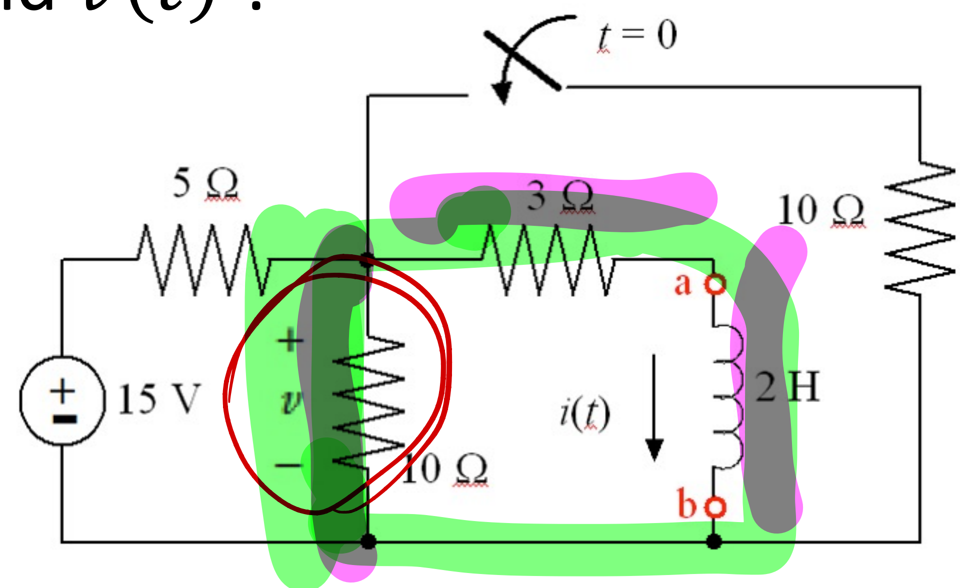
1^{st} Order Transients – 4

other circuit variables

Question – How do we find $v(t)$?



- Method 1:
  Propagate $i(t)$ from earlier class
  KVL:

