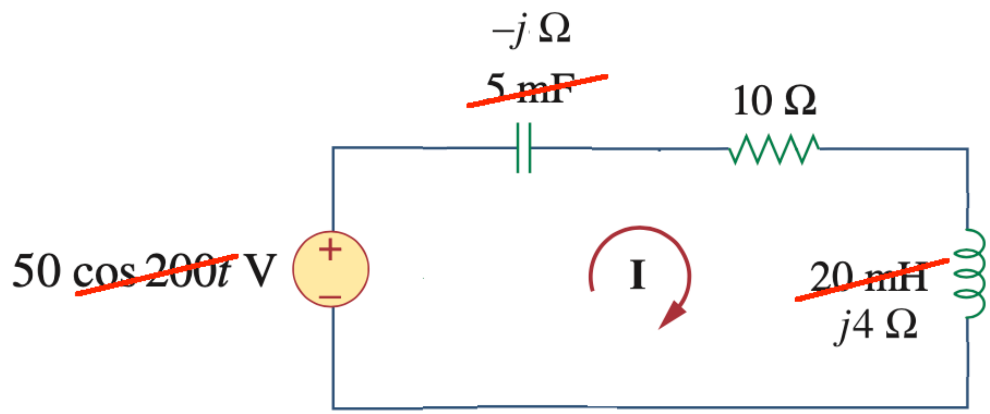
$$p(t) = v(t)\,i(t)$$

$$= 22.9\cos(200t - 16.7°)\cos(200t - 107°)\ \text{W}$$

$$= 11.5\ \cos(400t - 124°)\ \text{W}$$

So $P_{average} = 0$

- The source:



$\mathbf{I} = 4.59 - j1.38$

$$i(t) = 4.79\cos(200t - 16.7°)\ A$$

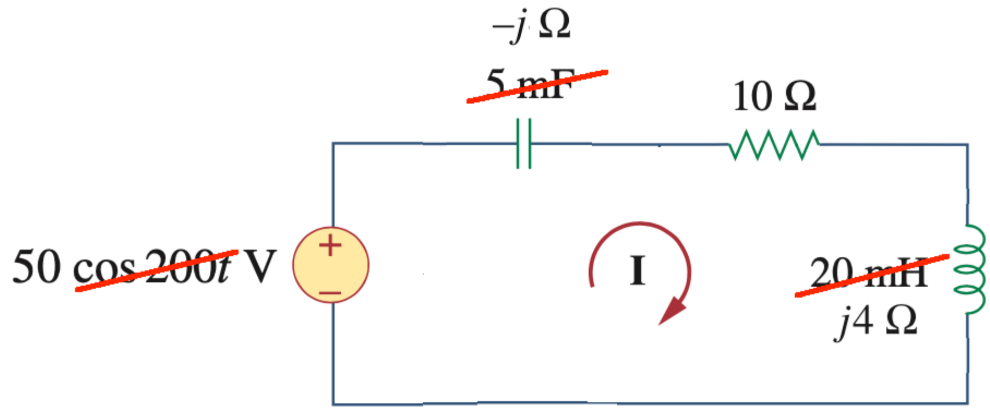
$\mathbf{V} = -50$

$$V_s(t) = 50\cos 200t\ \text{V}$$

$$p(t) = V_s(t)\, i(t)$$
$$= -239\cos 200t\ \cos(200t - 16.7°)\ \text{W}$$
$$= -115 - 120\cos(400t - 16.7°)\text{W}$$

So $P_{average} = -115\ \text{W}$

- Summary:



Real power
P = 115 Watts

$$P_R(t) = 115 + 115\ \cos(400t - 33.4°)\ \ \text{W}$$
$$P_L(t) = \quad 46.0\ \cos(400t + 56.6°)\ \text{W}$$
$$P_C(t) = \quad 11.5\ \cos(400t - 124°)\ \text{W}$$
$$P_S(t) = -115 + 120\cos(400t - 16.7°)\ \ \text{W}$$

Reactive power
Q = 34.5 VARs

Time varying portions sum to zero

Complex Power
$S = P + jQ$

Complex Power

- Can get these quantities directly from phasors

$$S = P + jQ = \frac{\mathbf{V}\,\mathbf{I}^*}{2} = \begin{cases} \dfrac{(\mathbf{Z}\,\mathbf{I})\,\mathbf{I}^*}{2} = \dfrac{\mathbf{Z}\,\mathbf{I}\,\mathbf{I}^*}{2} = \dfrac{|\mathbf{I}|^2}{2}\mathbf{Z} \\ \dfrac{\mathbf{V}\,(\mathbf{V}/\mathbf{Z})^*}{2} = \dfrac{\mathbf{V}\,\mathbf{V}^*}{2\,\mathbf{Z}^*} = \dfrac{|\mathbf{V}|^2}{2\,\mathbf{Z}^*} \end{cases}$$

