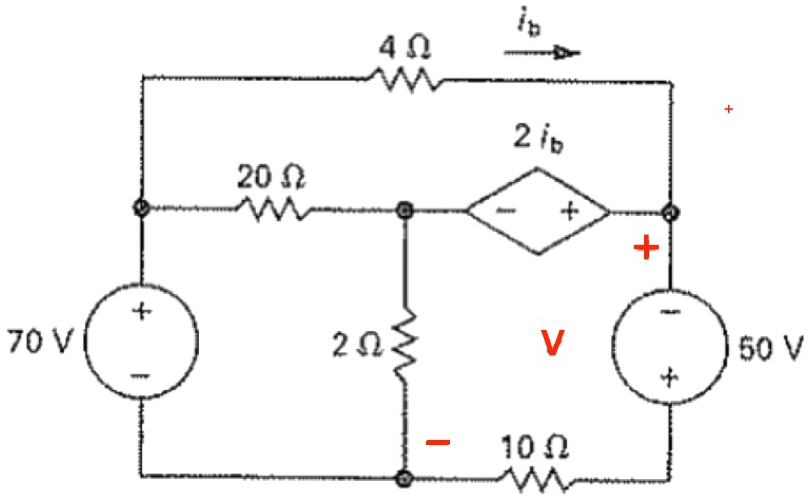
Mesh Analysis

concept; examples

General Methods to Analyze Circuits

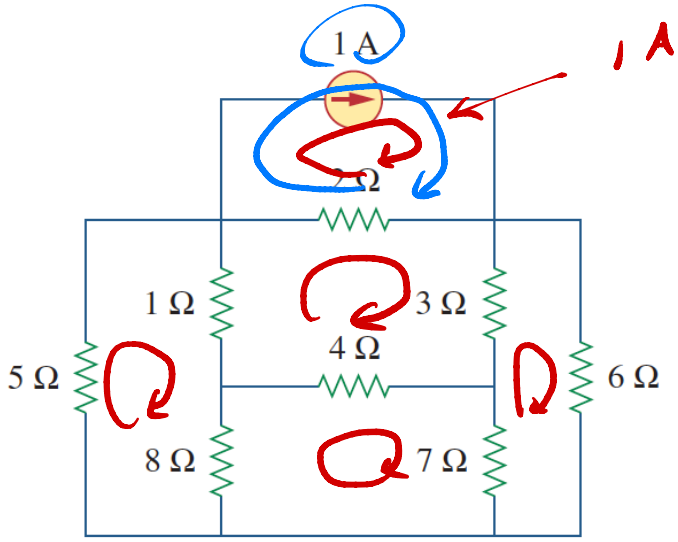
- What to do first?
  - KVL?
  - KCL?
  - Ohm's Law?



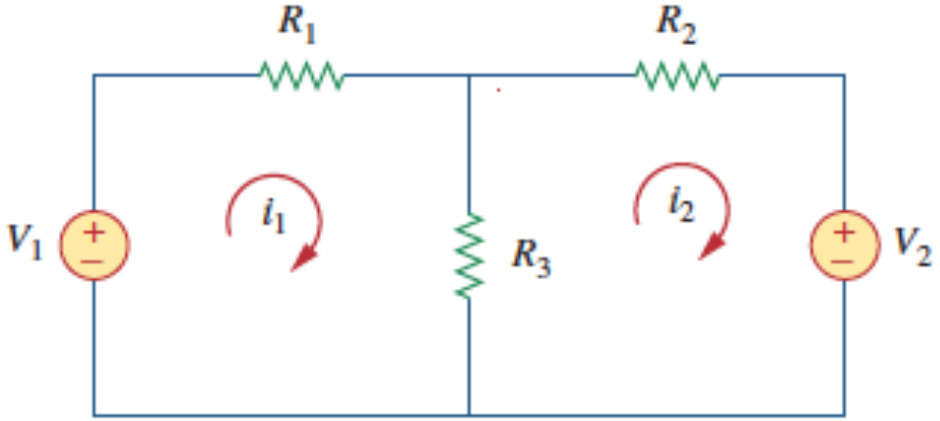
- We need a more direct approach:
  - Nodal analysis (KCL based)
  - Mesh analysis (KVL based, **NOW**)

Mesh Analysis

- Mesh refers to the simple loops visible in a circuit



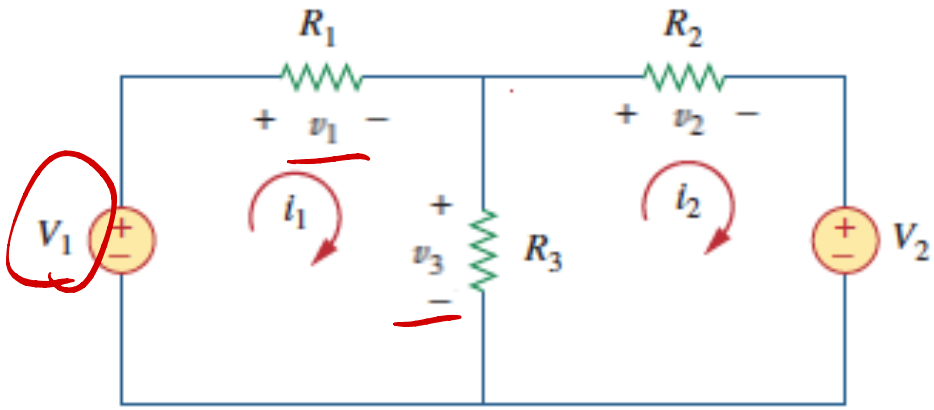
- Method – define the "mesh" currents



- Write KVL on these meshes

$$v_1 + v_3 = V_1$$

$$v_3 = v_2 + V_2$$



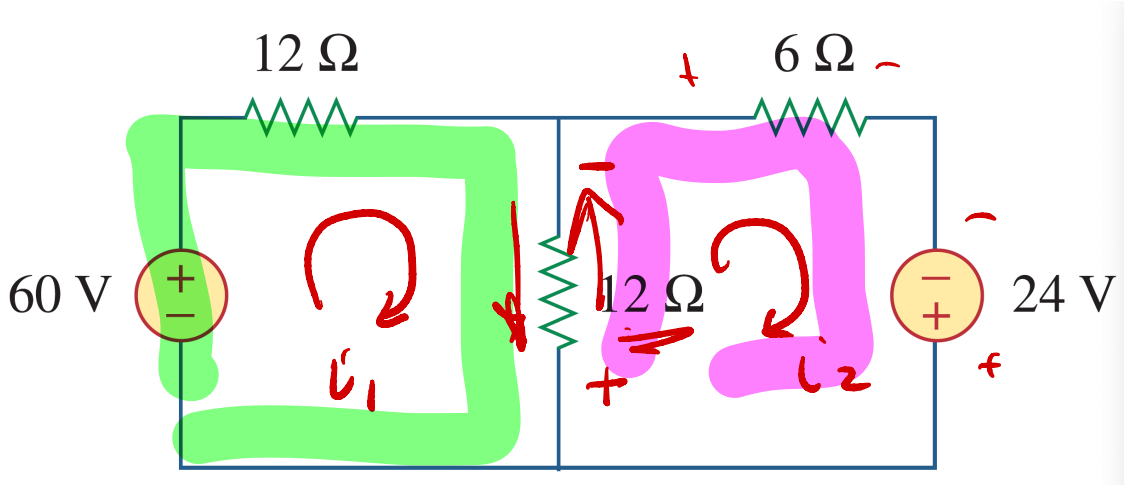
- Use Ohm's Law for voltages on resistive branches

$$v = R\, i$$

  - Careful on current directions

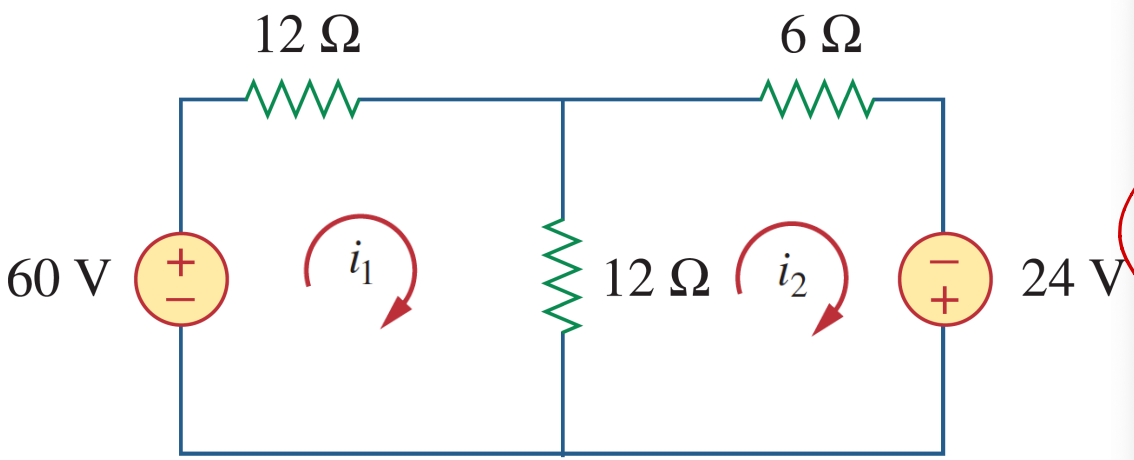
- Result is a set of simultaneous equations to solve