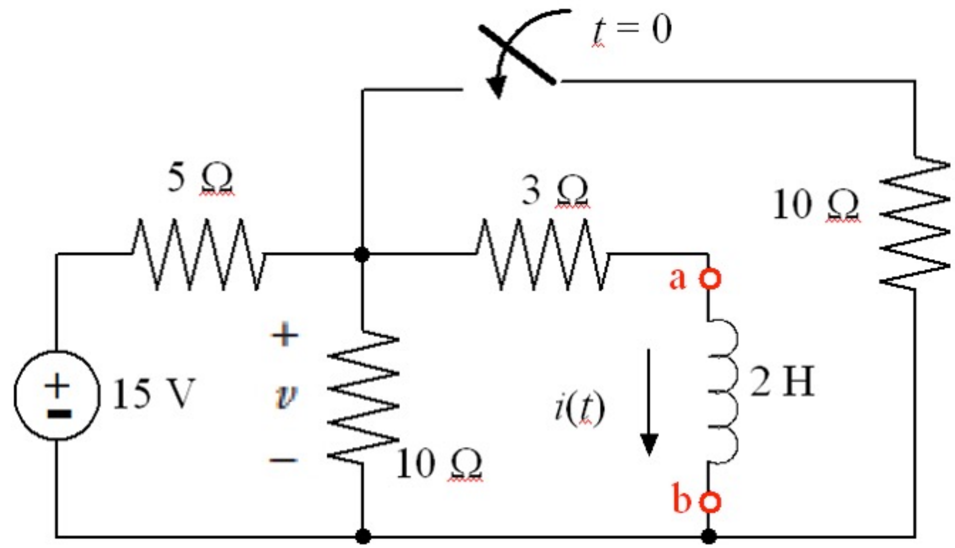
$$v(t) = 3\,i(t) + 2\frac{di(t)}{dt}$$

$$= 3(0.22\,e^{-2.75\,t} + 1.36)$$
$$+2(-2.75)(0.22\,e^{-2.75\,t})$$

$$= -0.55\,e^{-2.75\,t} + 4.09 \quad \text{volts}$$

- Method 2

  Direct solution



$$v(t) = (v_0 - v_\infty)\, e^{-t/\tau} + v_\infty$$

$$\tau = \frac{1}{2.75} \text{ sec}$$

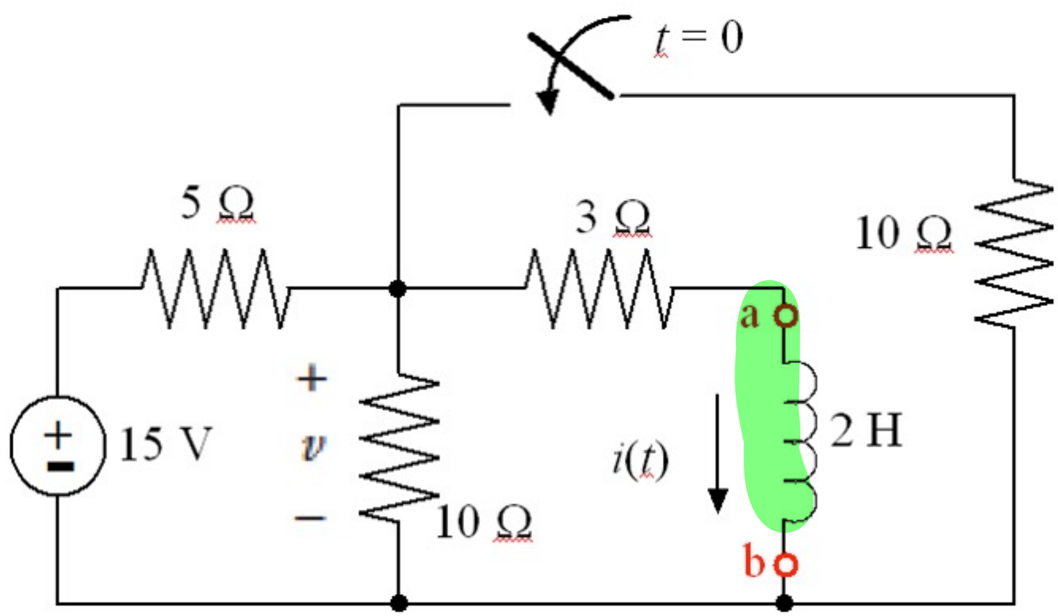
Same time constant !!

$$v_\infty = 15 \frac{10||10||3}{10||10||3 + 5} = 4.09$$

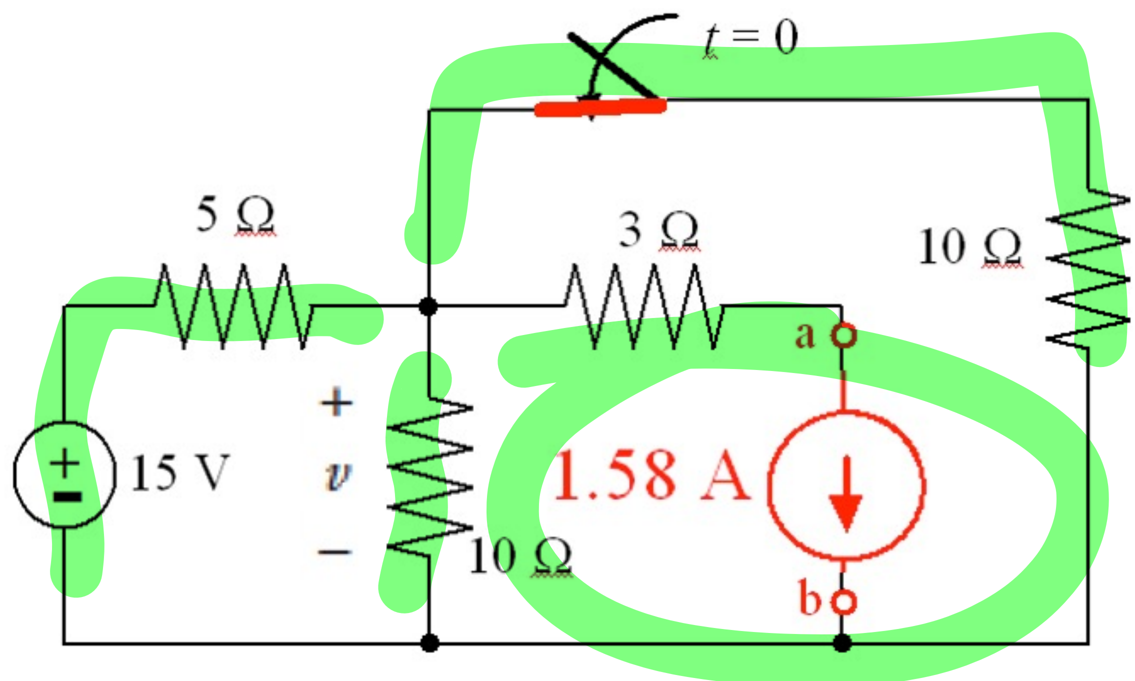
Voltage division (same idea)