- Our example

$$S = \frac{50\,(4.59 - j1.38)^*}{2} = 115 + j34.5$$

- Consider a load $\boldsymbol{Z} = |Z|e^{j\theta}$ and that we have the phasor relationship $\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}}$
  - The angle between the current and voltage phasors (vectors) is θ

- Further, recall the vector dot product so that the average power is

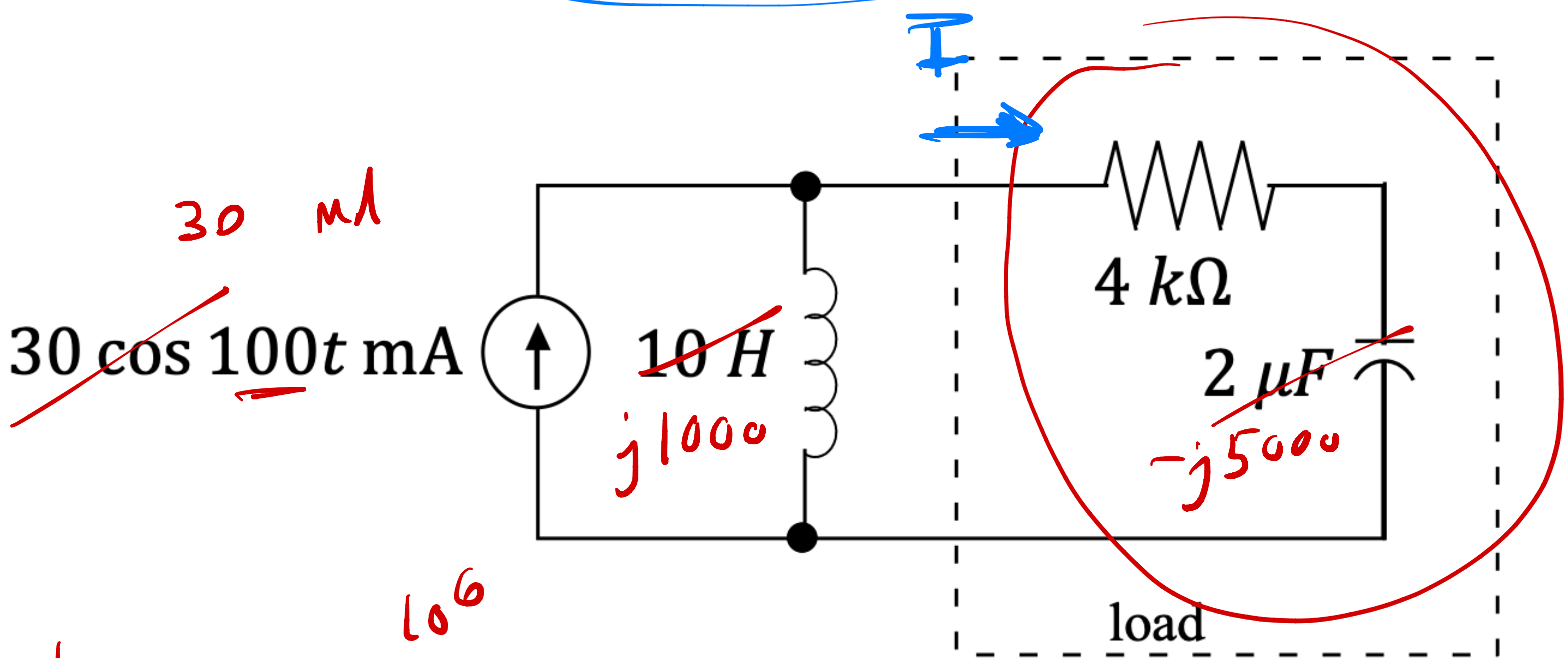
$$P = \frac{\mathbf{V}\,\mathbf{I}^*}{2} = \frac{|\mathbf{V}||\boldsymbol{I}|\cos\theta}{2} = \frac{|\mathbf{V}|}{\sqrt{2}}\frac{|\boldsymbol{I}|}{\sqrt{2}}\cos\theta$$

$$= V_{RMS}\, I_{RMS} \cos\theta$$

"power factor"

Example problem: Find the real and reactive powers absorbed by the load in this circuit.

$4000 - j5000$



$$-j\frac{1}{\omega C} = -j\frac{10^6}{100 \cdot 2}$$

$$I = .03\,\frac{j1000}{4000 - j5000 + j1000} = \frac{j1000}{4000 - j4000}$$

$$= \frac{j}{4(1-j)}\,\frac{1+j}{1+j} = \frac{-1+j}{8}$$

$$P = \frac{|I|^2}{2} Z = \frac{\frac{2}{64}(4000 - j5000)}{2}$$

$$I = \frac{-1 + j}{8} \qquad |I| = \frac{\sqrt{2}}{8}$$

$$Z = 4000 - j5000$$

$56.25\ mW, 56.25\ mVARs$