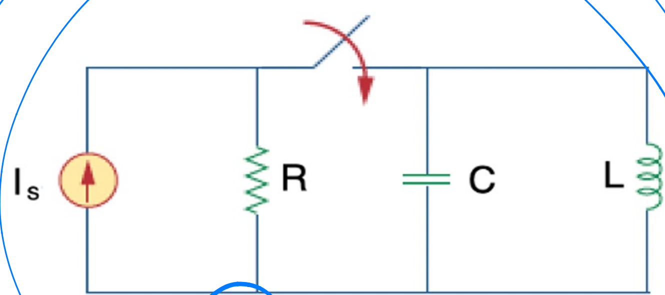
2^{nd} Order Transients – 2

Form; more resistances;
initial and final conditions

So far, for "simple" RLC circuits

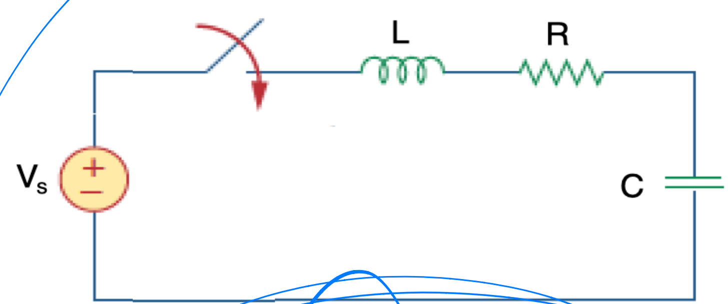
- Step 1 – identify type and form characteristic polynomial

Parallel



$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

Series



$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

- Step 2 – based on real vs complex roots, identify form of solution

Real roots:

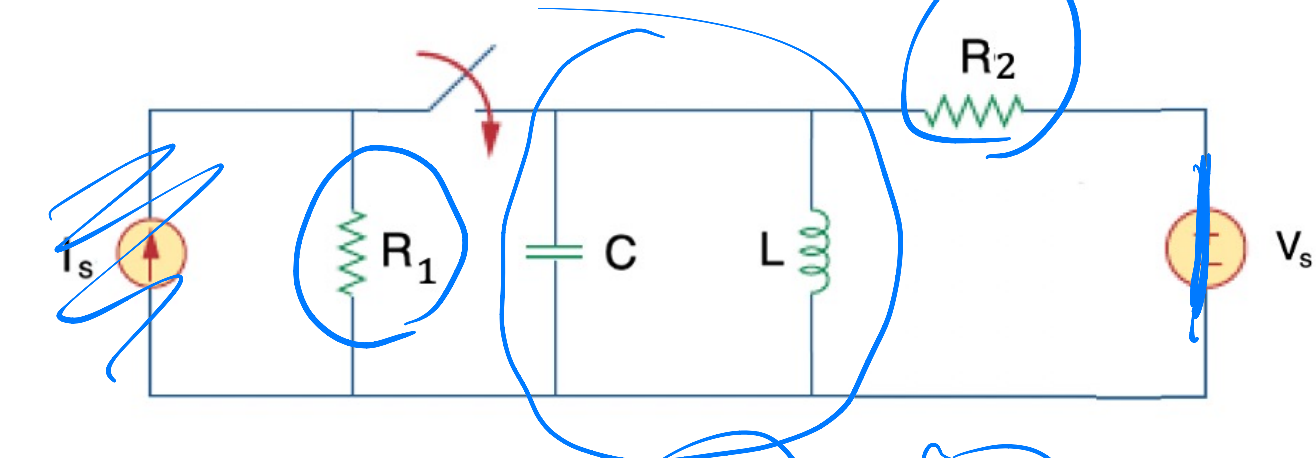
$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + x_\infty$$

Complex roots:

$$x(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t + x_\infty$$

- Step 3 – use final value to evaluate x_∞
- Step 4 – the other constants?