Example (details on next slide)



$$60 - 12 i_1 - 12 (i_1 - i_2) = 0$$

$$-12(i_2 - i_1) - 6 i_2 + 24 = 0$$



$$60 - 12i_1 - 12(i_1 - i_2) = 0$$
$$-12(i_2 - i_1) - 6i_2 + 24 = 0$$

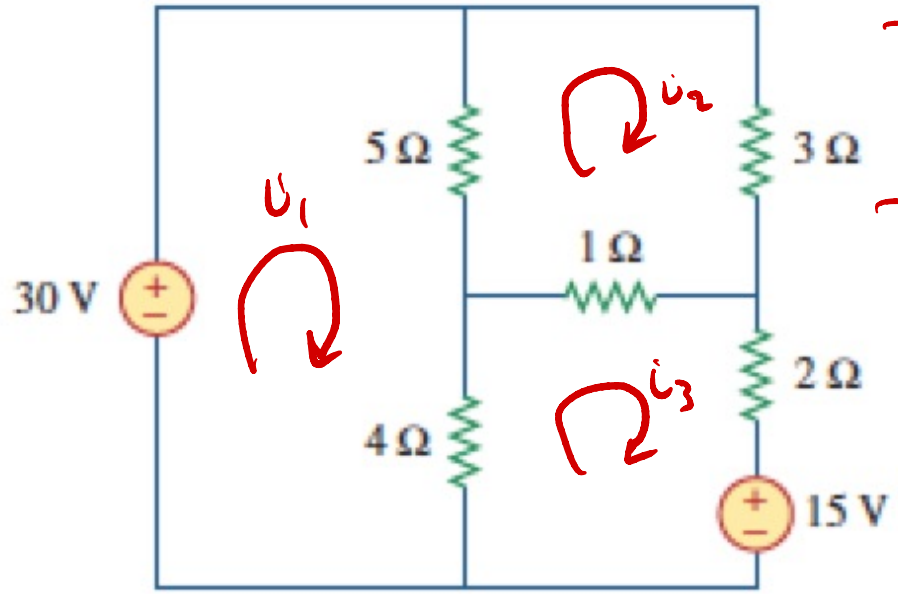
$$\mathbf{24}i_1 - \mathbf{12}i_2 = \mathbf{60}$$

$$\mathbf{-12}i_1 + \mathbf{18}i_2 = \mathbf{24}$$

$$i_1 = \frac{\begin{vmatrix} 60 & -12 \\ 24 & 18 \end{vmatrix}}{\begin{vmatrix} 24 & -12 \\ -12 & 18 \end{vmatrix}} = \frac{1368}{288} = 4.75 \text{ amps}$$

$$i_2 = \frac{\begin{vmatrix} 24 & 60 \\ -12 & 24 \end{vmatrix}}{\begin{vmatrix} 24 & -12 \\ -12 & 18 \end{vmatrix}} = \frac{1296}{288} = 4.5 \text{ amps}$$

Example:



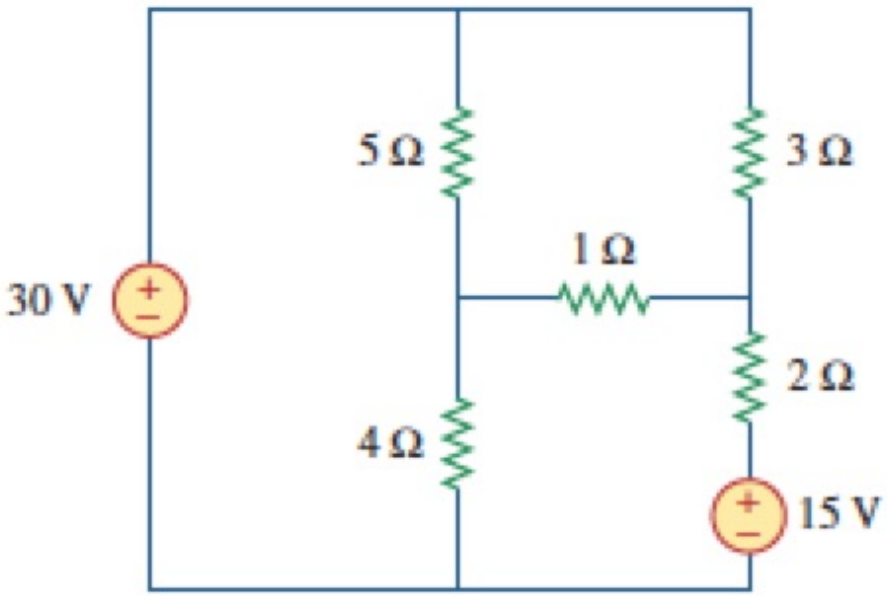
$$30 - 5(i_1 - i_2) - 4(i_1 - i_3) = 0$$

$$-5(i_2 - i_1) - 3i_2 - 1(i_2 - i_3) = 0$$

$$-4(i_3 - i_1) = 1(i_3 - i_2)$$

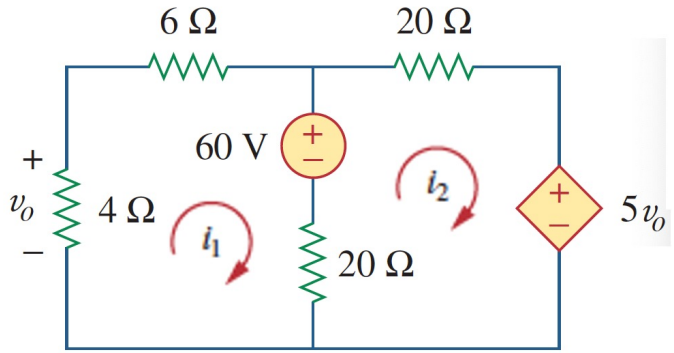
$$-2i_3 - 15 = 0$$

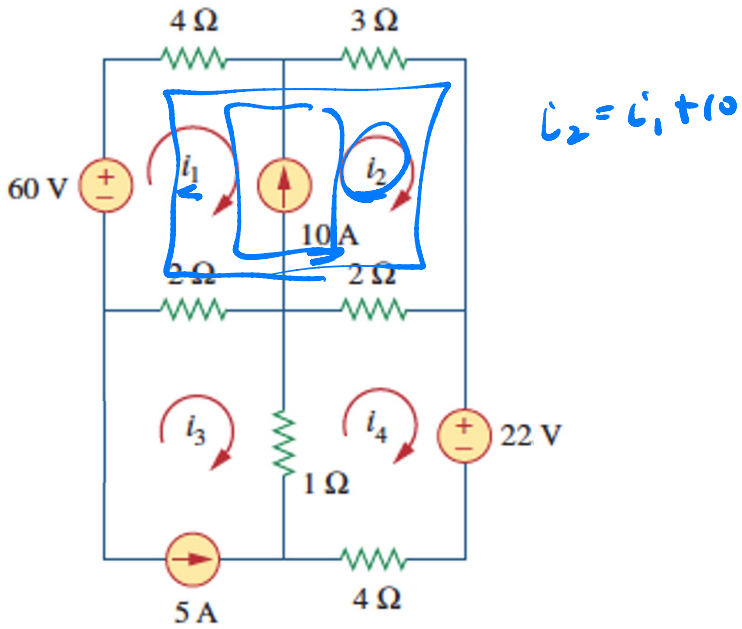
$i_L = 6.26\ A, i_T = 3.69\ A, i_B = 1.96\ A$

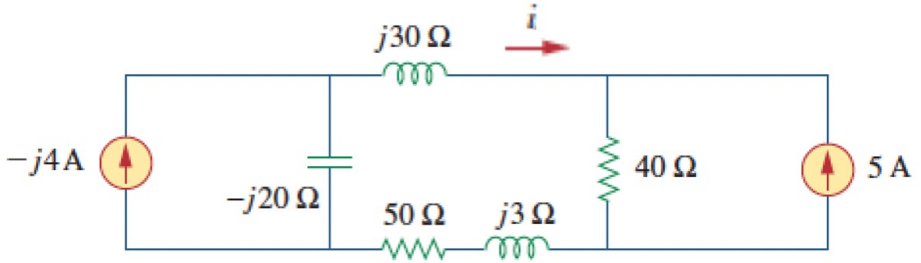


Extensions

- Treat dependent sources the same way as in node analysis
- Current sources are either trivial or require a “supermesh”
- Works for phasors