- How do we find v_0 ?
  - **Cannot** use voltage before the switch (IC) since it need not be continuous
  - **Can** exploit the fact that

$$i_L(0) = 1.58 \quad \text{amps}$$



At time $t = 0$:

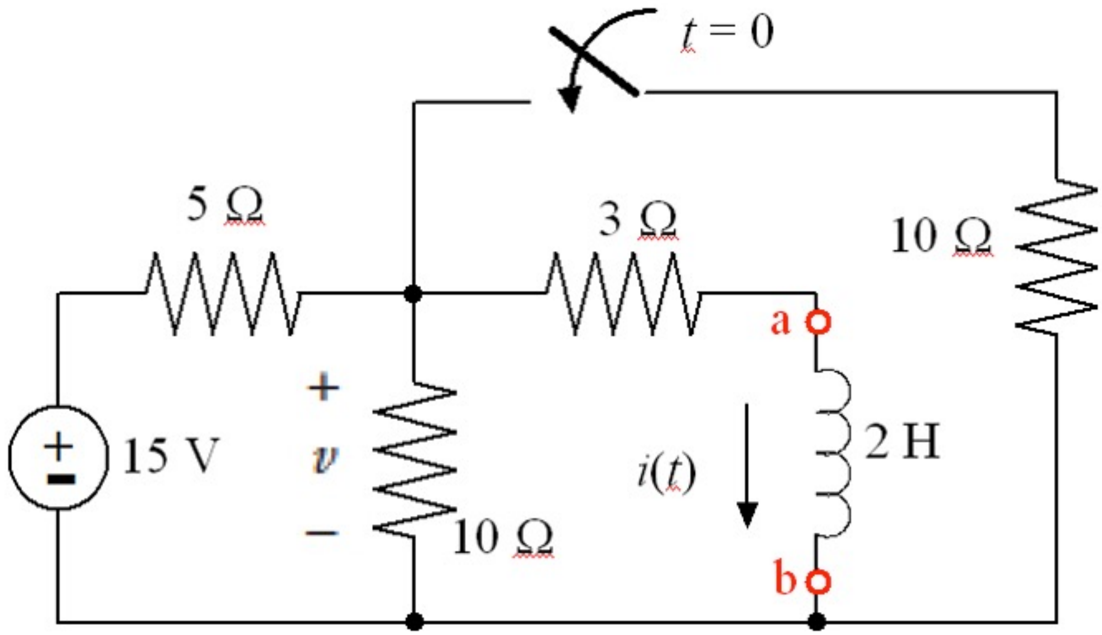


Node analysis:

$$\frac{v_0 - 15}{5} + \frac{v_0}{10} + \frac{v_0}{10} + 1.58 = 0$$

$$v_0 = 3.55 \quad \text{volts}$$

- So

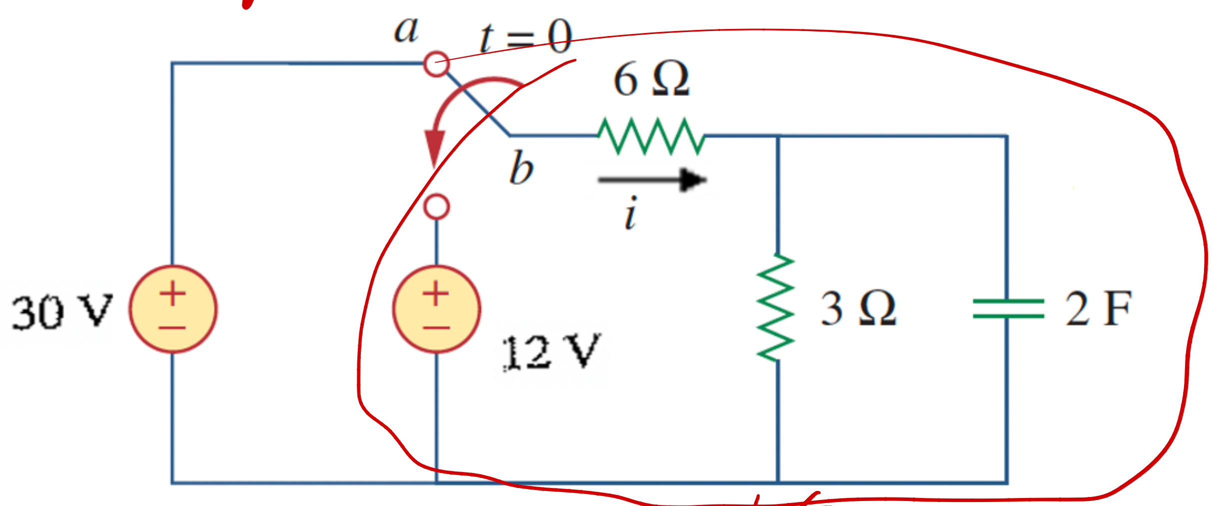


$$
\begin{aligned}
v(t) &= (v_0 - v_\infty)\, e^{-\frac{R}{L}t} + v_\infty \\
&= (3.55 - 4.09)\, e^{-2.75\,t} + 4.09 \\
&= -0.55\, e^{-2.75\,t} + 4.09 \quad \text{volts}
\end{aligned}
$$

- Also, for $t < 0$, $v(t) = \frac{90}{13} = 4.74$ volts, so we see a "jump" at $t = 0$