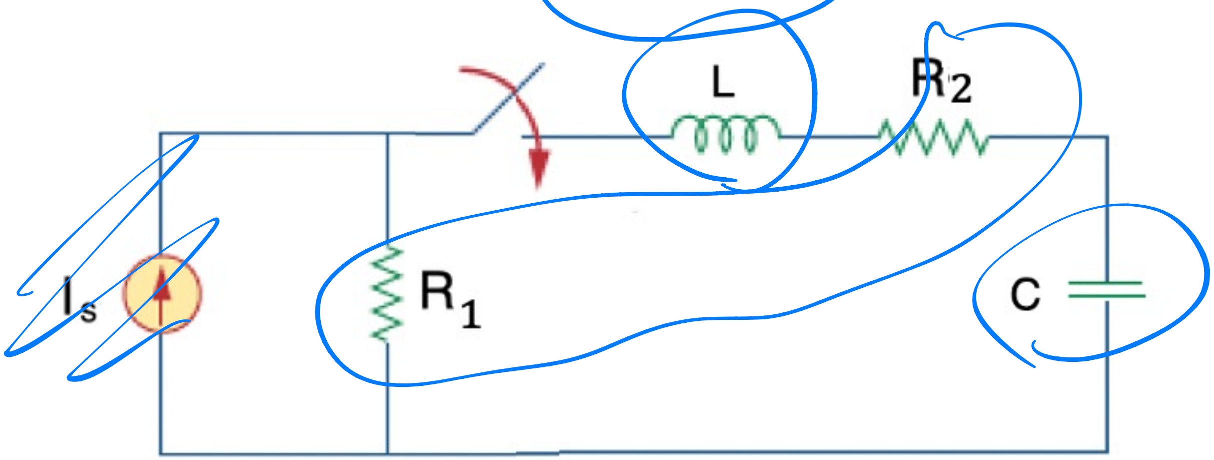
- Aside – what if we add an extra resistor?