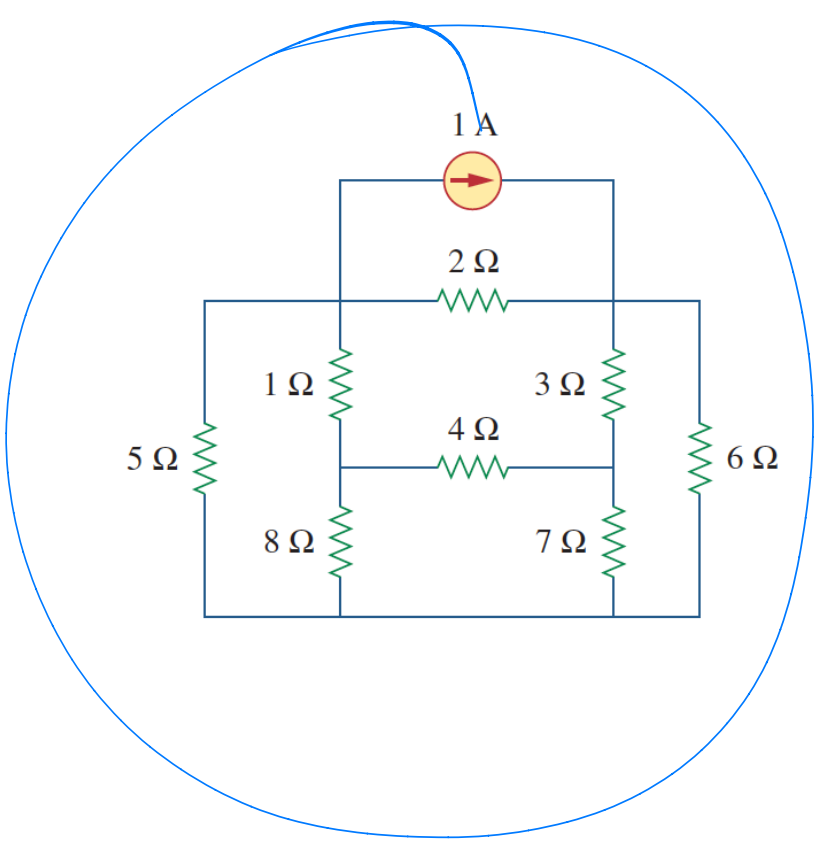


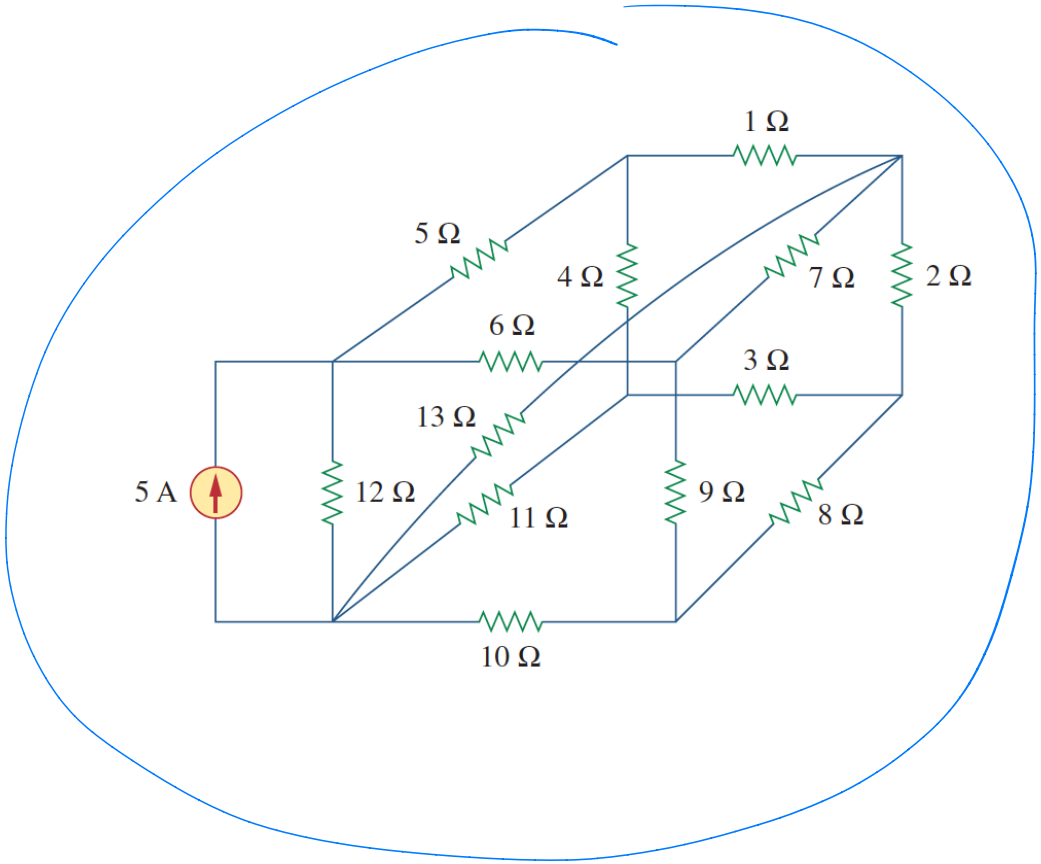




Limitation

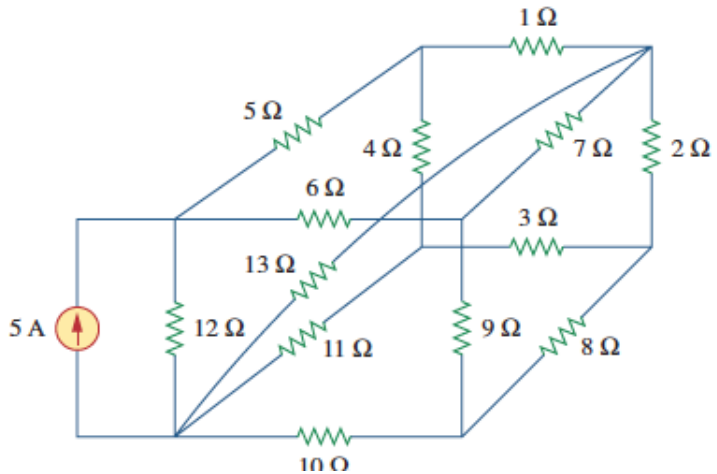
- Circuit must be **“planar”**



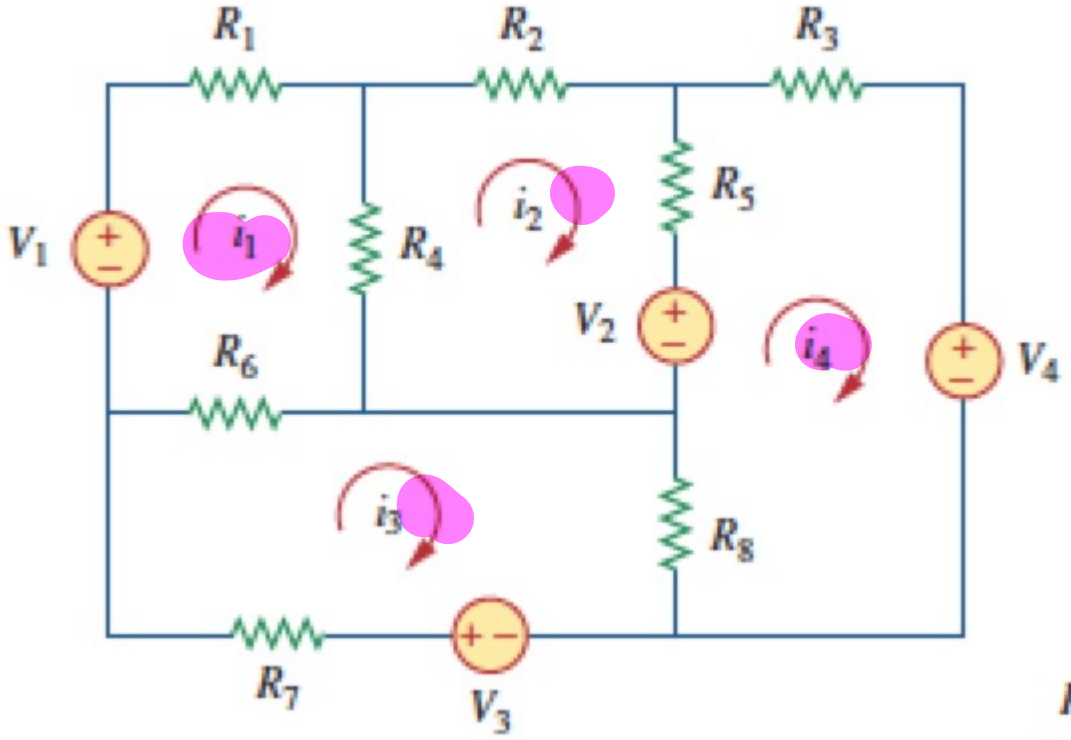


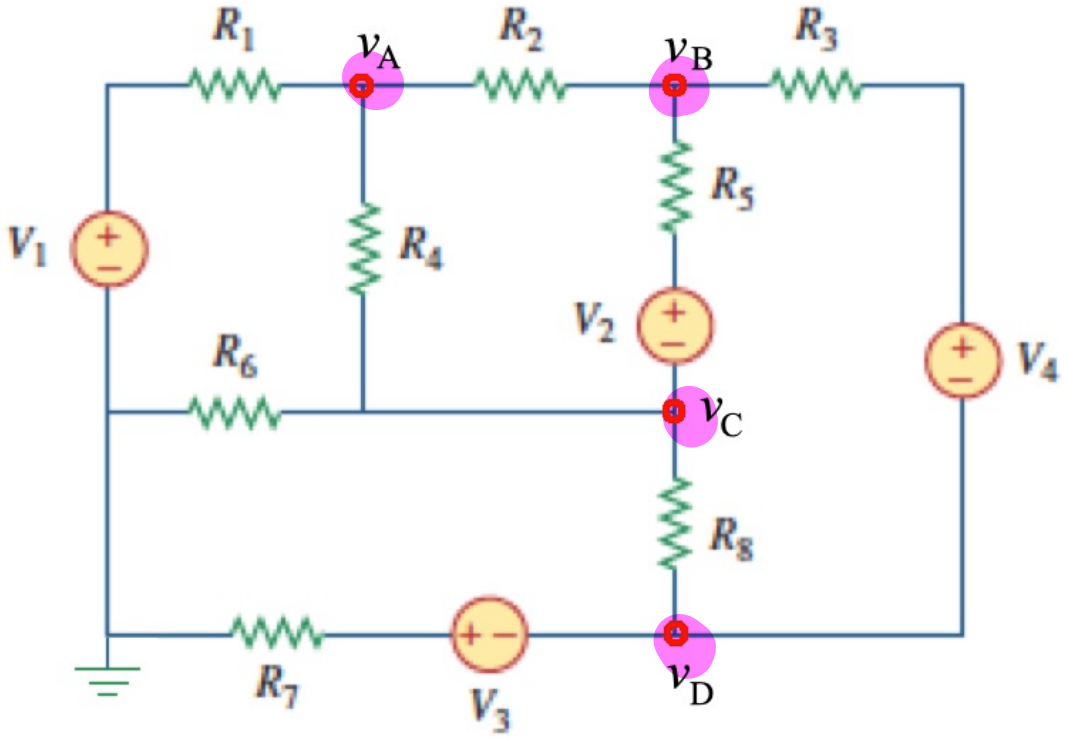