Example: find $i(t)$

$i_\infty + [i_0 - i_\infty] e^{-t/4}$ Am

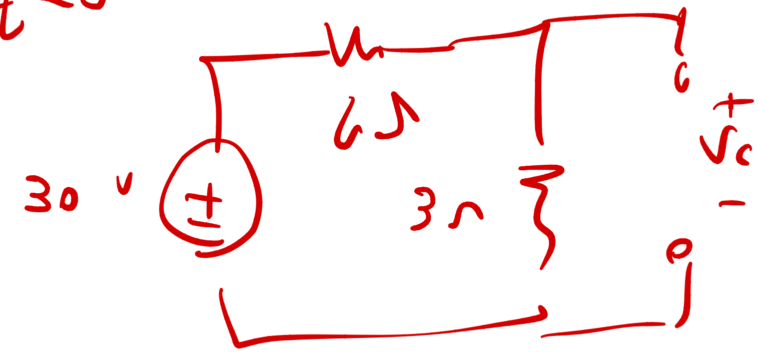


Form: $i(t) =$ $i_\infty + [i_0 - i_\infty] e^{-t/RC}$ Am

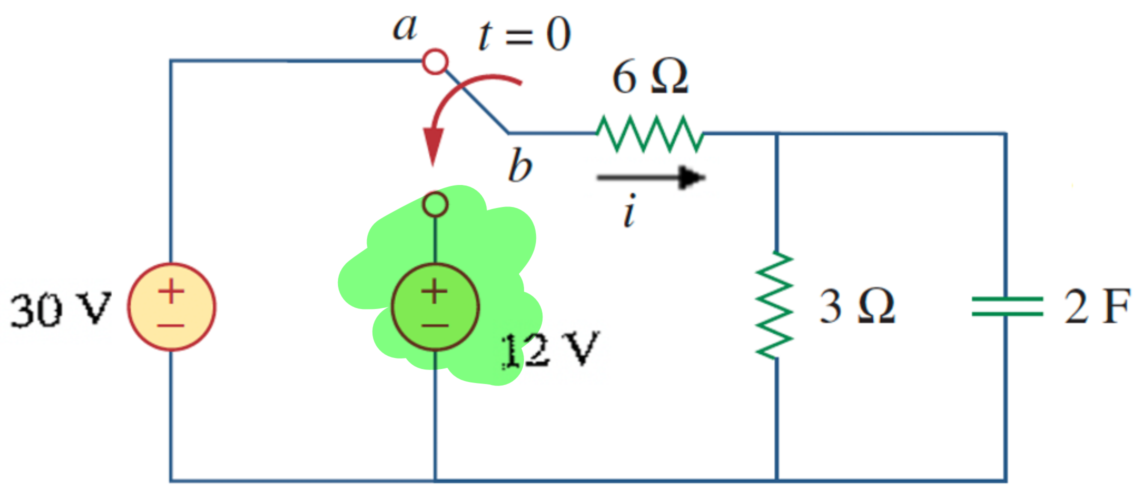
Time constant: $\tau = RC =$ $2 \cdot 2 = 4$

Final value: $i_\infty =$ $\frac{4}{3}$ A

$t<0$



Initial value: use $v_c(0)$

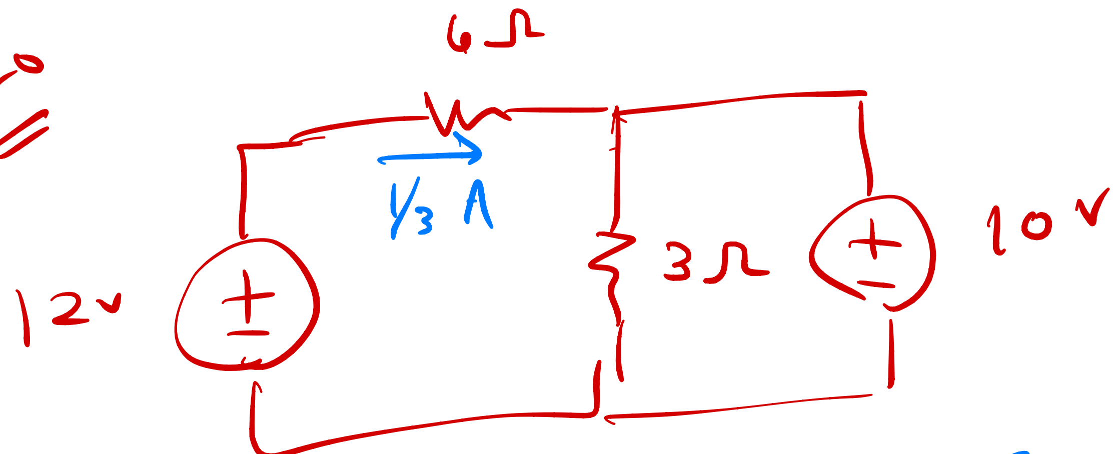


voltage div.