Parallel form

$R = R_1 \parallel R_2$

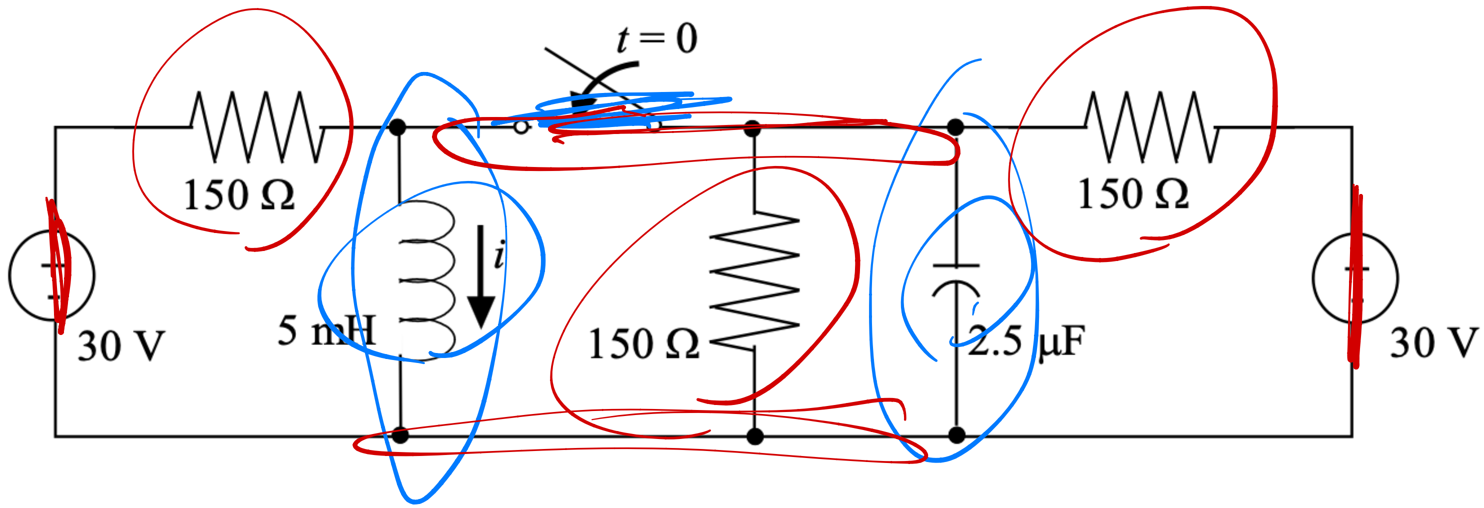
$s^2 + \frac{1}{RC}s + \frac{1}{LC}$

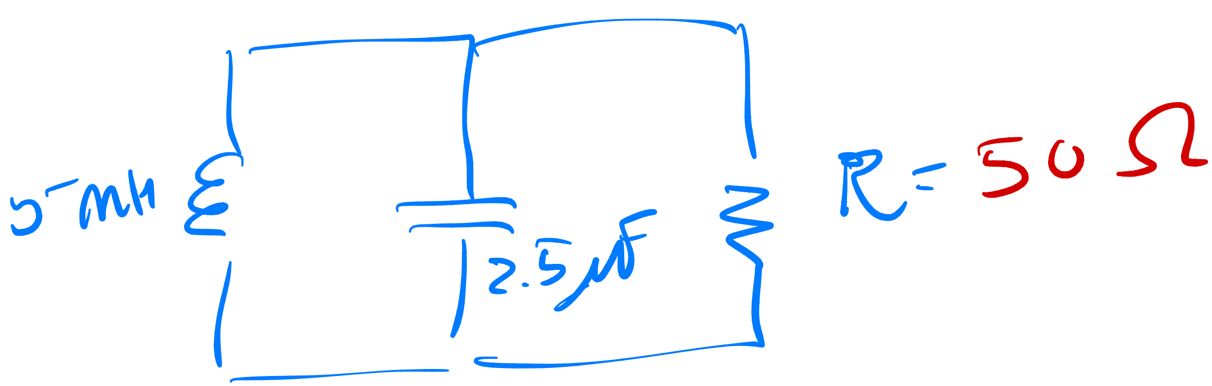


$R = R_1 + R_2$

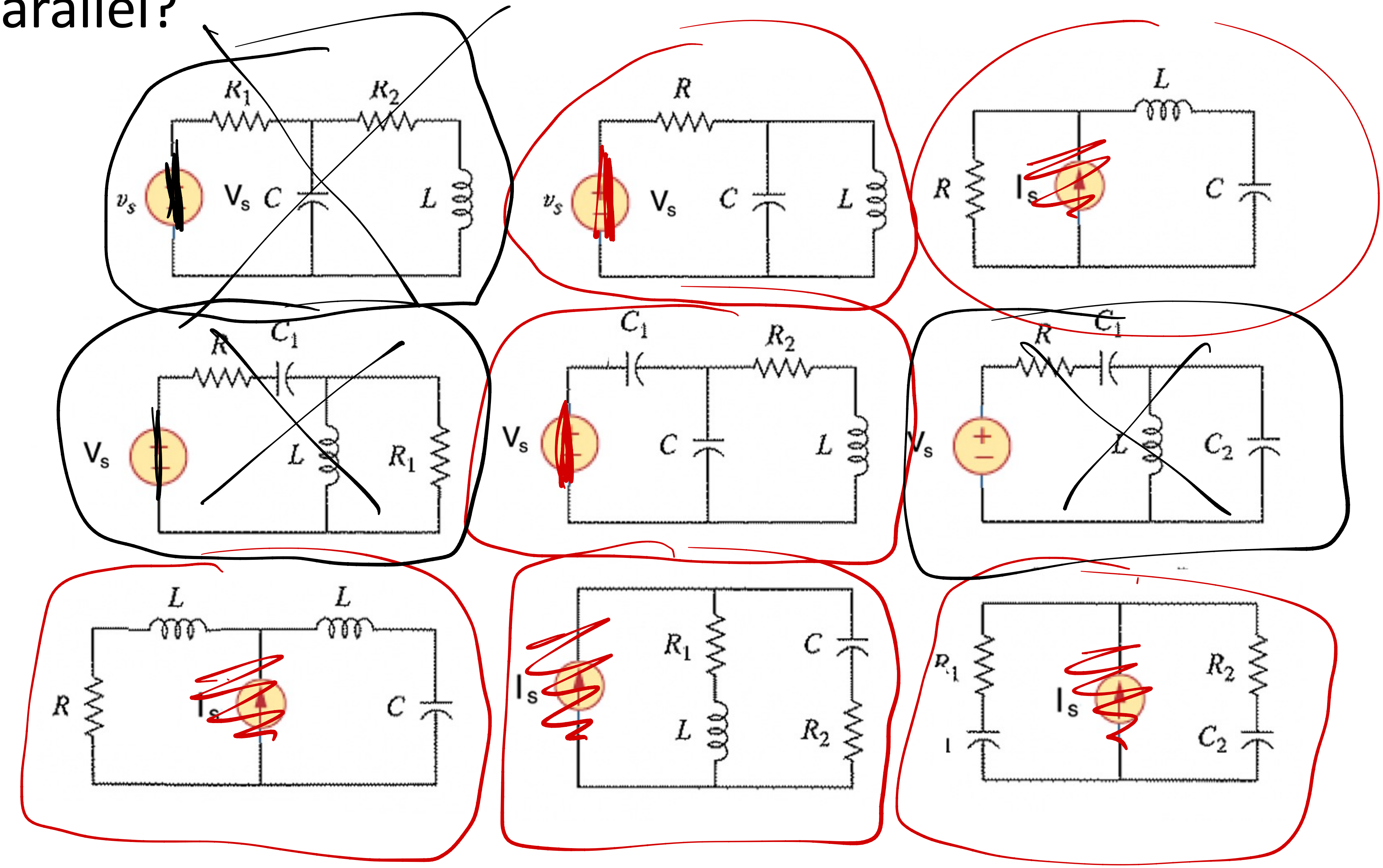
$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

- Or several?