Node vs Mesh?

- Non-planar → node only

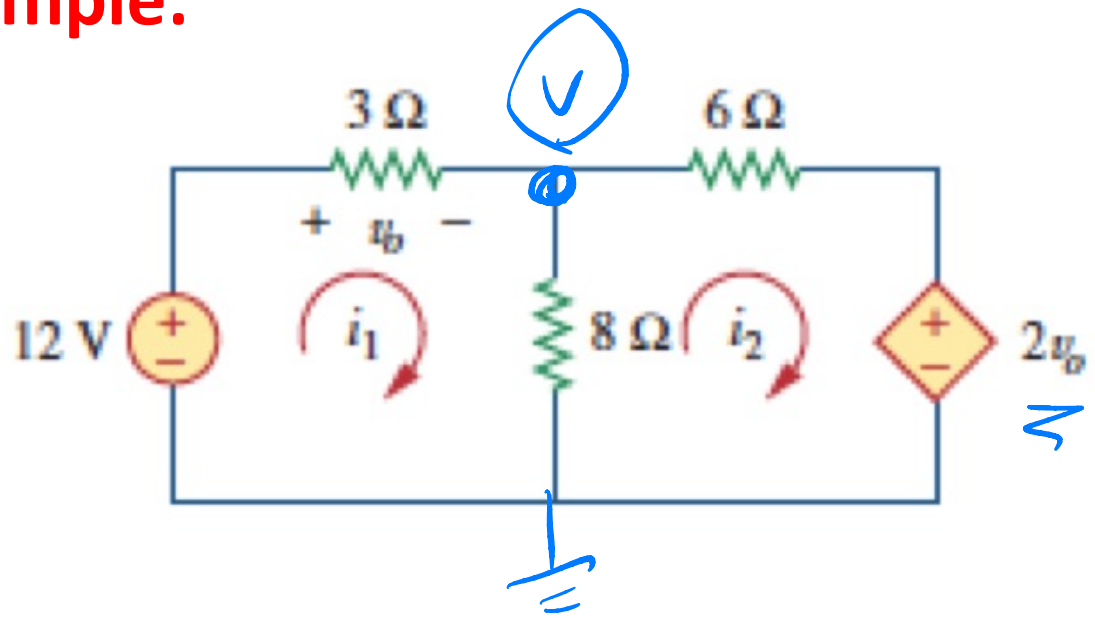


- Could count # of nodes/loops; select smaller
- Personal preference


R1
R2
R3
V1
i1
R4
i2
R5
V2
i4
V4
R6
i3
R8
R7
V3


R1
vA
R2
vB
R3
R5
V1
R4
V2
V4
R6
vC
R8
R7
V3
vD

Example:

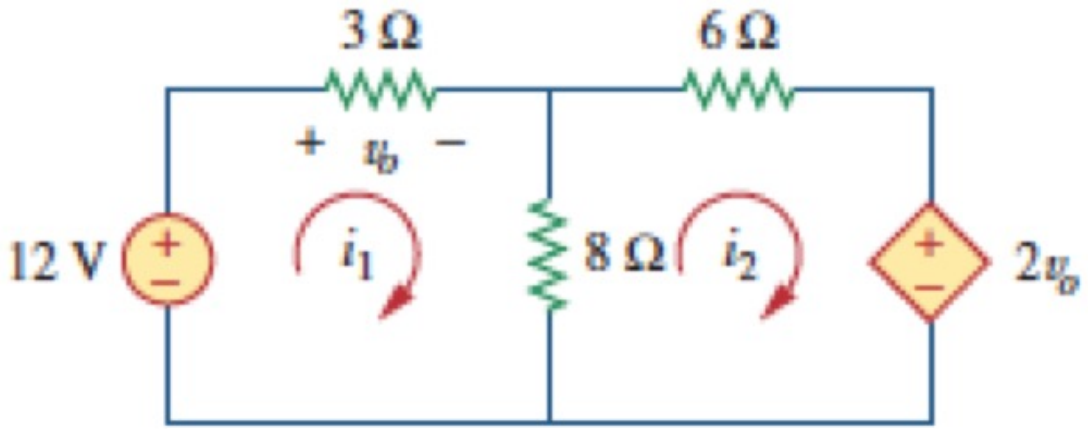


mesh

$v_o = 3 i_1$

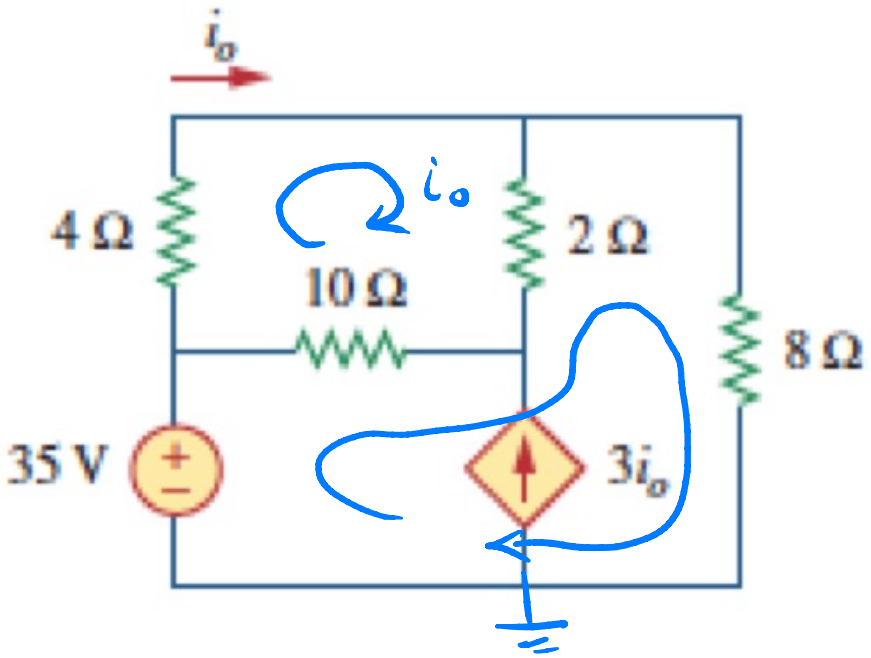
node

$v_o = 12 - v$



$i_1 = 1.22\ A, i_2 = 0.174\ A$

Example: find i_o

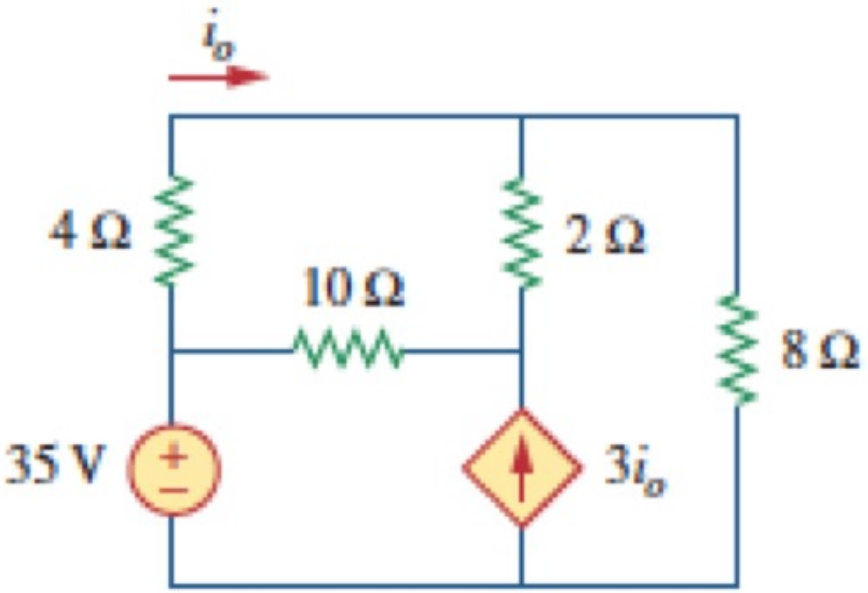


$i_o = \frac{35 - V_2}{4}$

+ 2 node eqs

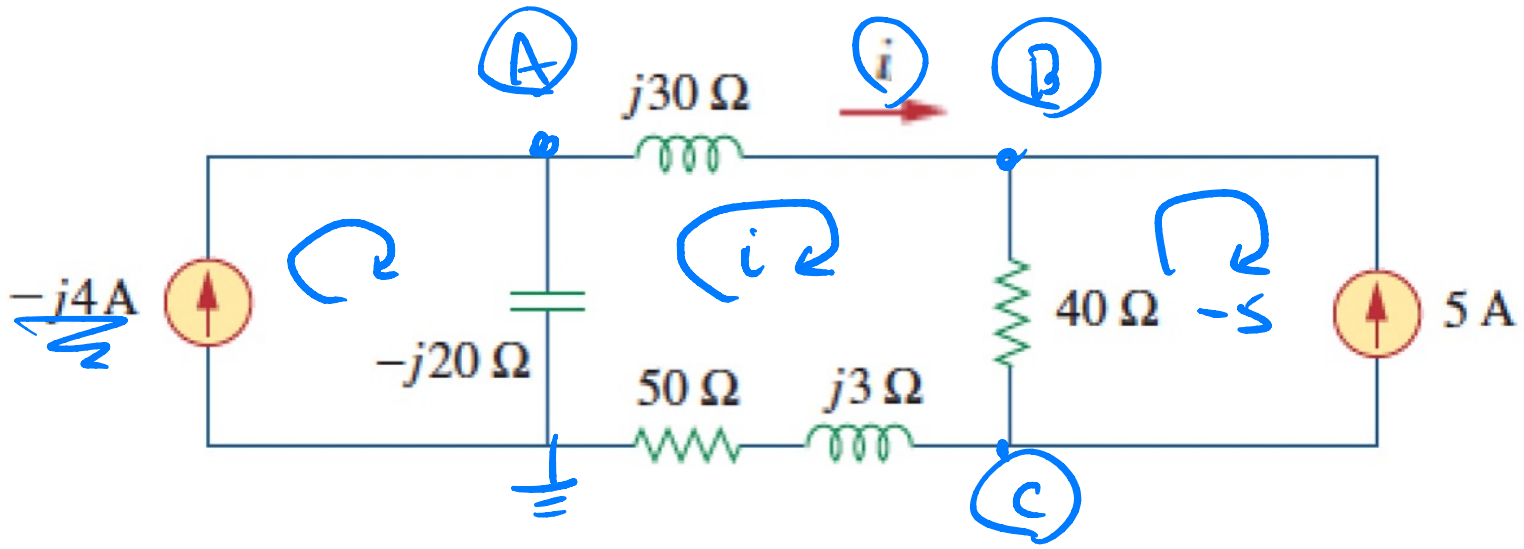
2 mesh eqs

node



$$i_o = \frac{105}{104}\ A$$

Example: find i