$v_c(0) = 10$ V

$i_0 = \frac{1}{3}$

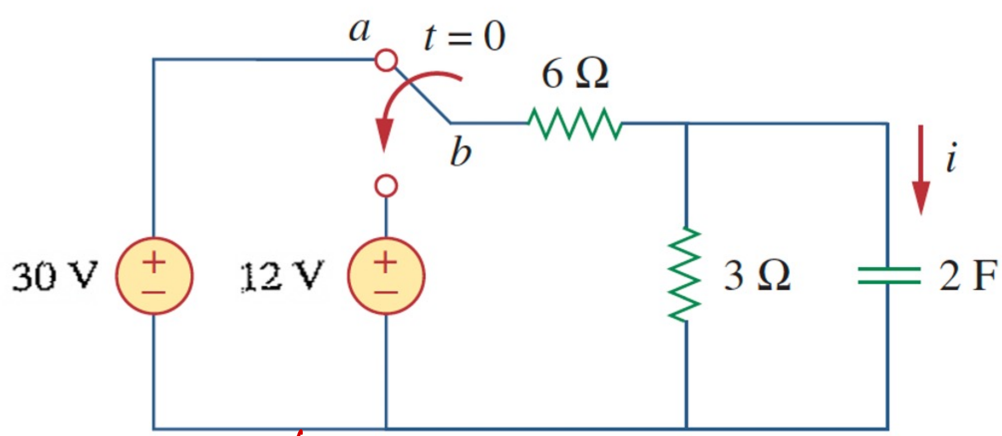
$t>0$



$i(t) = \frac{4}{3} + \left[\frac{1}{3} - \frac{4}{3}\right] e^{-t/4}$ A

$\boxed{\frac{4}{3} - 1e^{-t/4}}$

Example: same circuit, different variable

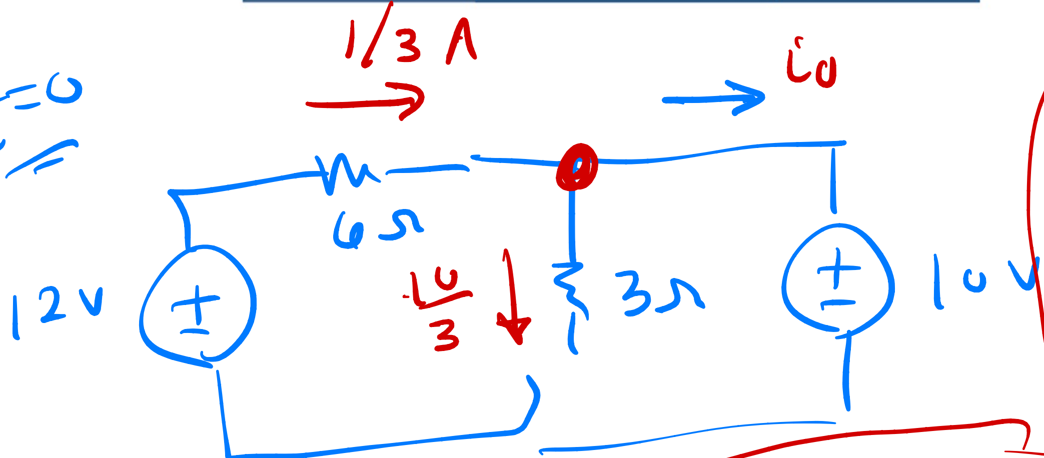


$$i(t) = i_\infty + [i_0 - i_\infty] e^{-t/RC}$$

$RC = 4$

$i_\infty = 0$

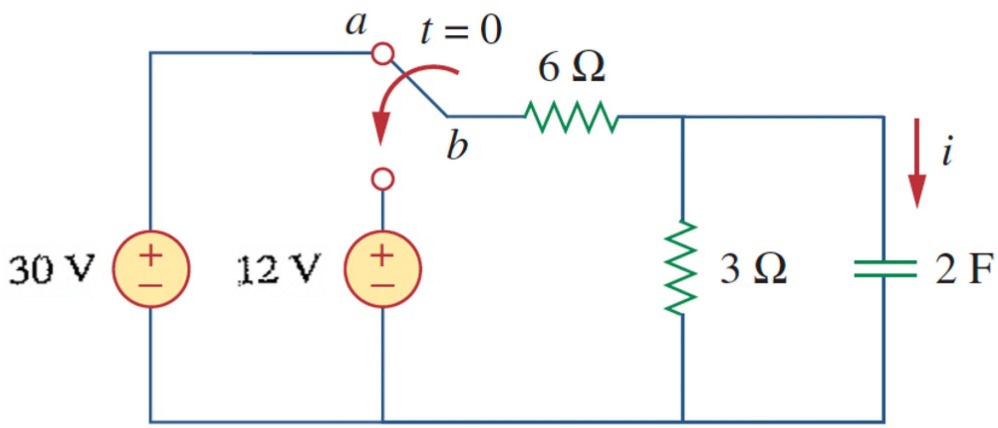
$t=0$



$$i_0 + \frac{10}{3} = \frac{1}{3}$$