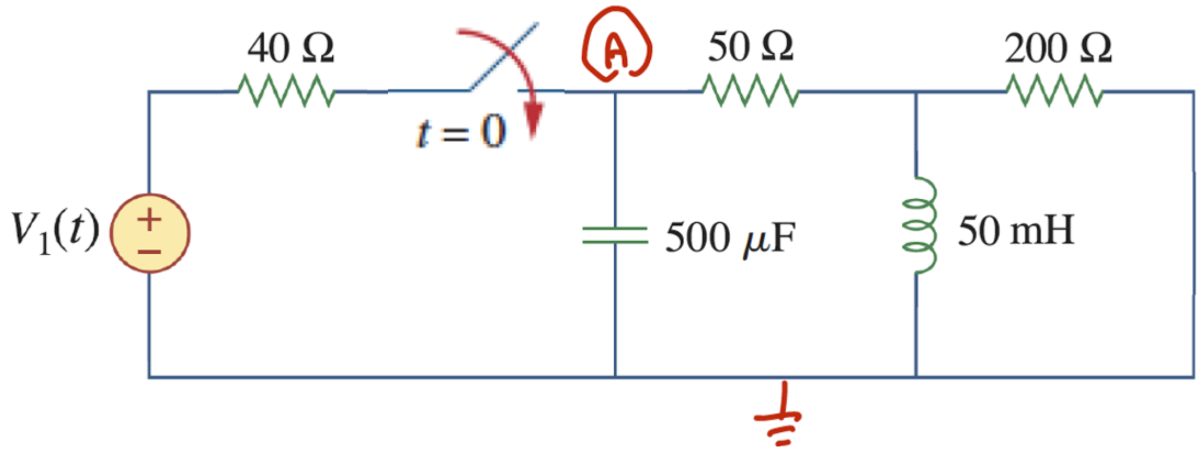
Question: Which of these circuits match our assumed 2nd order RLC circuit form? If yes, which form, series or parallel?



Initial and Final Conditions

- Just like the 1st order case:
  - From a DC analysis based on "open" or "short" models for C and L both before and after the switch event
    - Before switching event yields initial values
    - After switching event yields final values

Example:



$$A(t) = a_0 + a_1 e^{-94.3t} + a_2 e^{-764t}$$

- $A(0) = 0$
- $A(\infty) = \frac{5}{9} V_1$

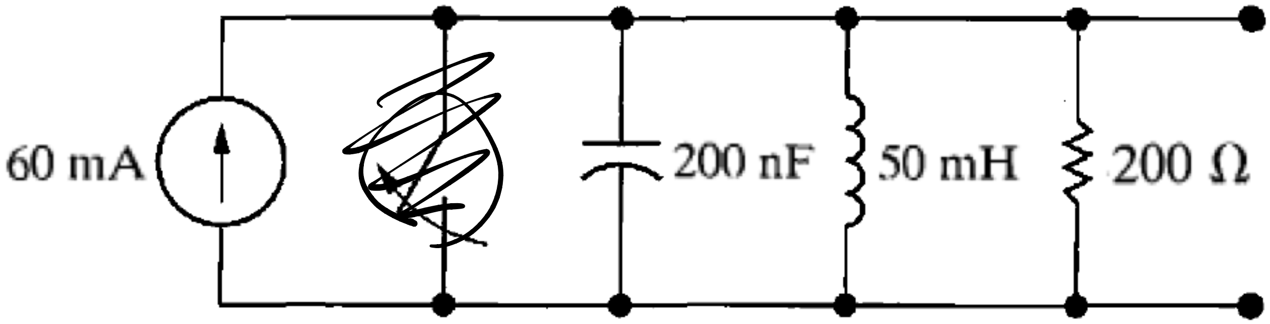
$$a_0 = \frac{5}{9} V_1$$

$$a_0 + a_1 + a_2 = 0$$

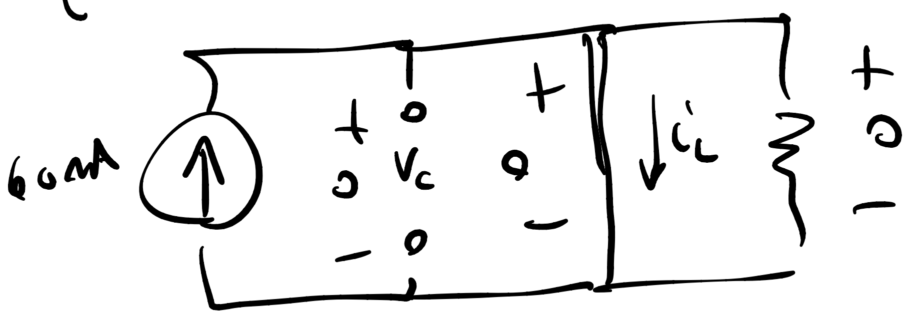
Example: Find the initial/final conditions