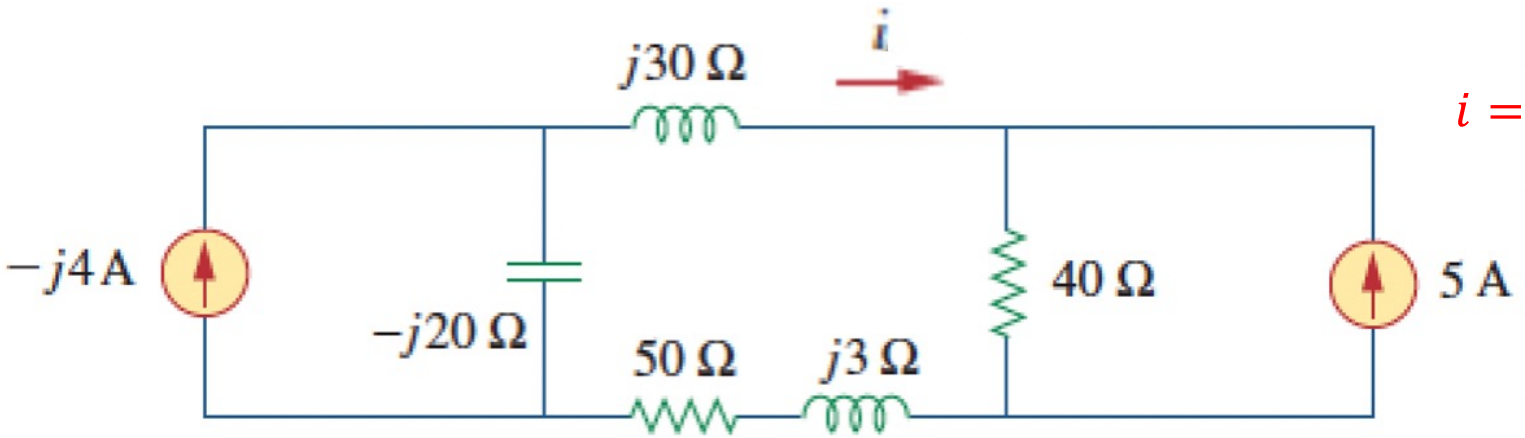


node

3 node eqs $i = \frac{A-B}{j30}$

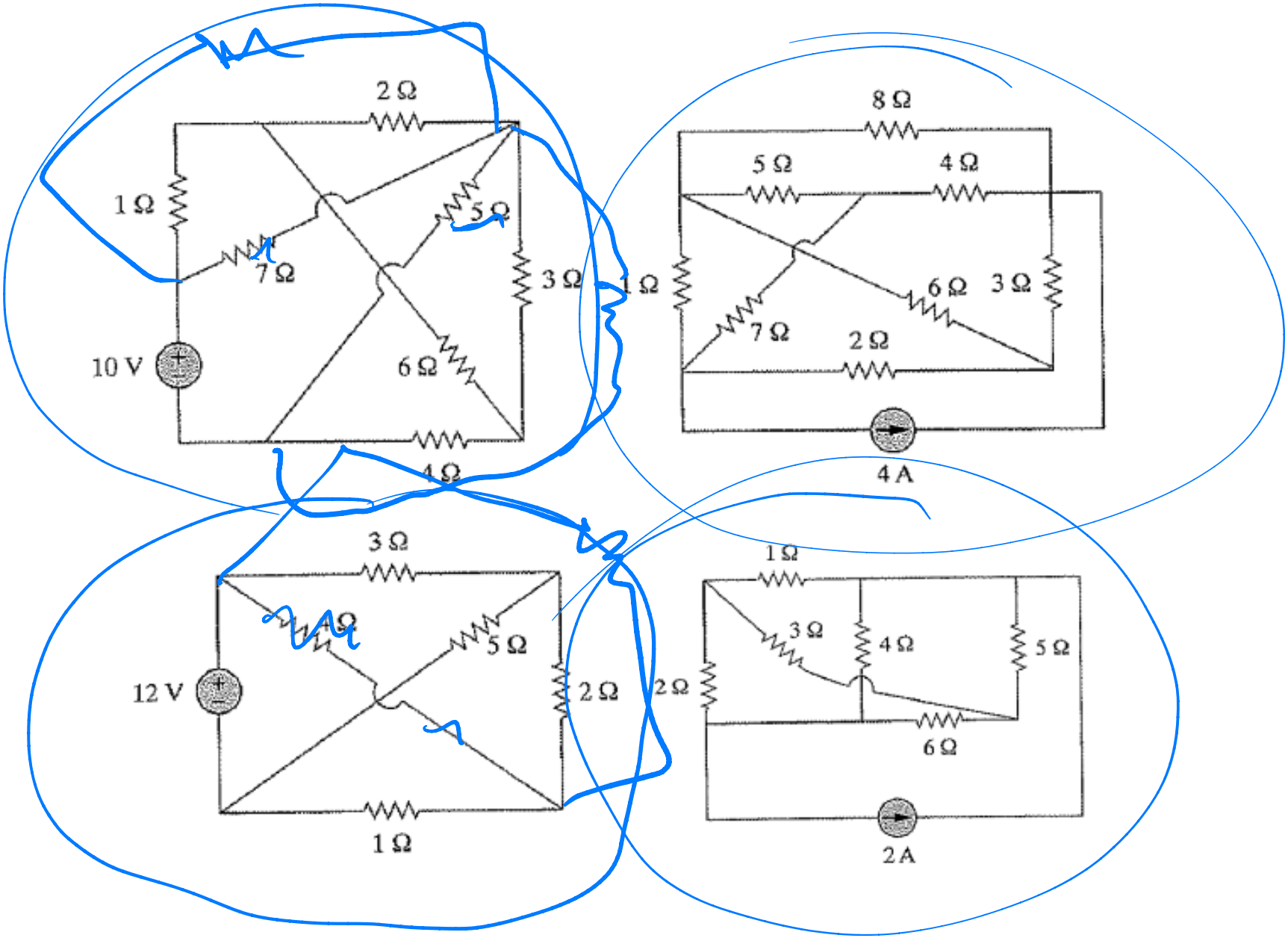
mesh

1 mesh eq in i

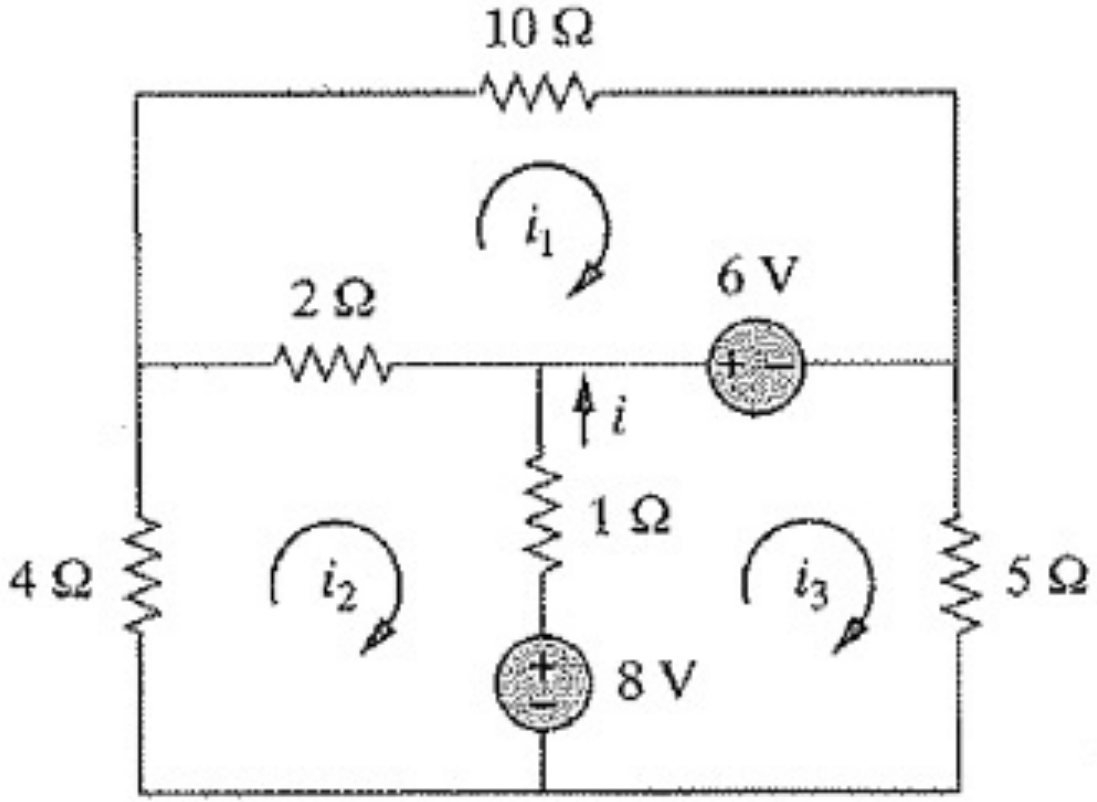


$$i = -\frac{112}{17} + \frac{j28}{17}\ A$$

Practice problem: Which of these circuits are planar (i.e. would allow for mesh analysis)?



Practice problem: Find the currents i_1, i_2, and i_3.