$$i_0 = -\frac{9}{3} = -3\ A$$

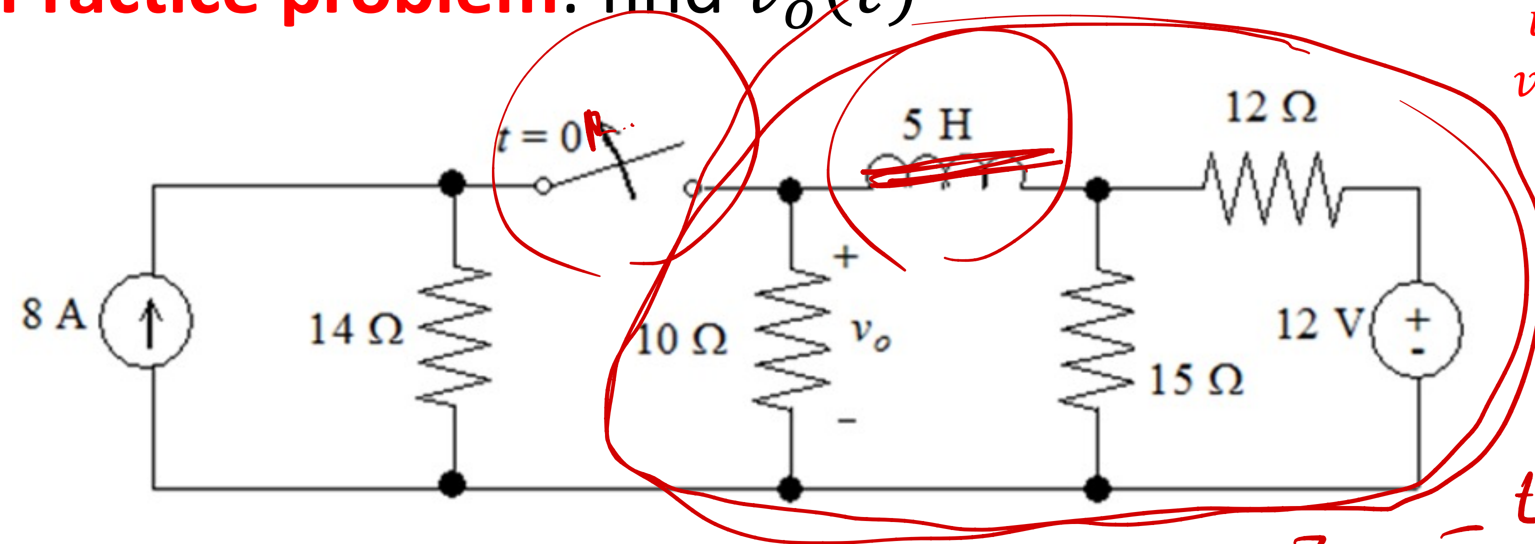
$$i(t) = -3e^{-t/4}\ A$$



$$R = 2\ \Omega$$
$$i(\infty) = 0\ A$$
$$v_C(0) = 10\ V$$
$$i(0) = -3\ A$$

Practice problem: find $v_o(t)$

$R = 16.7\ \Omega$
$v(\infty) = 4\ V$
$i_L(0) = 3.2A$
$v(0) = -32\ V$



$v_o(t) = v_R(\infty) + [v_o(0) - v_o(\infty)]\, e^{-t/\tau}$ v_o Hs

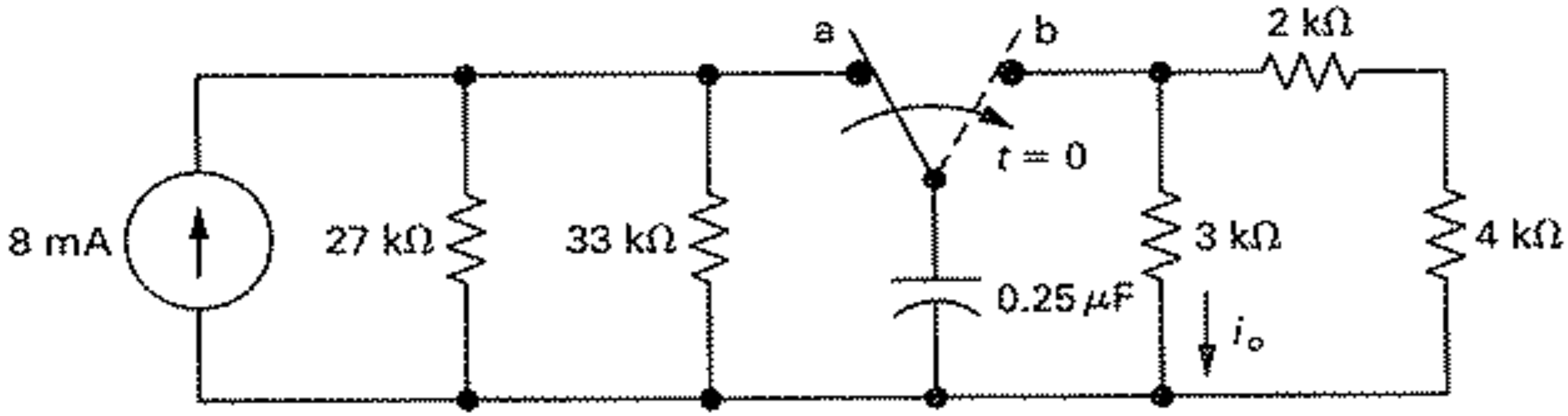
$\tau = \frac{L}{R} = \frac{5}{R_{TH}}$