$t \to \infty$

$t = 0^-$



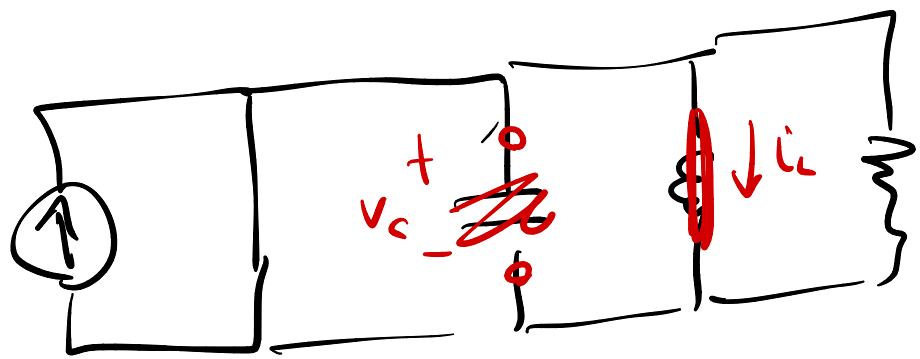
$t \to \infty$



$v_C(\infty) = 0$

$i_L(\infty) = 60\ \text{mA}$

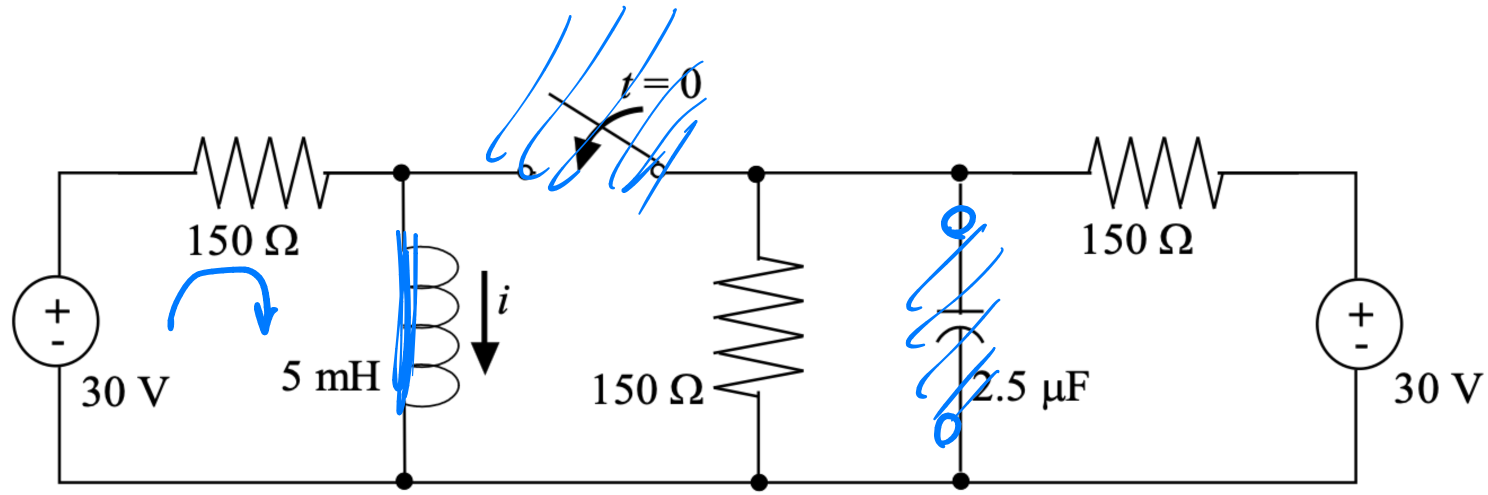
$t = 0^-$



$i_L(0^-) = 0$

$v_c(0^-) = 0$

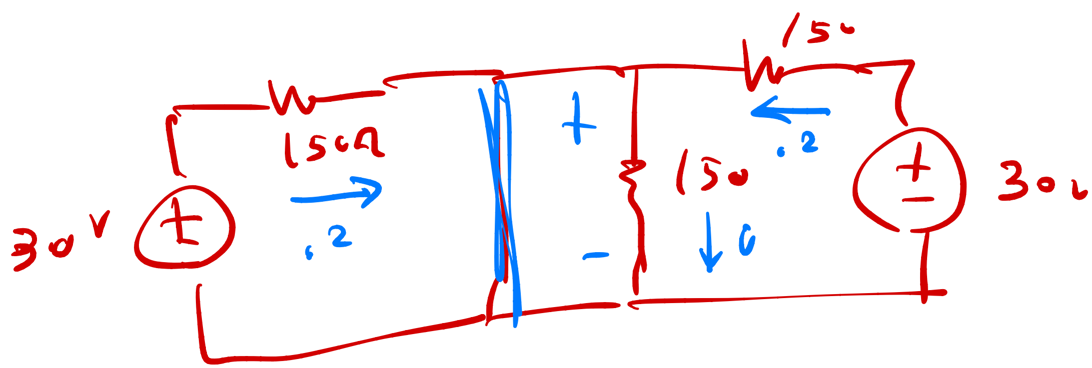
Example: Find the initial/final conditions