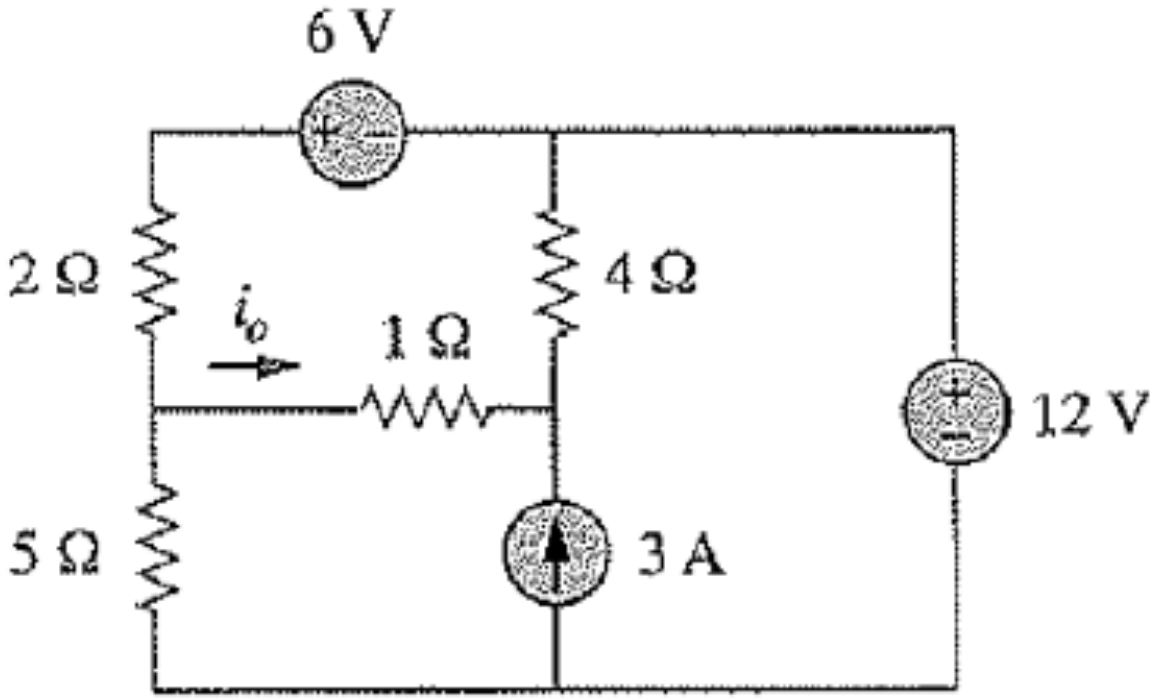
$$i_1 = \frac{77}{234} A$$

$$i_2 = -\frac{240}{234} A$$

$$i_3 = \frac{38}{234} A$$

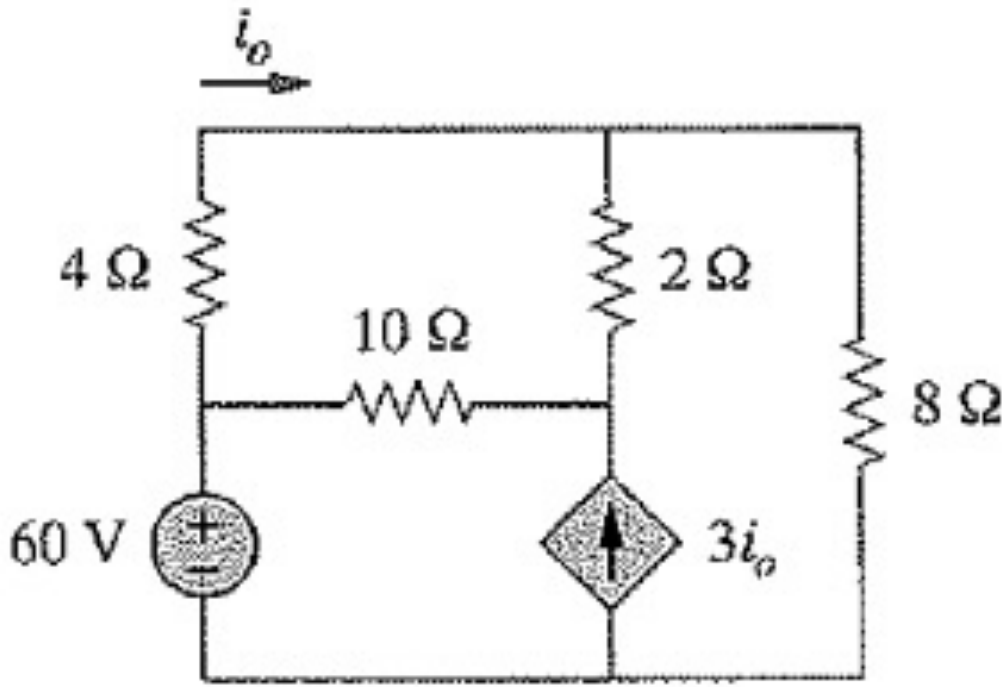
Practice problem: Find the current i_0

$$i_0 = -\frac{26}{15}\ A$$

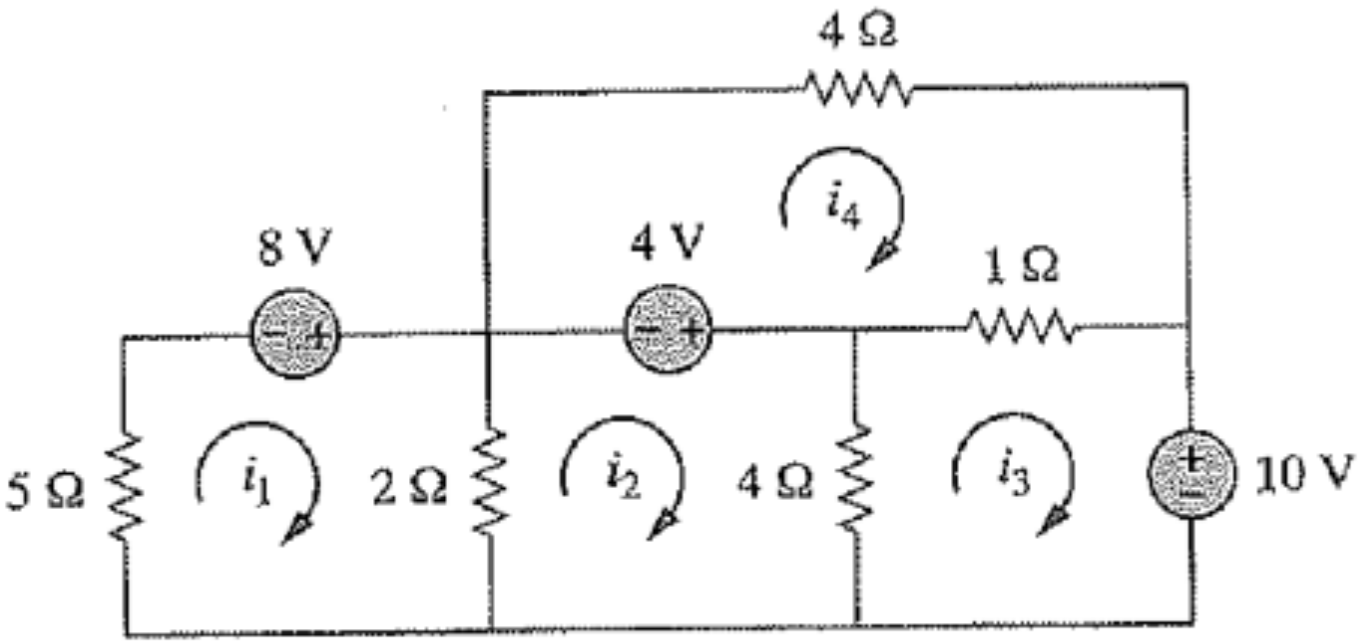


Practice problem: Find the current i_0

$i_0 = \frac{45}{26}\ A$



Practice problem: Find the currents i_1, i_2, i_3, and i_4



$$i_1 = \frac{68}{88}\ A$$

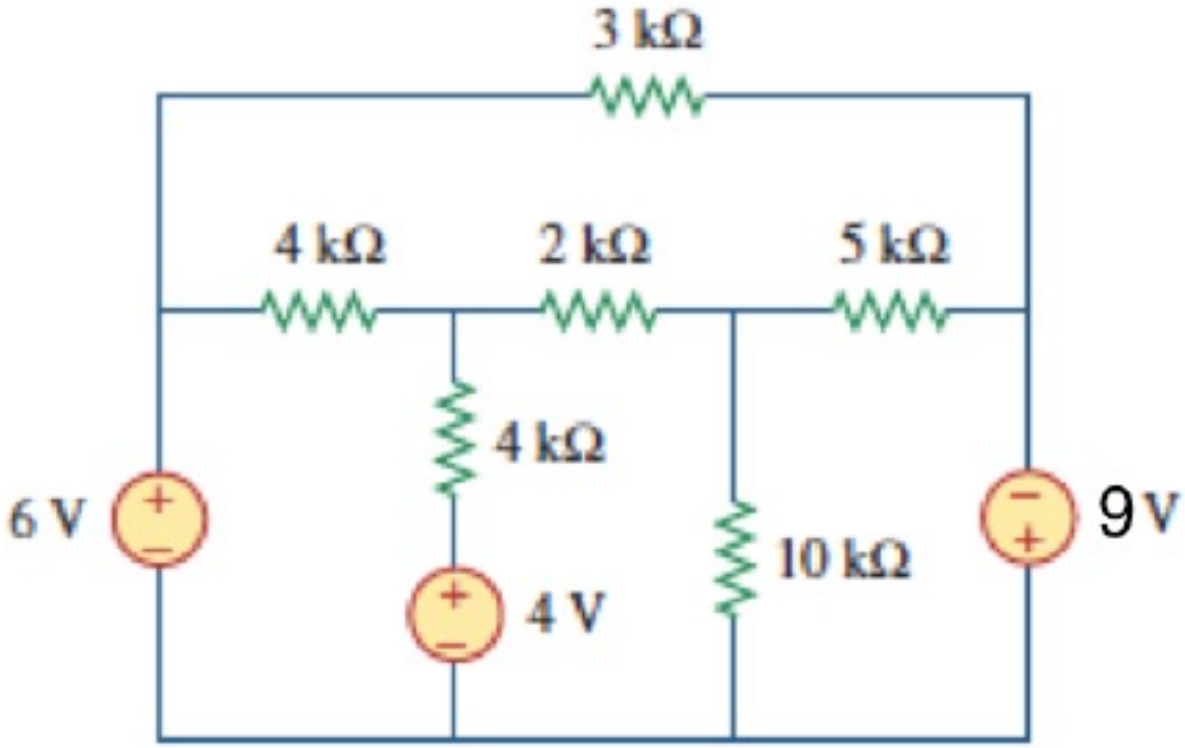
$$i_2 = -\frac{114}{88}\ A$$

$$i_3 = -\frac{293}{88}\ A$$

$$i_4 = -\frac{129}{88}\ A$$