$\boxed{R_{TH} = 10 + 15 \parallel 12}$

$\frac{v_o - 12}{12} + \frac{v_o}{15} + \frac{v_o}{10} = 0 \Rightarrow \boxed{v_o(\infty) = 4V}$

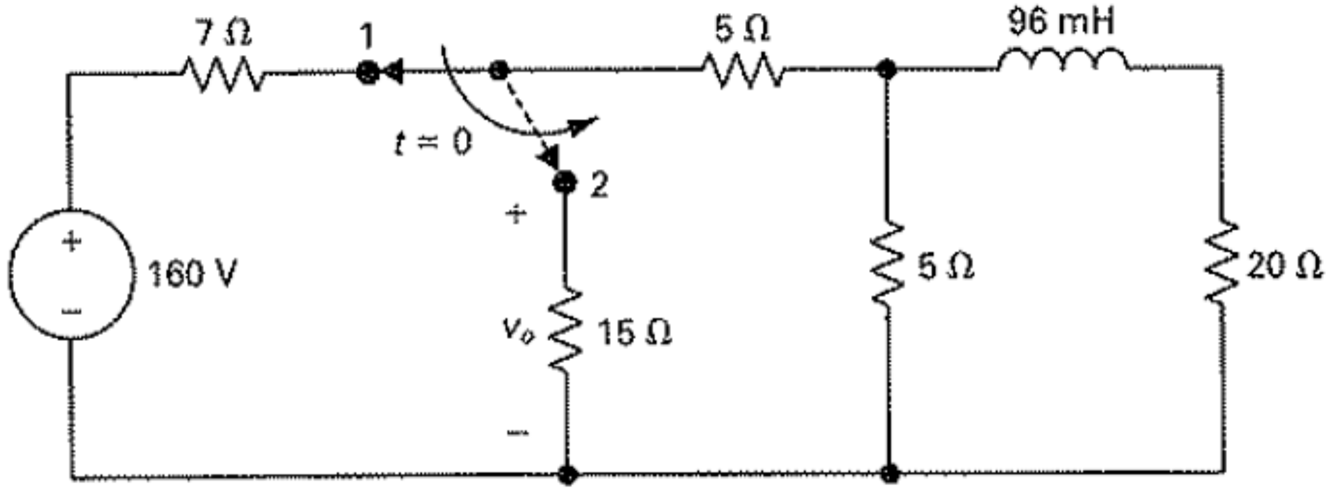
Practice problem: find $i_o(t)$

$R = 2\ k\Omega$
$i(\infty) = 0\ V$
$v_C(0) = 118.8\ V$
$i(0) = 39.6\ mA$



$R = 24\ \Omega$
$v(\infty) = 0\ V$
$i_L(0) = 2A$
$v(0) = -6\ V$

Practice problem: find $v_o(t)$



$-60e^{-80,000t}\ V$

Practice problem: find $v_o(t)$