$t \to \infty$

$i_L(\infty) = .4\ A$

$v_C(\infty) = 0$

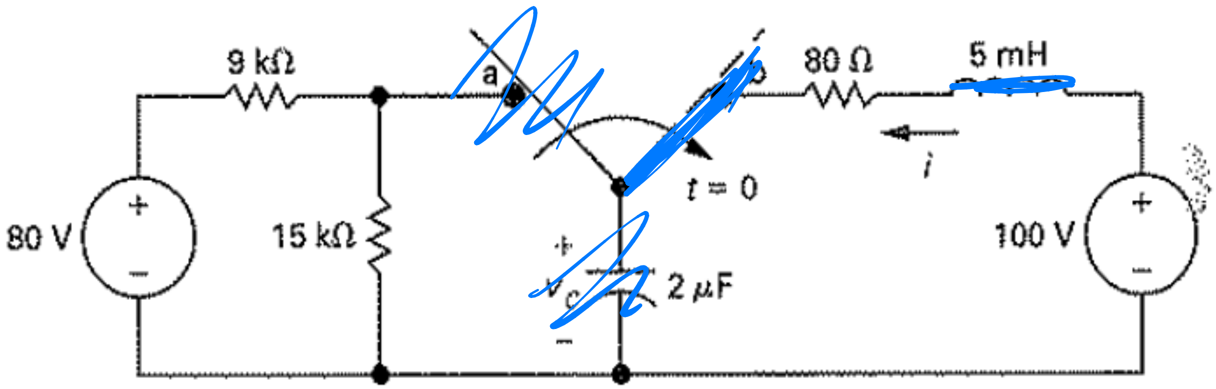


$t = 0^-$

$i_L(0^-) = \frac{30}{150} = .2\ A$

$v_C(0^-) = 15\ V$

Example: Find the initial/final conditions



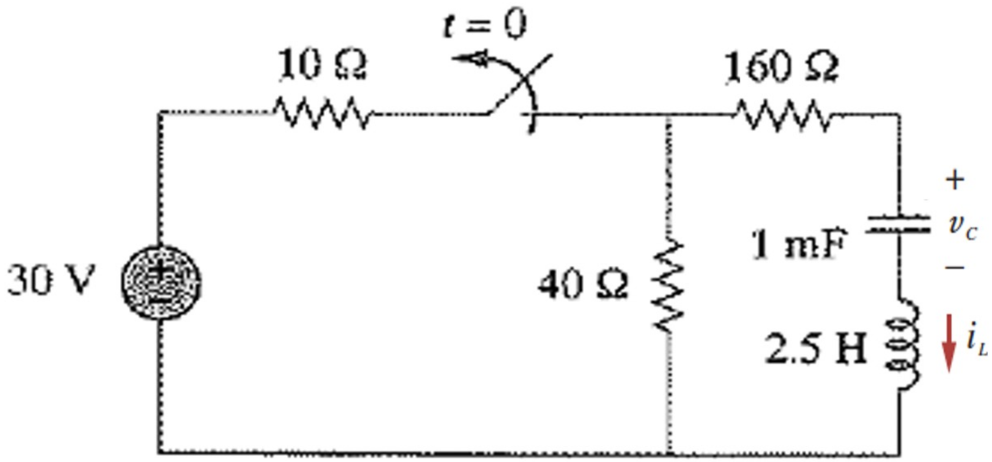
$i_L(0^-) = 0$ A

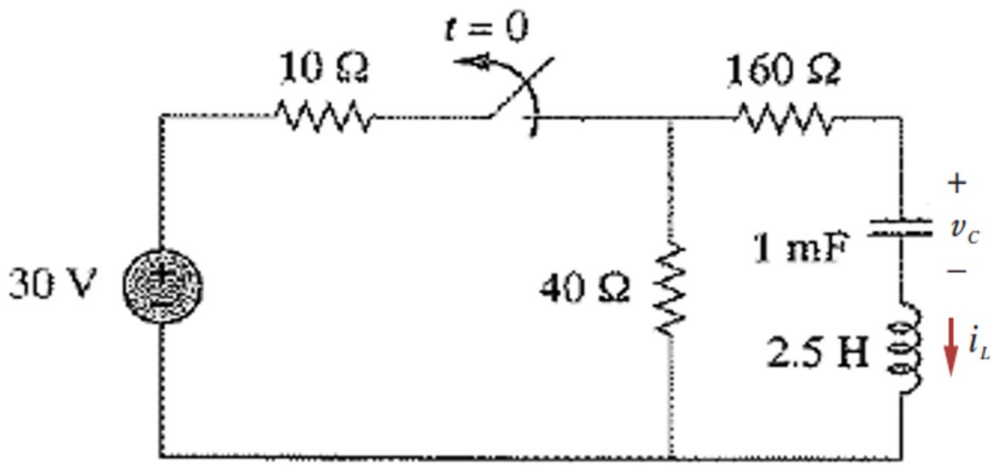
$v_C(0^-) = 80 \cdot \frac{15}{15+9}$