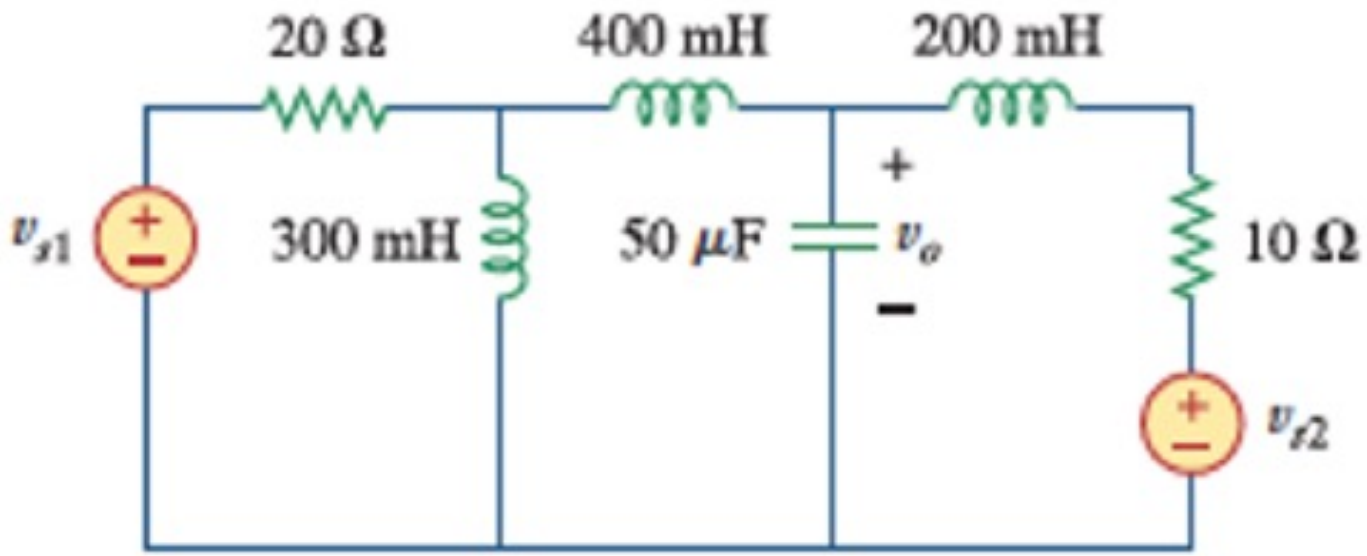
Practice problem: For the circuit below which method appears easier, mesh or node? Use your preference to find the power dissipated in the 10 kΩ resistor.

$100\ \mu W$

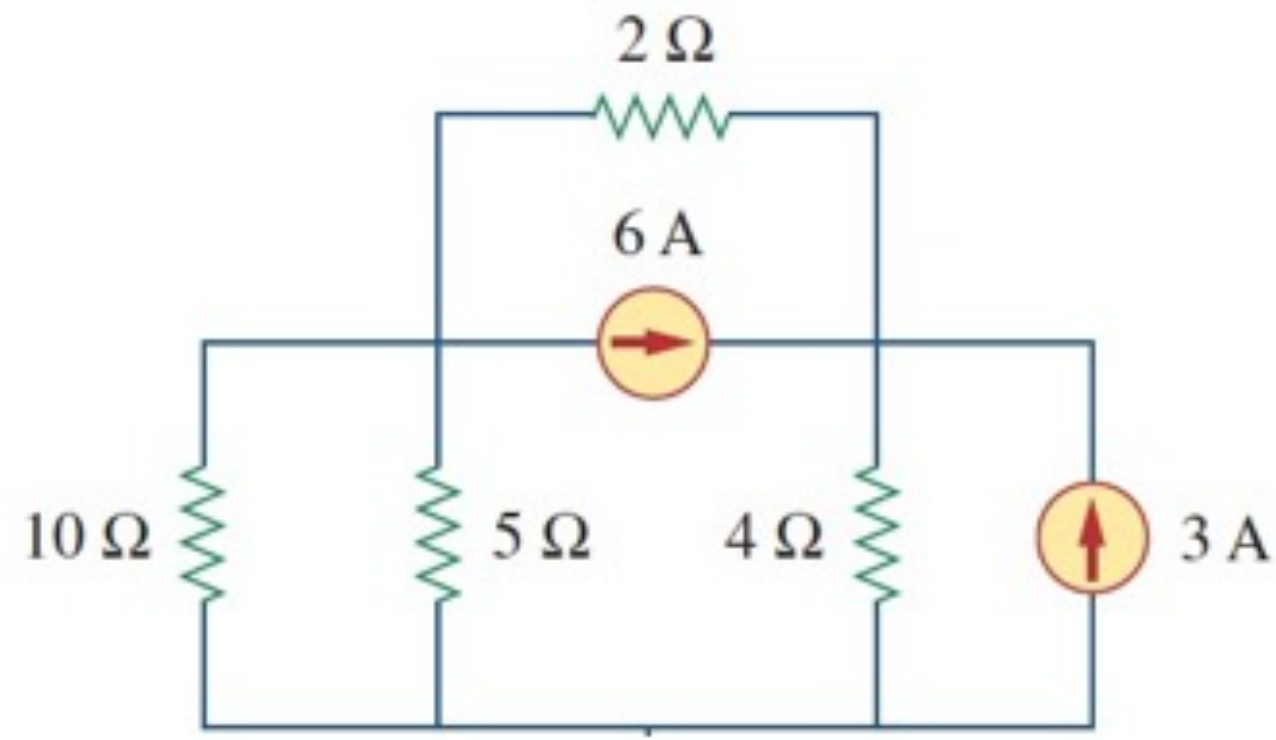


Practice problem: Find $v_0(t)$ assumin that $v_1(t) = 120\cos(100t + 90°)$ V and $v_2(t) = 80\cos 100t$ V

$v_o(t) = 29.9\cos(100t + 46°)\ V$

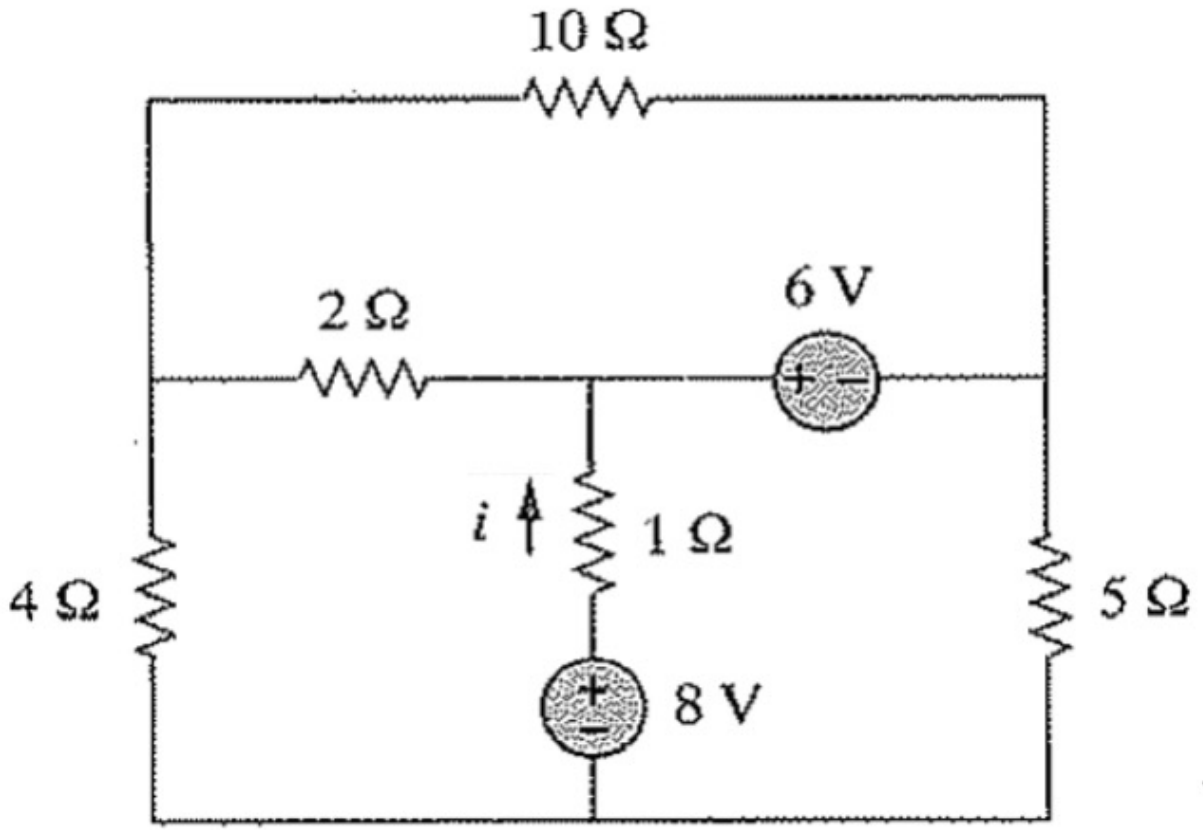


Practice problem: Find the power in the 10 Ω resistor $0\ W$



$i = 1.18\ A$

Practice problem: Find i



$i_o = 0.4\ A$

Practice problem: Find i_0