$i_L(\infty) = 0$ A

$v_C(\infty) = 100$ V

Practice problem: Find the form of solution and the initial/final conditions

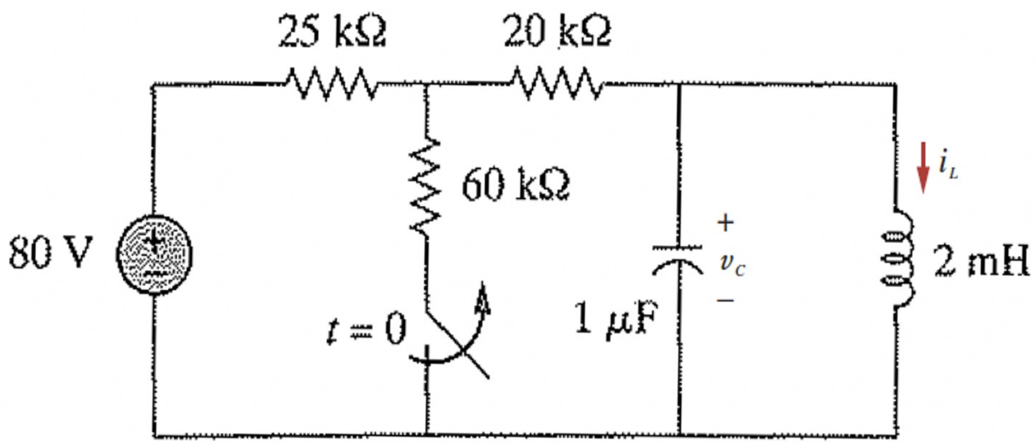


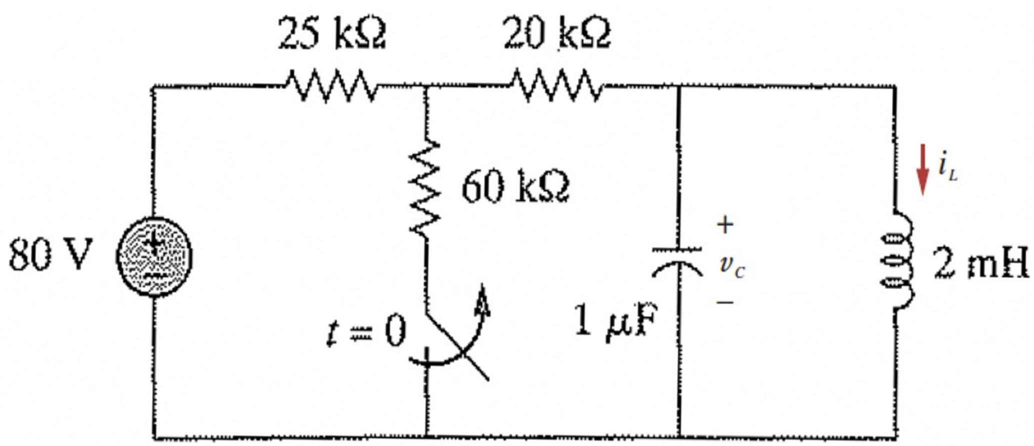


$$x(t) = A_1 e^{-5.36t} + A_2 e^{-74.6t} + x_\infty$$

$$i_L(0) = 0\ A$$
$$i_L(\infty) = 0\ A$$
$$v_C(0) = 24\ V$$
$$v_C(\infty) = 0\text{V}$$

Practice problem: Find the form of solution and the initial/final conditions





$$i_L(0) = 1.5\ A$$

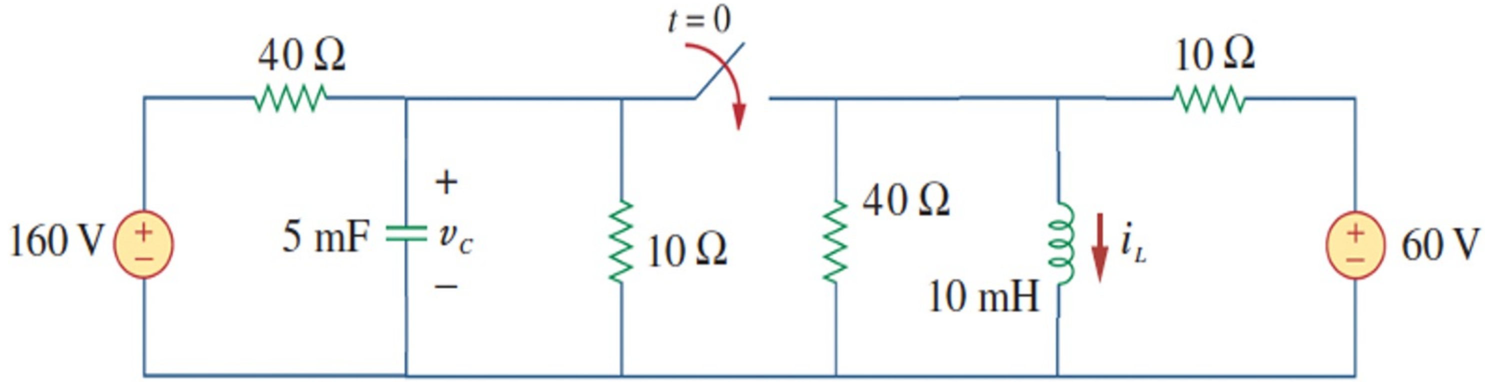
$$i_L(\infty) = \frac{16}{9}\ A$$

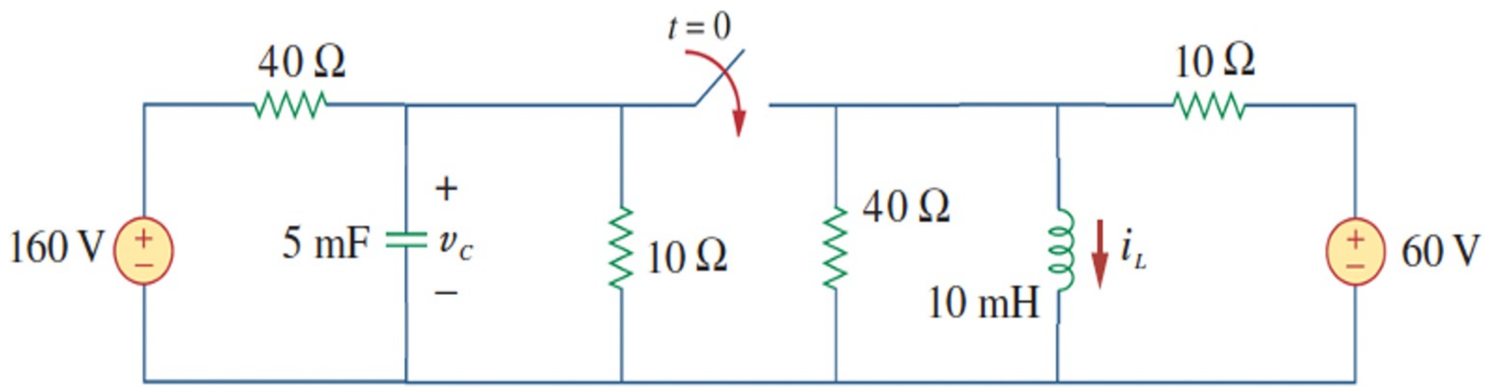
$$v_C(0) = 0\ V$$

$$v_C(\infty) = 0\ \mathrm{V}$$

$$x(t) = B_1 e^{-11.1t}\cos 22361t + B_2 e^{-11.1t}\sin 22361t + x_\infty$$

Practice problem: Find the form of solution and the initial/final conditions





$$x(t) = B_1 e^{-25t} \cos 139t + B_2 e^{-25t} \sin 139t + x_\infty$$

$$i_L(0) = 6\ A$$
$$i_L(\infty) = 10\ A$$
$$v_C(0) = 32\ V$$
$$v_C(\infty) = 0\ \mathrm{V}$$